W. P. Conant

HAGAR'S COMMON SCHOOL ARITHMETIC

PHILADELPHIA,
COWPERTHWAIT & CO.

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A COMMON SCHOOL ARITHMETIC

BY

D. B. HAGAR,

PRINCIPAL OF STATE NORMAL SCHOOL, SALEM, MASS.

PHILADELPHIA:
COWPERTHWAIT & COMPANY.
1872.

HAGAR'S
MATHEMATICAL SERIES.

I. PRIMARY LESSONS IN NUMBERS.
II. ELEMENTARY ARITHMETIC.
III. COMMON SCHOOL ARITHMETIC.

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To be followed by other Books of a Complete Series.

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Printers, Philada.

INTRODUCTION.

This Common School Arithmetic is designed to be a complete manual for learners who may be prepared to advance beyond the first lessons in Numbers.

It has been constructed with a view to the most rapid and thorough progress of the pupil by the use of the least number of books possible, and by the greatest economy of time.

It combines mental and written exercises in a practical system. All obsolete and valueless material and all merely puzzling problems have been excluded, but no pains have been spared to embody valuable modern methods of computation and topics having direct relation to business as it is transacted at the present day.

The work is sufficiently comprehensive to render the use of a higher arithmetic quite unnecessary. It is ample enough in its range of subjects and exercises to qualify the learner for a skillful and prompt solution of all ordinary problems of a commercial character, and at the same time to subserve the purposes of mental discipline.

The Primary Lessons in Numbers and the Elementary Arithmetic, of this series, it is believed, form a valuable compendious course sufficient for a majority of pupils. The Primary Lessons and the Common School Arithmetic likewise form a two-book course, but full and complete.

SUGGESTIONS TO TEACHERS.

A TEACHER should never undertake a recitation in Arithmetic without a full understanding of the subject of the lesson. Preparation should be made for the elucidation of difficulties, and for making plain the way of the learner, whenever required, by happily-chosen, familiar illustrations.

In forming classes, pupils of the same attainments, as nearly as possible, should be placed together. Vary the exercises in a class, so as to secure animation and interest and retain the attention of each member during the entire recitation.

Tolerate no indefinite answering of questions. Require all principles and rules to be recited exactly, and all forms of solution to be logically and concisely expressed. Let all answers to Test Questions be definite and prompt.

Do not overlook the importance of mental arithmetic. The plan of this book is to combine mental and written exercises, and to require a reason for every process. The proficiency of the learner should be often tested with problems not found in the book. Require, as an occasional test, the formation of problems and their solution without regard to rules.

Seek, above all, to make the arithmetical exercise useful in the cultivation of the invaluable habit of self-reliance. Endeavor to give such a practical character to the instruction in the science of Numbers, that the knowledge acquired may be found readily available in the many computations required in business. To attain this object, good books are aids, but can never perform the duties or assume the responsibilities of the teacher.

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Common School Arithmetic.

SECTION I.

PRELIMINARY DEFINITIONS.

ARTICLE 1.—A **Unit** is one, or a single thing of any kind.

2. A **Number** is a unit, or a collection of units.

Thus, one, two, three, four, five, are numbers.

3. The **Unit** of a number is one of the collection forming that number.

Thus, one is the unit of six, one book is the unit of six books.

4. An **Integer** is a number formed wholly of entire units.

Thus, three, five, six, nine, are integers. Integers are also called *integral* or *whole* numbers.

5. **Similar Numbers** are those which have the same unit.

Thus, three yards and five yards are similar numbers.

6. **Dissimilar Numbers** are those which do not have the same unit.

Thus, three yards and three books are dissimilar numbers.

7. A **Concrete Number** is one that names the kind of unit numbered.

Thus, five bushels, in which the kind of unit is named, is a concrete number.

8. An **Abstract Number** is one that does not name the kind of unit numbered.

Thus, five, in which the kind of unit is not named, is an abstract number.

9. **Arithmetic** is the science of numbers and the art of computing by them.

10. A **Solution** in Arithmetic is the process of answering a question which requires computation.

11. A **Proof** of a solution is the process of testing its correctness.

12. A **Problem** is a question for solution.

13. A **Principle** is a general truth.

14. A **Rule** is a concise statement of the method of solving a problem.

15. An **Example** is a problem which is used to illustrate a principle or rule.

16. An **Exercise** is a problem which is intended to render knowledge familiar by drill or practice.

EXERCISES.

1. How many units in one? In one dollar? Three is a collection of how many units?
2. What is the unit of two books? Of four? Of five pounds? Of seven houses?
3. Are two cents and five cents similar or dissimilar numbers? Why are three men and five books dissimilar numbers?
4. Is four yards a concrete or an abstract number? Three? Two boys?
5. Why is one mile the unit of four miles? Why is one the unit of six?
6. Why is two houses a concrete number? Why is four an abstract number?
7. Of the two numbers, two miles and ten miles, what is the unit?
8. Of the two numbers, seven dollars and nine dollars, what is the unit?

SECTION II.

NUMERATION AND NOTATION.

17. The **Naming** of numbers is performed by means of a small number of words.

A single thing is named *one;* one and one is named *two;* one and one and one is named *three;* and so we have the separate names,

One, two, three, four, five, six, seven, eight, nine, ten.

18. Ten, by being regarded as forming a set or collection of units, may be treated as a single thing, or as a unit equal to ten ones.

One and ten, two and ten, three and ten, four and ten, etc., by change of form, give the familiar names,

Eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

Two tens, three tens, four tens, etc., by change of form, give the names,

Twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

Twenty and one, twenty and two, etc.; thirty and one, thirty and two, etc., to ninety and nine, by change of form, give the names,

Twenty-one, twenty-two, etc.; *thirty-one, thirty-two,* etc., to *ninety-nine.*

19. One Hundred is the name given to a collection of ten tens.

One hundred and one hundred, two hundred and one hundred, etc., form

Two hundred, three hundred, etc., to *nine hundred.*

FIGURES.

20. Figures are the characters commonly used to represent numbers. They are as follows—

PRINTED,	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
WRITTEN,	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
NAMED,	Zero,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

The figure 0 is sometimes called a *Cipher*, or *Naught*, because when written alone it expresses no value, or the absence of number; and the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *Numerals*, or *Significant Figures*, because each expresses as many ones as are denoted by its name.

Numbers greater than nine are expressed by repeating or combining two or more of the ten figures.

21. Exact Tens are written with the figure expressing the number of tens at the left of 0, which marks the absence of ones; and tens and ones are written with the figure expressing the tens at the left of the figure expressing the ones. Thus,

Ten,	or 1 ten,	is written	*10,*
Eleven,	or 1 ten and 1 one,	"	*11,*
Twelve,	or 1 ten and 2 ones,	"	*12,*
Thirteen,	or 1 ten and 3 ones,	"	*13,*
Twenty,	or 2 tens,	"	*20,*
Twenty-one,	or 2 tens and 1 one,	"	*21,*
Thirty,	or 3 tens,	"	*30,*

and so on to ninety-nine, or 9 tens and 9 ones.

22. Exact Hundreds are written with the figure expressing the hundreds at the left of two zeros.

Hundreds, tens and ones are expressed together in a number by writing the figure expressing the tens at the left of the figure expressing the ones, and the figure expressing the hundreds at the left of that expressing the tens. Thus,

One hundred, or 1 hundred 0 tens 0 ones, is written *100.*

Two hundred, or 2 hundreds 0 tens 0 ones, is written *200.*

Four hundred ten, or 4 hundreds 1 ten 0 ones, is written *410.*

Five hundred six, or 5 hundreds 0 tens 6 ones, is written *506.*

Nine hundred seventy-eight, or 9 hundreds 7 tens 8 ones, is written *978.*

23. Numeration is the method of naming numbers, and of reading numbers expressed by figures.

24. Notation is the method of writing numbers, or of expressing numbers by figures.

WRITTEN EXERCISES.

Write in figures—

1. One hundred thirty-six; two hundred thirteen.
2. Four hundred forty-four; one hundred eleven.
3. Three hundred twenty-five; five hundred ten.
4. Six hundred seventeen; two hundred twenty.
5. Seven hundred five; eight hundred fifteen.
6. Nine hundred nine; seven hundred four.
7. Five hundred; eight hundred seventy-one.
8. Six hundred; three hundred eighty.
9. Five hundred twenty-two; nine hundred ninety-nine.
10. Seven hundred one; three hundred twenty-five.

ORDERS AND PERIODS OF UNITS.

25. Orders of Units are denoted by the successive figures used in expressing a number.

Thus, in 365, the 5, which expresses 5 ones, denotes units of the *First Order;* the 6, which expresses 6 tens, denotes units of the *Second Order;* and the 3, which expresses 3 hundreds, denotes units of the *Third Order.*

26. In naming numbers, the first three orders of units are regarded as forming a group, called the **Class,** or **Period,** of **Units,** having ones, tens and hundreds.

Thus, 425 forms a period composed of 425 units.

27. Ten hundreds form *One Thousand;* ten thousands form *One Ten-Thousand;* and ten ten-thousands form *One Hundred-Thousand.*

These three orders of units form a group, called the **Period of Thousands,** having ones, tens and hundreds.

Thus, 363425 is composed of 363 *thousands* 425 *units,* or of two periods, and is read three hundred sixty-three thousands four hundred twenty-five.

406007 is composed of 406 *thousands* 007 *units,* or of two periods, and is read four hundred six thousands seven.

In like manner are formed and read other periods.

THE DECIMAL SYSTEM.

28. Simple or **Primary Units** are the units expressed by a numeral when written alone, or, by the order of ones in the period of units, when in a collection.

In writing numbers, the order of simple units may be marked by placing a point (.), called the **Decimal Point,** at the right of the units' period; and the different periods may be separated by a *Comma* (,).

Thus, 215 thousands 463 units may be written, 215,463.

Units are understood to be Primary Units when not otherwise indicated by the expression or its connection.

29. The Names of the Orders of Units, and the Names of the Periods, are given in the following

TABLE.

PERIODS.	6th Period.	5th Period.	4th Period.	3d Period.	2d Period.	1st Period.
NAMES OF PERIODS.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
	of	of	of	of	of	of
ORDERS OF UNITS.	Hundreds. Tens. Ones.	Hundreds. Tens. Ones.	Hundreds. Tens. Ones.	Hundreds. Tens. Ones.	Hundreds. Tens. Ones.	Hundreds. Tens. Ones.
NUMBER.	4 2 0,	7 3 5,	8 0 0,	6 1 2,	3 0 9,	2 5 4.

The number expressed is *Four hundred twenty* QUADRILLIONS, *seven hundred thirty-five* TRILLIONS, *eight hundred* BILLIONS, *six hundred twelve* MILLIONS, *three hundred nine* THOUSANDS, *two hundred fifty-four* UNITS.

In reading numbers the name of the units' period is not usually given, since when omitted it is readily understood.

30. The **Periods** above Quadrillions, in order, are—

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novendecillions, Vigintillions, etc.

31. The **Scale** of numbers is the arrangement of their units.

In the ordinary system of notation, or that which has been explained, the scale is *ten*, because the units are so arranged that ten ones are one ten, ten tens are one hundred, ten hundreds are one thousand, etc.

32. The method of expressing numbers by ten figures is termed the **Arabic Notation,** from its having been introduced into Europe by the Arabs.

33. The method of expressing numbers by the scale of ten is termed the **Decimal System,** from the Latin *decem,* which signifies *ten;* and the scale of ten is, for the same reason, termed the *Decimal Scale.*

Principles of Numeration and Notation.

34.—1. *Ten units of any order in a number are always equal to one unit of the next higher order.*

2. *The same figure represents invariably the same number of units.*

3. *The name and value of the units represented by a figure in a number are always those of its order in that number.*

4. *The absence of units in any order in a number is marked by a cipher.*

5. *The order of simple units in a number may be known by having the decimal point expressed or understood at the right of that order.*

EXERCISES IN NUMERATION.

35.—Ex. 1. Write and read 56073402.

Solution.—56073402, separated into periods, is 56,073,402, or 56 millions 73 thousands 402 units, and is read fifty-six millions seventy-three thousands four hundred two.

2. Write and read 735467005. Write and read 93606121.

36. Rule for Numeration.—*Beginning with the lowest order of units, separate the figures of the number into periods of three figures each.*

In reading begin at the left; read the hundreds, tens and ones of each period, and give the name of each period, except the last, after its ones.

PROBLEMS.

Write and read—

1. *314.*	12. *132401.*
2. *1780.*	13. *3000835.*
3. *2344.*	14. *92416512.*
4. *16110.*	15. *732534902.*
5. *70008.*	16. *7324768291.*
6. *134020.*	17. *4444444444.*
7. *68110.*	18. *56073014211597.*
8. *89000.*	19. *313134405678012.*
9. *143211.*	20. *1413248687901132 6.*
10. *456104.*	21. *5944463213200 7955.*
11. *215779.*	22. *356789003853190 0210.*

EXERCISES IN NOTATION.

37.—Ex. 1. Write in figures twenty-two millions four hundred six thousands.

22,406,000

SOLUTION.—Writing 22 for the tens and ones of *millions*, 406 for the hundreds, tens and ones of *thousands*, 000 for the absence of hundreds, tens and ones of *units*, gives 22,406,000 as the required expression.

2. Write in figures three hundred sixty-five millions nine hundred twenty-five thousands seven hundred seventy-five.

3. Write in figures nine hundred thirty-two thousands four hundred forty-seven.

4. Write in figures four hundred eighteen millions eight hundred sixty-three thousands two hundred three.

38. Rule for Notation.—*Write the figures representing the hundreds, tens and ones of each period in their order.*

Mark by a cipher any order in the number which has no units given.

PROBLEMS.

Write in figures—

1. Three hundred fourteen; four hundred ten.
2. Five hundred six; nine hundred seventy-seven.
3. Sixteen thousand ninety-one; twenty-five thousand one hundred.
4. One hundred eighty-three thousand; two hundred nine thousand ninety-nine.
5. Nine thousands seven hundreds three tens four ones.
6. Four millions six; ten millions; five hundred five millions.
7. Five hundred thousands four hundred six; one hundred one thousands one hundred one.
8. Thirty-seven millions one hundred seventy-one thousands eleven.
9. Two hundred forty-nine millions; seventeen billions nine millions.
10. Ninety-three thousands one hundred eighty-six.
11. One hundred fifty-two millions four hundred twenty-five thousands three hundred thirty-three.
12. Seven hundred fifty-five trillions one hundred six billions four hundred fifteen millions one hundred five units.
13. One quintillion twenty-five quadrillions one hundred fifteen trillions seven billions eight hundred eighty-eight millions five hundred fifty units.
14. Eight hundred eighty-eight quintillions six thousand six hundred six.
15. Three hundred thirty-seven billions four hundred forty-nine millions two thousands three hundred eleven.
16. Five decillions one hundred six nonillions eight octillions four septillions one hundred nineteen sextillions six hundred seventy-nine quintillions four quadrillions three hundred fifteen trillions seven hundred twenty billions forty-six millions three thousands one.

TEST QUESTIONS.

39.—1. What is a UNIT? A number? Name some number. What is the unit of a number? Give an illustration of the unit of a number. What is an integer?

2. What are SIMILAR NUMBERS? Name two. What are dissimilar numbers? Name two. What is a concrete number? What is an abstract number? Give an illustration of a concrete number. Of an abstract number.

3. What is ARITHMETIC? A solution? A proof? A problem? A principle? A rule? An example? An exercise?

4. How is the NAMING of numbers performed? Give the names of the first ten numbers. How may ten be regarded? What numbers do we get by combining ten with each of the first nine numbers, and changing their form? Two tens, three tens, etc., by change of form give what numbers? Twenty and one, twenty and two, etc.?

5. What are FIGURES? What does 0 written alone express? What is 0 called? What does each of the other nine figures express when written alone? What are they called? How are exact tens written? How are the numbers between the tens written? How are exact hundreds written? How are hundreds, tens and ones expressed together in a number?

6. What is NUMERATION? What is notation? How are orders of units denoted in expressing a number?

7. What are SIMPLE UNITS, or units of the first order? Units of the second order? Units of the third order? How many simple units does 2 of the first order express? 2 of the second order? 2 of the third order?

8. How many orders compose a PERIOD? What is the name of the first period? Of the second? Name the orders of each. How many units in the period of units equal one unit in the period of thousands? Give the names of the first six periods in their order. Name in order periods higher than the period of quadrillions. How may the order of simple units be marked? How may the different periods be separated?

9. What is the SCALE of numbers? What is the scale in the ordinary system? What is the method of expressing numbers by figures called? Of expressing numbers by the scale of ten? What is the scale of ten termed?

10. Recite the PRINCIPLES of numeration and notation. The rule for numeration. The rule for notation.

SECTION III.

ADDITION.

40.—Ex. 1. If you have 5 dollars and your brother has 4, how many dollars have both?

2. If James has 6 books and John has 7, how many have both?

3. How many cents are 9 cents and 3 cents?

4. Add by 2's from 1 to 21.

SOLUTION.—1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21.

5. Add by 3's from 1 to 22. Add by 3's from 22 to 52.
6. Add by 4's from 2 to 34. Add by 4's from 34 to 62.
7. Add by 5's from 3 to 33. Add by 5's from 33 to 68.
8. Add by 6's from 4 to 34. Add by 6's from 34 to 70.
9. Add by 7's from 5 to 40. Add by 7's from 40 to 75.
10. Add by 8's from 6 to 46. Add by 8's from 46 to 94.
11. Add by 9's from 7 to 52. Add by 9's from 52 to 88.
12. What is the sum of 8 dollars and 7 dollars?
13. What is the unit of 8 dollars and 7 dollars?
14. Why are 8 dollars and 7 dollars similar numbers? What is the unit of their sum?
15. Are 9 books and 8 dollars similar or dissimilar numbers?
16. Why cannot 9 books and 8 dollars be united into one number?

Because 9 books and 8 dollars are neither 17 books nor 17 dollars.

17. Only what kind of numbers, then, can be united so as to form one number?

18. In my garden there are 5 roses upon one bush, 7 upon another and 2 upon another. How many roses are there in all?

19. How many ones are 5, 7 and 2? 7, 5 and 2? 7, 2 and 5? 2, 7 and 5? 2, 5 and 7?

20. When the same numbers are added in different orders, is the result changed?

21. 4, 3, 5, 1 and 2 are how many?

22. 8, 1, 9, 3, 2 and 5 are what number?

DEFINITIONS.

41. **Addition** is the process of uniting two or more numbers to find their sum.

42. The **Sum** is the result of the addition. It contains as many ones as all the numbers added.

43. A **Sign** is a mark used for abbreviating an expression.

44. The **Sign of Addition** is $+$, and is called *plus*. When placed between two numbers, it means that they are to be added.

45. The **Sign of Equality** is $=$, and is read *equals* or *equal*. When placed between arithmetical expressions it denotes that they are equal.

Thus, $8 + 5 = 13$, is read, *eight plus five equals thirteen*, and means that the sum of eight and five is thirteen.

Principles of Addition.

46.—1. *Only similar numbers can be added.*

2. *The sum and the numbers added must be similar.*

3. *The sum of numbers will be the same in whatever order they are added.*

CASE I.

When the Sum of all the Units of each Order is Less than Ten.

47.—Ex. 1. What is the sum of 3142, 2320 and 516?

$$\begin{array}{rr} \textit{Numbers added,} & \left\{\begin{array}{r} 3142 \\ 2320 \\ \underline{516} \end{array}\right. \\ \textit{Sum,} & 5978 \end{array}$$

SOLUTION.—For convenience in adding, write the numbers so that figures representing units of the same order stand in the same column.

Add the ones, tens, hundreds, and thousands separately.

6 ones + 0 ones + 2 ones are 8 ones, which write for the ones of the sum.

1 ten + 2 tens + 4 tens are 7 tens, which write for the tens of the sum.

5 hundreds + 3 hundreds + 1 hundred are 9 hundreds, which write for the hundreds of the sum.

2 thousands + 3 thousands are 5 thousands, which write for the thousands of the sum.

Hence, the sum is 5 thousands 9 hundreds 7 tens 8 ones, or 5978.

Write and add—

(2.)	(3.)	(4.)	(5.)
61	*187*	*42*	*423*
25	*12*	*55*	*354*

6. What is the sum of 13, 173 and 202? *Ans.* 388.

7. How many dollars are 101 dollars, 65 dollars and 113 dollars?

8. How many books are 341 books, 113 books and 202 books? *Ans.* 656.

CASE II.

When the Sum of all the Units of any Order is Greater than Ten.

48.—Ex. 1. What is the sum of 5591, 1428 and 2335?

$$\text{Numbers added,}\begin{cases} 5591 \\ 1428 \\ 2335 \end{cases}$$
$$\text{Sum,}\quad 9354$$

SOLUTION.—Write the numbers as in the preceding case.

Begin to add with the ones, so that when the sum of any of the orders of units is greater than nine, its tens may be conveniently added with the units of the next higher order.

5 ones + 8 ones + 1 one are 14 ones, or 1 ten and 4 ones. Write the 4 ones for the ones of the sum, and reserve the 1 ten to add with the tens.

1 ten + 3 tens + 2 tens + 9 tens are 15 tens, or 1 hundred, and 5 tens. Write the 5 tens for the tens of the sum, and reserve the 1 hundred to add with the hundreds.

1 hundred + 3 hundreds + 4 hundreds + 5 hundreds are 13 hundreds, or 1 thousand and 3 hundreds. Write the 3 hundreds for the hundreds of the sum, and reserve the 1 thousand to add with the thousands.

1 thousand + 2 thousands + 1 thousand + 5 thousands are 9 thousands, which we write for the thousands of the sum.

Hence, the sum is 9 thousands 3 hundreds 5 tens 4 ones, or 9354.

The explanation may be abbreviated by naming only results. Thus,

Five, thirteen, fourteen; write 4 and reserve 1. One, four, six, fifteen; write 5 and reserve 1. One, four, eight, thirteen; write 3 and reserve 1. One, three, four, nine; write 9. *Ans.* 9354.

PROOF.—The correctness of the solution may be proved by reviewing the work carefully, or by adding the numbers downward. If the work is correct, the result in each case will equal the first result, since the sum of numbers must be the same in whatever order they may be added. (Art. 46—3.)

Add and prove—

(2.)	(3.)	(4.)	(5.)	(6.)
1683	467	1062	114	7703
456	305	3457	590	4104
312	587	2004	309	3492

7. What is the sum of 615 + 3045 + 5000?
8. How many are 3600, 7240 and 797?
9. A farmer raised from one field 1213 bushels of wheat; from a second, 1308 bushels; from a third, 2230 bushels; and from a fourth, 244 bushels. How much wheat did he raise?
Ans. 4995 bushels.

49. Rule for Addition.—*Write the numbers so that all figures of the same order shall stand in the same column, and draw a line under them.*

Begin at the right, add the units of each order separately, and write the sum, if less than ten, under the column added.

If the sum is ten or more, write the figure standing for its ones, and add its tens with the units of the next higher order.

Write the whole sum of the units of the highest order.

PROOF.—Add the numbers a second time, in a different order. If the work is correct, the result will be the same by both methods.

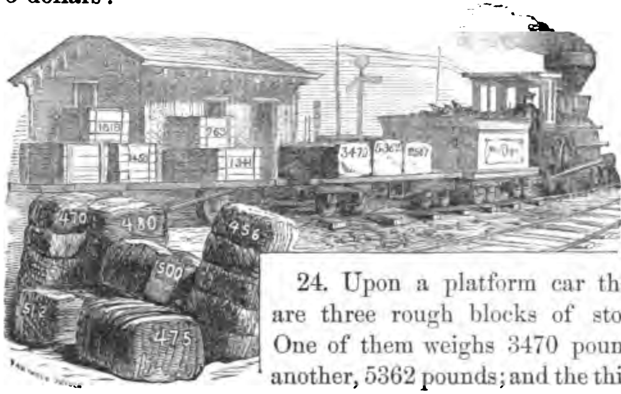
PROBLEMS.

Add and prove—

(1.)	(2.)	(3.)	(4.)	(5.)
4306	172	3334	146	4455
4507	903	678	504	777
8421	129	105	810	91

6. What is the sum of 563, 491 and 708? *Ans.* 1762.
7. What is the sum of 423, 567 and 385?
8. Find the sum of 785, 584, 175 and 145. *Ans.* 1689.

9. Find the sum of 2385, 385 and 500. *Ans.* 3270.
10. Find the sum of 1603, 495, 1708 and 793.
11. How many are 11063, 4461, 1030 and 309?
12. How many are 64004, 15300, 1008 and 488?
13. How many are 3560, 1246, 8556 and 2451?
14. How many are $2063 + 950 + 805 + 470$? *Ans.* 4288.
15. How many are 1623, 2045, 705 and 3435?
16. How many are 4000, 7891, 1631, 500 and 19?
17. $600 + 419 + 6663 + 1000 + 317 =$ what number?
18. $1798 + 4444 + 6666 + 1333 + 122 =$ how many?
19. $20120 + 3290 + 3167 + 532 + 499 =$ how many?
20. Add 138935, 113467 and 12506. *Ans.* 264908.
21. Add 38394, 22957 and 601826. *Ans.* 663177.
22. Add 2352 yards, 3800 yards and 1785 yards.
23. How many dollars are 13177 dollars, 7346 dollars and 1275 dollars?



24. Upon a platform car there are three rough blocks of stone. One of them weighs 3470 pounds; another, 5362 pounds; and the third, 2567 pounds. What is the weight of the three?

Ans. 11399 pounds.

25. Six bales of cotton contain, respectively, 470, 480, 456, 500, 512 and 475 pounds. What is their entire weight?

26. If four boxes weigh, respectively, 1618, 1450, 963 and 1341 pounds, what is their entire weight?

27. A merchant bought at one time 513 barrels of flour; at another, 763 barrels; and at another, 1347 barrels. How many barrels did he buy in all? *Ans.* 2623.

28. A gardener took from one tree 348 pears; from another, 316; from another, 159; and from another, 96. How many did he take from the whole? *Ans.* 919.

29. In a certain school there are in the first section 31 pupils; in the second, 39; in the third, 42; in the fourth, 47; and in the fifth, 64. How many pupils are there in the five sections? *Ans.* 223.

30. Thompson has in his farm 150 acres; Reed in his farm, 317 acres; Howland in his farm, 72 acres; and two others have each 875 acres. How many acres have they all?

31. A large work consists of five volumes: there are 612 pages in the first volume, 709 in the second, 691 in the third, and 1357 in the remaining two together. How many pages in the entire work? *Ans.* 3369.

32. The distance from New York to Chicago by railroad is 911 miles; from Chicago to Omaha, 401 miles; from Omaha to Ogden, 1101 miles; from Ogden to Sacramento, 743 miles; from Sacramento to San Francisco, 117 miles. What is the whole distance from New York to San Francisco?

33. A lumber-dealer has 37270 feet of boards in one yard, 9536 in another, 45098 in vessels, and 8876 at a mill. How many feet of boards has he in all these places?

34. I bought a house for 6236 dollars; I paid 869 dollars for repairs, and 300 dollars for painting it. For what price must it be sold to gain 325 dollars? *Ans.* 7730 dollars.

35. The sailing distance from New Orleans to Charleston is 1167 miles; from Charleston to Norfolk, 431 miles; from Norfolk to New York, 308 miles; and from New York to Boston, 356 miles. How many miles must a steamer sail, that shall touch at all these places, in going from New Orleans to Boston? *Ans.* 2262.

36. The mariner's compass was invented in China 1120 years before Christ; America was discovered by Columbus 1492 years after Christ; and steam was first applied by Fulton to propelling boats 315 years after the discovery of America. How many years after the invention of the mariner's compass was steam first applied to propelling boats?

Copy, add and prove—

(37.)	(38.)	(39.)	(40.)	(41.)
131	500	180	4811	11319
256	176	316	1141	4121
702	31	701	7891	3006
91	45	9	123	723
61	39	114	416	345

42. Alaska contains 577,390 square miles; California, 188,981; and Oregon, 95,274. How many square miles do the three contain? *Ans.* 861,645.

43. In 1868 there were employed in the United States, as cultivators of the land, 3,219,495 persons; as day-laborers, 960,000; as servants, 560,000; as mechanics and manufacturers, 480,905; and as merchants, 123,564. How many persons were employed in all of the occupations named?

44. The area of America is 14,491,000 square miles; of Europe, 3,760,000 square miles; of Asia, 16,313,000; of Africa, 10,936,000; and of Oceanica, 4,500,000. What is the area of the whole? *Ans.* 50,000,000 square miles.

45. If the population of America is 70,677,000; of Europe, 295,954,000; of Asia, 558,562,000; of Africa, 71,604,000, and of Oceanica, 23,261,000; what is the population of the whole?

TEST QUESTIONS.

50.—1. What is ADDITION? The sum or amount? A sign? The sign of addition? The sign of equality? Illustrate the use of the sign of addition. Of the sign of equality.

2. What are PRINCIPLES of addition? Why cannot 5 dollars and 4 books be added? Why can 4 dollars and 5 dollars be added? Show that the sum of numbers is the same in whatever order they are added.

3. What is the RULE for addition? Why are the numbers in addition written so that all figures of the same order shall stand in the same column? Why begin at the right to add? When the sum of any column is ten or more, why add its tens with units of the next higher order? Give the proof of addition. The reason for the proof.

SECTION IV.

SUBTRACTION.

51.—Ex. 1. If John has 7 books, how many more must he obtain to have 12?

2. In my purse there were 11 dollars; 6 dollars have since been taken out. How many remain?

3. William is 17 years old, and his brother is 8. What is the difference in their ages?

4. Subtract by 2's from 21 to 1.

SOLUTION.—19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

5. Subtract by 3's from 22 to 1. By 3's from 52 to 22.
6. Subtract by 4's from 32 to 4. By 4's from 60 to 32.
7. Subtract by 5's from 33 to 3. By 5's from 68 to 33.
8. Subtract by 6's from 34 to 4. By 6's from 70 to 34.
9. Subtract by 7's from 40 to 5. By 7's from 75 to 40.
10. Subtract by 8's from 46 to 6. By 8's from 94 to 46.
11. Subtract by 9's from 52 to 7. By 9's from 88 to 52.

12. What is the unit of 12 and 7? What is the unit of the difference of 12 and 7?

13. What is the unit of the difference of 13 dollars and 9 dollars?

14. Are 13 dollars and 9 dollars and their difference similar or dissimilar numbers?

15. Why can you not subtract 9 books from 13 dollars?

Because the numbers have no common unit.

16. Only what kind of a number, therefore, can be taken from another number?

17. If you take 7 cents from 16 cents, what number will be the difference? What number added to 7 will make 16?

DEFINITIONS.

52. Subtraction is the process of taking one number from another.

53. The **Difference** is the result obtained by the subtraction.

54. The **Minuend** is the number from which the subtraction is to be made.

55. The **Subtrahend** is the number which is to be subtracted.

The subtrahend cannot be greater than the minuend. When the subtrahend is equal to the minuend, the difference is 0.

56. The **Sign of Subtraction** is —, and is called *minus*. When placed between two numbers it denotes that the one on the right is to be taken from that on the left.

Thus, 13 — 5 is read, *thirteen minus five*, and means that five is to be taken from thirteen.

Principles of Subtraction.

57.—1. *Only similar numbers can be subtracted.*

2. *The minuend, subtrahend and difference must be similar numbers.*

3. *The sum of the difference and subtrahend must equal the minuend.*

CASE I.

When the Units of the Subtrahend do not Exceed those of the same Orders in the Minuend.

58.—Ex. 1. From 569 subtract 235.

Minuend,	569
Subtrahend,	235
Difference,	334

Solution.—For convenience in subtracting, write the figures of the subtrahend under figures representing the same orders in the minuend. Subtract each order of units separately: 9 ones — 5 ones are 4 ones, which write for the ones of the difference.

6 tens — 3 tens are 3 tens, which write for the tens of the difference.

5 hundreds — 2 hundreds are 3 hundreds, which write for the hundreds of the difference.

Hence, the difference is 3 hundreds 3 tens 4 units, or 334.

	(2.)	(3.)	(4.)	(5.)
From	496	873	98	3318 dollars,
Subtract	365	651	54	208 dollars.

In Ex. 5, as there are no units of the thousands' order in the subtrahend, we proceed as if the subtrahend had been written 0208.

6. A person born in the year 1819 was how old in 1871?

7. Johnson had 765 dollars, and paid away 351 dollars. How many dollars had he left? *Ans.* 414.

8. What is the difference between 3467 and 1352?

9. What is the difference between 25674 and 5473?

10. Andrew Holt bought a plantation for 14630 dollars, and sold it for 15750 dollars. How many dollars did he gain by the operation? *Ans.* 1120.

CASE II.

When the Units of one or more Orders of the Subtrahend Exceed those of the Same Orders of the Minuend.

59.—Ex. 1. From 8662 take 4734.

	7.16.5.12.
Minuend,	*8662*
Subtrahend,	*4734*
Difference,	*3928*

SOLUTION.—Write the numbers as in the preceding case.

Since 4 ones cannot be subtracted from 2 ones, take 1 ten from the 6 tens, leaving 5 tens; and add 10 ones, which are equal to the 1 ten, to the 2 ones, making 12 ones; 12 ones — 4 ones are 8 ones, which write for the ones of the difference.

5 tens — 3 tens are 2 tens, which write for the tens of the difference.

Since 7 hundreds cannot be subtracted from 6 hundreds, take 1 thousand from the 8 thousands, leaving 7 thousands, and add the 10 hundreds that equal the 1 thousand to the 6 hundreds, making 16 hundreds; 16 hundreds — 7 hundreds are 9 hundreds, which write for the hundreds of the difference.

7 thousands — 4 thousands are 3 thousands, which write for the thousands of the difference.

Hence, the difference is 3 thousands 9 hundreds 2 tens 8 units, or 3928.

The explanation may be abbreviated thus: 2 + 10 are 12; 12 — 4 are 8, which write. 6 — 1 are 5; 5 — 3 are 2, which write. 6 + 10 are 16; 16 — 7 are 9, which write. 8 — 1 are 7; 7 — 4 are 3, which write.

Ans. 3928.

We begin at the right to subtract, so that when any figure of the subtrahend denotes a greater number than the corresponding figure of the minuend, 1 may be taken from the next higher order of the minuend.

Subtrahend,	*4734*
Difference,	*3928*
Minuend,	*8662*

PROOF.—To test the solution, we add the difference and subtrahend, and since the result is equal to the minuend, the work is known to be correct. (Art. 57—3.)

Solve and prove—

	(2.)	(3.)	(4.)	(5.)
From	*4167*	*5483*	*1922*	*8769* pounds,
Subtract	*3058*	*938*	*1415*	*6718* pounds.

6. What is the difference between 1634 and 1329?

7. I bought goods for 9363 dollars, and sold them for 10382 dollars. How much did I gain by the transaction?

Ans. 1019 dollars.

60. Rule for Subtraction.—*Write the subtrahend under the minuend, so that figures of the same order shall stand in the same column.*

Begin at the right, subtract the units of each order of the subtrahend from units of the same order of the minuend, if possible, and write the difference beneath.

When the units of any order of the minuend are less than those of the same order of the subtrahend, increase their number by adding ten, the value of a unit taken from the next higher order of the minuend, and subtract; then consider the units of that higher order of the minuend one less.

PROOF.—Add the difference and subtrahend. If the work is correct, the result will equal the minuend.

PROBLEMS.

Solve and prove—

	(1.)	(2.)	(3.)	(4.)
From	*4592*	*9863*	*34061*	*46603* soldiers,
Take	*3683*	*416*	*12341*	*928* soldiers.

5. How many are 69351 less 40402? *Ans.* 28949.
6. How many are 32443 less 965?
7. How many are 17630 less 8542? *Ans.* 9088.
8. What is the difference between 92334 and 13403?
9. What is the difference between 46304 and 37413?
10. How much more is 32045 than 862? *Ans.* 31183.
11. How much less is 40920 than 91605? *Ans.* 50685.

12. 36841 — 27777 = what number? *Ans.* 9064.

13. 81710 — 70809 = what number? *Ans.* 10901.

14. What is the difference between 4000 and 1321?

	3.9.9.10.
Minuend,	*4000*
Subtrahend,	*1321*
Difference,	*2679*

SOLUTION.—As there are no units expressed in the three lower orders of the minuend, reduce one of the four thousands to hundreds, then one of the hundreds to tens, and then one of the tens to ones. The minuend will then be 3 thousands 9 hundreds 9 tens 10 ones, from which the subtrahend can easily be taken.

Then, 10 ones — 1 one are 9 ones, which write for the ones of the difference. 9 tens — 2 tens are 7 tens, which write for the tens of the difference. 9 hundreds — 3 hundreds are 6 hundreds, which write for the hundreds of the difference. 3 thousands — 1 thousand are 2 thousands, which write for the thousands of the difference. Hence, the answer is 2679.

Solve and prove—

	(15.)	(16.)	(17.)	(18.)
From	*3400* men,	*5002*	*8003*	*20000* bushels,
Take	*2306* "	*2151*	*7004*	*4242* "

19. Subtract 199 from 1000. *Ans.* 801.

20. Subtract 3003 from 30030. *Ans.* 27027.

21. How many are 19000 men less 567 men?

22. How many yards are 31005 yards less 1777 yards?

23. How many tons are 51067 tons less 45075 tons?

24. From 100000 take 16898. *Ans.* 83102.

Find the difference—

25. Between 36512 and 18735. *Ans.* 17777.

26. Between 12701 and 4110.

27. Between 7125 and 3647. *Ans.* 3478.

28. Between 38217 and 9548. *Ans.* 28669.

29. Between 1000000 and 107701.

30. Between 500807 and 480705. *Ans.* 20102.

31. If a man should buy a farm for 8000 dollars, and should make a payment of 5925 dollars upon it, how many dollars would he still owe? *Ans.* 2075.

32. The Pilgrims landed at Plymouth in the year 1620.

How long was it from that time to the Declaration of Independence, in the year 1776?

33. A merchant bought 63100 bushels of corn, and sold 17734 bushels. How many bushels had he left?

34. I bought a piece of property for 95565 dollars, and sold it for 100000 dollars. What was the gain? *Ans.* 4435 dollars.

35. In a certain election A received 25316 votes and B 21925 votes. What was A's majority? *Ans.* 3391.

36. By selling a farm for 7535 dollars I gained 1675 dollars. What was its cost?

37. At a certain election the successful candidate received 25316 votes, which was 3191 votes more than were given to the rival candidate. How many votes did the latter receive?

38. The population of a town is 2763; ten years ago it was 2595. What was the increase in the ten years? *Ans.* 168.

39. A mill sawed 427563 feet of boards one year and 385895 the year before. How many more feet did it saw in the one year than in the other? *Ans.* 41668.

40. St. Peter's Church, Rome, has standing-room for 54000 persons, and the New York Academy for 1326 persons. How many more persons can stand at the same time in the one than in the other?

41. There are 41823360 seeds in a bushel of timothy, and 16400960 seeds in a bushel of clover. How many more seeds in a bushel of the one than in a bushel of the other?

TEST QUESTIONS.

61.—1. What is SUBTRACTION? The difference? The minuend? The subtrahend? When the subtrahend and minuend are equal, what is their difference? What is the sign of subtraction? Illustrate its use.

2. What are the PRINCIPLES of subtraction? Why cannot 5 apples be subtracted from 8 dollars? If the minuend and subtrahend express dollars, what will the difference express? If the minuend is 8, what must be the sum of the subtrahend and difference?

3. What is the RULE for subtraction? Why is the subtrahend written under the minuend so that figures shall stand in the same column? Why begin at the right to subtract? What is the method of proof? When the subtrahend and difference are given, how may the minuend be found?

SECTION V.

REVIEW PROBLEMS.

MENTAL EXERCISES.

62.—Ex. 1. How many tens and ones in 96? In 78?

2. How many hundreds, tens and ones in 365? In 431? In 987?

3. How many thousands and units in 55625? In 107308?

4. John has 14 dollars, William 10 dollars and Henry 9 dollars. How many dollars have they all?

5. A farmer has 26 cows in one field, 15 in another and 23 in a third. How many cows has he in the three fields?

SOLUTION.—He has as many cows in the three fields as the sum of 26, 15 and 23. $26 + 10 = 36$; $36 + 5 = 41$; $41 + 20 = 61$; $61 + 3 = 64$. Therefore he has 64 cows in the three fields.

6. A man paid 18 dollars for a coat, 33 dollars for an oversack and 9 dollars for pantaloons. How much did he pay for all?

7. A drayman moved 41 boxes on one day, 17 on another day, 34 on a third day and 23 on a fourth day. How many boxes did he move in all?

8. A certain orchard contains 28 apple trees, 18 pear trees, 32 peach trees and 11 cherry trees. How many trees does it contain altogether?

9. If I had 9 dollars more I should have 27 dollars. How much money have I?

10. John had 48 cents, but has since spent 29 cents. How many cents has he remaining?

SOLUTION.—He has as many cents remaining as the difference between 48 and 29. $48 - 9 = 39$; $39 - 20 = 19$. Therefore he has 19 cents remaining.

11. Robert is 15 years old, and his father is 53 years old. What is the difference in their ages?

12. Susan is 13 years old. In how many years will she be 42 years old?

13. Willie gathered 27 nuts in the forenoon, and in the after-

noon as many more less 19. How many did he gather in the afternoon?

14. A drover bought 16 sheep at one time and 112 at another; he then sold 18. How many had he left?

15. I had 54 cents in my purse, but took out 15 cents at one time and 16 cents at another. How many cents then remained in the purse?

WRITTEN EXERCISES.

63.—Ex. 1. A man paid 700 dollars for a span of horses, 450 dollars for a carriage and 175 dollars for a double harness. What sum did he pay for the whole? *Ans.* 1325 dollars.

2. What is the sum of one million five hundred seventy-five thousand three hundred twenty-two, plus four hundred nineteen thousand three hundred sixty-five, plus one hundred thirty-two thousand three hundred fifty-five?

3. I bought some property for 5390 dollars, and sold it for 6585 dollars. How much did I gain? *Ans.* 1195 dollars.

4. If the smaller of two numbers is 5390 and their difference 1195, what is the larger number?

5. Mount St. Elias is 18087 feet in height and Mount Washington 6225 feet. How much higher is Mount St. Elias than Mount Washington? *Ans.* 11862 feet.

6. If the minuend is four hundred seventeen thousand seven hundred twelve, and the subtrahend is one hundred thousand ninety-three, what is the difference? *Ans.* 317619.

7. A man's salary is 1200 dollars, and his income from other sources is 655 dollars. His expenses are 850 dollars. How much is his net income?

	1200	dollars.
	655	"
Income,	*1855*	dollars.
Expenses,	*850*	"
Net Income,	*1005*	dollars.

Solution. — His income must be the sum of his salary and his income from other sources. $1200 + 655 =$ 1855 dollars.

His net income must be his income less his expenses. 1855 dollars — 850 dollars $=$ 1005 dollars. His net income, therefore, is 1005 dollars.

8. A merchant bought goods to the amount of 7835 dollars. He then sold a part for 9563 dollars, and the balance for 361 dollars. How much did he gain by the transaction?

9. The great bell of Moscow weighs 432000 pounds, and the bell of City Hall, New York, weighs 22300 pounds. What is the difference in their weight?

10. A grain-dealer had 9867 bushels of corn. He sold 3479 bushels at one time and 5372 bushels at another time. How many bushels had he then left?

3479 bushels.	*9867* bushels.
5372 "	*8851* "
8851 bushels.	*1016* bushels.

SOLUTION.—If he sold 3479 bushels at one time and 5372 bushels at another time, he sold in all 3479 bushels + 5372 bushels, or 8851 bushels.

If he had 9867 bushels, and sold 8851 bushels, he had left 9867 bushels — 8851 bushels, or 1016 bushels. *Ans.* 1016 bushels.

11. Kaspar sold 516 barrels of flour at one time, 419 barrels at another time, and had 316 remaining. How many barrels had he at first? *Ans.* 1251.

12. A man had 16745 acres of land. To A he sold 5304 acres and to B 6319 acres. How much had he then unsold?

13. Andrew Jones bought a piece of land for 18068 dollars. He sold a part for 9563 dollars, and the remainder for 9385 dollars. How much did he gain by the operation?

14. A man had 15675 dollars; afterward he expended 5000 for a house and 1225 for furniture, and then earned 963 dollars. How much money had he then? *Ans.* 10413 dollars.

15. The sailing distance from New York to Cape Horn is 7232 miles; from Cape Horn to Canton, 8840 miles; from Canton to Cape Good Hope, 7000 miles; and from Cape Good Hope to New York, 6790 miles. The sailing distance from Canton to San Francisco is 6090 miles, and the distance by railroad from San Francisco to New York, 3273 miles. How much shorter is the route from Canton to New York, by way of San Francisco, than by either Cape Good Hope or Cape Horn? *Ans.* 4427 miles shorter than by Cape Good Hope; 6709 miles shorter than by Cape Horn.

SECTION VI.

MULTIPLICATION.

64.—Ex. 1. If a boy take from a library 4 books each day for 3 days, how many times does he take 4 books? How many books does he take in all?

2. Four boys have each 6 cents. How many cents have they in all? How many times must 6 cents be taken to make the number they have in all?

3. What is the difference between 6 cents + 6 cents + 6 cents + 6 cents and 4 times 6 cents?

4. How many times 9 cents will 3 pencils cost at 9 cents each?

5. Give the 2's from one 2 to six 2's.

SOLUTION.—One 2 is 2, two 2's are 4, three 2's are 6, four 2's are 8, five 2's are 10, and six 2's are 12.

6. Give the 2's from seven 2's to twelve 2's.
7. Give the 3's from three 3's to twelve 3's.
8. Give the 4's from five 4's to twelve 4's.
9. Give the 5's from five 5's to twelve 5's.
10. Give the 6's from six 6's to twelve 6's.
11. Give the 7's from seven 7's to twelve 7's.
12. Give the 8's from eight 8's to twelve 8's.
13. Give the 9's from nine 9's to twelve 9's.
14. Give the 10's from ten 10's to twelve 10's.
15. Give the 11's from eleven 11's to twelve 11's.
16. How many are 5 times 7? 4 times 8 dollars? Is the number which denotes the times another number is taken abstract or concrete?
17. When 7 dollars are taken 6 times, what is the result? To which of the given numbers is the result similar?
18. Is the result the same when we take 8 five times as when we take 5 eight times?
19. How many are 10 times 6? 10 times 9?
20. How many are 11 times 5? 11 times 10?
21. How many are 12 times 12? 12 times 11?

DEFINITIONS.

65. **Multiplication** is the process of taking one of two numbers as many times as there are units in the other.

66. The **Multiplicand** is the number taken or multiplied.

67. The **Multiplier** is the number which shows how many times the multiplicand is to be taken.

68. The **Product** is the result of the multiplication.

69. The multiplier and multiplicand are called **Factors** of the product.

When the factors are more than two, the multiplication is called *Continued Multiplication*, and the result a *Continued Product*.

70. The **Sign of Multiplication** is $\times$, and is read, *times*, or *multiplied by*. When placed between two numbers, it denotes that they are to be multiplied together.

Thus, 8×5 is read, *eight multiplied by five*, or *five times eight*.

Principles of Multiplication.

71.—1. *The multiplier is always an abstract number.*

2. *The product and the multiplicand are similar numbers.*

3. *In finding the product of two factors, either, abstractly considered, may be used as the multiplier.*

CASE I.

When the Multiplier consists of but One Order of Units.

72.—Ex. 1. Multiply 3427 by 6.

Multiplicand,	*3427*
Multiplier,	*6*
Product,	*20562*

Solution.—For convenience write the multiplier under the multiplicand, and begin at the right to multiply.

6 times 7 ones are 42 ones, or 4 tens and 2 ones. Write the 2 ones for the ones of the product, and reserve the 4 tens to add to the product of the tens.

6 times 2 tens are 12 tens, and 12 tens + 4 tens are 16 tens, or 1 hundred, and 6 tens. Write the 6 tens for the tens of the product, and reserve the 1 hundred to add to the product of the hundreds.

6 times 4 hundreds are 24 hundreds, and 24 hundreds + 1 hundred are 25 hundreds, or 2 thousands and 5 hundreds. Write the 5 hundreds for the hundreds of the product, and reserve the 2 thousands to add to the product of the thousands.

6 times 3 thousands are 18 thousands, and 18 thousands + 2 thousands are 20 thousands, or 2 ten-thousands and 0 thousands, which write.

Hence, the product is 2 ten-thousands 0 thousands 5 hundreds 6 tens 2 ones, or 20562.

We begin at the right to multiply, so that when the product exceeds nine, we may add its tens to the next product.

The explanation may be abbreviated thus: 6 times 7 are 42; write the 2 and reserve the 4. 6 times 2 are 12, and 4 are 16; write the 6 and reserve the 1. 6 times 4 are 24, and 1 is 25; write the 5 and reserve the 2. 6 times 3 are 18, and 2 are 20, which write. *Ans.* 20562.

The correctness of the work may be tested or proved by carefully reviewing the whole process.

Solve and prove—

	(2.)	(3.)	(4.)	(5.)
Multiply	*645*	*916* men.	*1125*	*3246* yards.
By	*4*	*5*	*7*	*3*

6. Multiply 30874 by 6. *Ans.* 185244.

7. What is the product of 73121 by 7? *Ans.* 511847.

8. What will 8 farms cost at 13035 dollars each?

9. How many square miles in 9 townships, if each contain 24375 square miles? *Ans.* 219375.

CASE II.

When the Multiplier consists of More than One Order of Units.

73.—Ex. 1. Multiply 962 by 34.

Multiplicand,	*962*
Multiplier,	*34*
Partial Products,	*3848*
	2886
Product,	*32708*

Solution.—Since 34 is equal to 3 tens + 4 ones, 34 times 962 must equal 4 times 962 + 3 tens times 962.

4 times 962, or the product of 962 by the ones of the multiplier, is 3848, the first partial product.

3 tens × 2 ones are 6 tens, which write for the tens of the second partial product. 3 tens × 6 tens are 18 tens of tens, or 18 hundreds, or 1 thousand and 8 hundreds; write the 8 hundreds for the hundreds of the partial product, and reserve the 1 thousand for the thousands. 3 tens × 9 hundreds are 27 tens of hundreds,

or 27 thousands, and 27 thousands + 1 thousand are 28 thousands, or 2 ten-thousands and 8 thousands, which write in the partial product, making the second partial product 2886 tens, or 28860.

The entire product, or the sum of the partial products, 3848 + 28860, is 32708.

The explanation may be abbreviated, thus: 962 × 4 is 3848 *units.* 3 times 2 are 6, which write for the tens. 3 times 6 are 18; write the 8 for the hundreds, and reserve the 1 for the thousands. 3 times 9 are 27, and 1 are 28, which write, making 2886 *tens.*

Adding the partial products, we have 32708, the answer required.

34
962
68
204
306
32708

PROOF.—Prove the solution by reversing the order of the factors in the multiplication, taking the 34 for the multiplicand and the 962 for the multiplier.

The result being the same as at first obtained, the work is presumed to be correct, since the product of two factors is the same, whichever is taken as the multiplier. (Art. 71—3.)

Solve and prove—

	(2.)	(3.)	(4.)	(5.)
Multiply	*8125*	*813*	*3104*	*614*
By	*23*	*55*	*16*	*207*
				4298
				1228
				127098

In Ex. 5 we omit to multiply by the 0 tens, since the product of any number multiplied by 0 is 0.

6. What is the product of 763 by 305? *Ans.* 232715.
7. What is the product of 706 by 408?
8. What is the product of 403 by 62? *Ans.* 24986.

CASE III,

When either Factor has One or More Ciphers on the Right.

74.—Ex. 1. Multiply 465 by 100.

Multiplicand,	*465*
Multiplier,	*100*
Product,	*46500*

SOLUTION.—Since 10 units of any order are always equal to 1 of the next higher order (Art. 34—1), the writing of a cipher on the right of a number, which removes its figures each an order to the left, must multiply it by 10. In like manner, the writing of two ciphers on the right of a number must multiply it by 100; the writing of three ciphers must multiply it by 1000, etc.

Hence, to 465×1, which is 465, we annex two ciphers, and have 100 times the number, or 46500, the answer required.

2. Multiply 465 by 500.

Multiplicand,	*465*
Multiplier,	*500*
Product,	*232500*

SOLUTION.—Since 500 is 100 times 5, 500 times 465 must be 100 times 5 times 465. 5 times 465 is 2325, and 100 times as much is 2325 with two ciphers annexed, or 232500, the answer required.

3. Multiply 4650 by 500.

Multiplicand,	*4650*
Multiplier,	*500*
Product,	*2325000*

SOLUTION.—5 times 4650 is 5 times 465 with a cipher annexed, or 23250; and 100 times as much is 23250 with two ciphers annexed, or 2325000, the answer required.

Multiply—

4. 516 by 10. *Ans.* 5160.
5. 1302 by 100.
6. 95 by 1000.
7. 254 by 600.
8. 75 by 300. *Ans.* 22500.

How many are—

9. 40 times 560? *Ans.* 22400.
10. 80 times 3400?
11. 200 times 500? *Ans.* 100000.
12. 120 times 4110?
13. 1000 times 1000?

14. If you can travel 12 miles in one hour, how far, at the same rate, can you travel in 100 hours?

15. What will 45 casks of molasses cost at 50 dollars each?

16. If there are 640 acres in a square mile, how many acres are there in 150 square miles? *Ans.* 96000.

75. Rule for Multiplication.—*If the multiplier consists of one order of units, multiply each order of the multiplicand, beginning at the right, by the multiplier. Write the units of each result in the product, and reserve the tens, if any, to be added to the next result.*

If the multiplier consists of more than one order of units, multiply each order of the multiplicand by each order of the multiplier, write the right-hand figure of each partial product under the order of the multiplier used, and add the partial products, *If either factor has one or more ciphers on the right, multiply without regard to these ciphers, and annex to the result as many ciphers as are on the right of both the factors.*

PROOF.—Review the work, or reverse the order of the factors and multiply. If the work is correct, the result will be the same by both methods.

PROBLEMS.

Multiply and prove—

1. 216 by 8. *Ans.* 1728.
2. 405 by 9.
3. 1315 by 6. *Ans.* 7890.
4. 116 by 1000.
5. 413 by 70. *Ans.* 28910.
6. 555 by 4.
7. 4163 by 7. *Ans.* 29141.
8. 3162 by 11.
9. 51003 by 90.
10. 18300 by 18.
11. 738 by 235.
12. 756 by 72. *Ans.* 54432.
13. 3216 by 5.
14. 248 by 19. *Ans.* 4712.
15. 160 by 30.
16. 365 by 37. *Ans.* 13505.
17. 1040 by 11.
18. 4561 by 603. *Ans.* 2750283.
19. 11140 by 13.
20. 5704 by 974. *Ans.* 5555696.
21. 4402 by 222.
22. 4561 by 4005.

23. How many are 312 times 4144? *Ans.* 1292928.
24. How many are 999 times 345?

$$\begin{array}{r} 345000 = 345 \times 1000 \\ \underline{345 = 345 \times 1} \\ 344655 = 345 \times 999 \end{array}$$

SOLUTION.—Since 1000 times any number, less once the number, must be 999 times the number, we here abridge the solution by taking the multiplicand 1000 times, or once too many, by annexing three ciphers, and then subtracting the multiplier. This method of abridgment applies whenever the multiplier is 1 less than 100, 1000, 10000, etc.

25. $4573 \times 99 =$ what number? *Ans.* 452727.
26. $1316 \times 999 =$ what number?
27. $1230 \times 9999 =$ what number? *Ans.* 12298770.
28. What is the product of 1036 by 990?

29. What is the product of 4455 by 105?

$$
\begin{array}{r}
445500 = 4455 \times 100 \\
\underline{22275} = 4455 \times \underline{5} \\
467775 = 4455 \times 105
\end{array}
$$

SOLUTION.—Here the solution may be abridged by writing the product of 4455 by the 1 hundred at once, by annexing two ciphers, and writing under it the product of 4455 by the 5 ones, and then adding the two partial products.

30. What is the product of 6307 by 1003?

31. If 5362 feet of boards can be sawed in a mill in one day, how many feet can be sawed in it in 313 days?
Ans. 1678306.

32. At 125 dollars a month, how many dollars can be earned in 12 months? *Ans.* 1500.

33. What will 3158 tons of coal cost at 8 dollars per ton?

$$
\begin{array}{r}
3158 \\
\underline{8} \\
25264
\end{array}
$$

SOLUTION.—At 8 dollars a ton, 3158 tons of coal will cost 3158 times 8 dollars, which is equal to 8 times 3158 dollars, or 25264 dollars.

8 dollars is the true multiplicand, but since 8 times 3158 gives the same product as 3158 times 8, we, for convenience, in the solution consider both factors as abstract numbers, and make the smaller factor the multiplier.

34. What will 8344 yards of cloth cost at 6 dollars a yard?
Ans. 50064 dollars.

35. How many oranges in 47 boxes when each box contains 279 oranges? *Ans.* 13113.

36. Two factors are 7312 and 7000. What is their product?
Ans. 51184000.

37. There were 6 drawers in a desk, 8 compartments in each drawer, and 87 dollars in each compartment. How many dollars did the desk contain?

$$
\begin{array}{rl}
87 & \text{dollars.} \\
\underline{48} & \text{No. of compartments.} \\
696 & \\
\underline{348} & \\
4176 & \text{dollars.}
\end{array}
$$

SOLUTION.—Since in the desk there were 6 drawers, and each had 8 compartments, there were in the desk 6 times 8, or 48 compartments.

Since there were 48 compartments in the desk, and 87 dollars in each, the desk contained 48 times 87 dollars, or 4176 dollars.

38. What is the continued product of the factors 6, 8 and 87?

39. If 15 pounds of hay are required by 1 horse for 1 day, how many pounds are required by 5 horses for 30 days?

40. How many gallons in 1025 casks, if each cask contain 63 gallons? *Ans.* 64575.

41. In a bushel of rye are 888390 seeds. How many seeds are there in 25 bushels? *Ans.* 22209750.

42. If the weight of a cubic foot of white-oak wood is 43 pounds, what is the weight of 128 cubic feet?

43. What is the continued product of the factors 12, 420 and 310? *Ans.* 1562400.

44. How many yards of cloth in 32 bales, each bale having 121 pieces, and each piece 31 yards? *Ans.* 120032.

45. What is the population of a State containing 8320 square miles, if each square mile has 91 inhabitants?

46. If Pennsylvania contain 46000 square miles, what will be its population at 75 persons to a square mile?

Ans. 3450000 persons.

47. Sound moves 1142 feet in a second. How far will it move in one hour, or 3600 seconds? *Ans.* 4111200 feet.

48. If a railroad, 1035 miles in length, should obtain government aid to the amount of 52400 dollars per mile, what would be the entire amount received?

TEST QUESTIONS.

76.—1. What is MULTIPLICATION? The product? The factors of a product? The multiplicand? The multiplier? What does the sign of multiplication denote when written between two numbers?

2. What are the PRINCIPLES of multiplication? Show that the multiplicand and product are similar numbers. Why must the multiplier be regarded always as an abstract number? Show that the product is the same in whatever order its factors are used.

3. Recite the RULE for multiplication. Give the proof. Why, in multiplication, do you begin at the right to multiply? When the multiplier consists of more than one order of units, how many partial products may there be? What will be the unit of the partial product when the multiplier is a number of simple units? When the multiplier is a number of tens? When there are partial products, how do you obtain the entire product?

SECTION VII.

DIVISION.

77.—Ex. 1. How many barrels, holding 3 bushels each, will be required to hold 30 bushels of apples?

2. If a farmer should raise 21 bushels of potatoes, and should wish to put them into barrels holding 3 bushels each, how many barrels would be required?

3. If you have 28 apples, and wish to distribute them equally among 7 boys, how many can you give to each boy?

4. How many times are 9 bushels contained in 36 bushels? What is the product of 9 bushels by 4?

5. How can you show that 9 bushels are contained in 36 bushels 4 times?

6. When you distribute 35 apples equally among 7 boys, do you find how many times 7 boys are contained in 35 apples, or do you find one of the 7 equal parts of 35?

7. When you find how many times 9 cents are contained in 72 cents, is the result a concrete or an abstract number?

8. When you find one of the 9 equal parts of 72 cents, is the result a concrete or an abstract number?

9. How many times are 9 dollars contained in 29 dollars, and how many dollars remain?

10. If you have 47 peaches to distribute among 7 boys, how many entire peaches can you give to each, and how many peaches will remain?

11. 7 times 6 peaches, and 5 peaches, are how many peaches?

DEFINITIONS.

78. **Division** is the process of finding how many times one number is contained in another; or,

Division is the process of separating one of two numbers into as many equal parts as there are units in the other.

79. The **Dividend** is the number to be divided.

80. The **Divisor** is the number by which to divide.

81. The **Quotient** is the result obtained by the division.

82. The **Remainder** is a part of the dividend remaining undivided.

83. The **Sign of Division** is $\div$, and is read, *divided by.* The dividend is placed at the left of the sign, and the divisor at the right of it.

Thus, $40 \div 8$ is read, *forty divided by eight.*

Division is sometimes denoted by placing the dividend over the divisor, with a line between them.

Thus, $\frac{16}{4}$ is read, *sixteen divided by four.*

Division is also denoted by a curved line,), with the divisor on the left and the dividend on the right.

Thus, 5)10 is read, *ten divided by five.*

84. A **Parenthesis** ( ), enclosing two or more numbers, or a **Vinculum**, $\overline{\quad\quad}$, drawn over them, denotes that the expression is to be treated as a single number.

Thus, $(16 + 4) \div 5$, or $\overline{16 + 4} \div 5$, denotes that the sum of 16 and 4, or 20, is to be divided by 5.

85. The **Names** of the equal parts into which a number may be divided differ according to their number.

A *half* of a number is one of *two* equal parts into which it is divided.

A *third* of a number is one of the *three* equal parts into which it is divided.

A *fourth* of a number is one of the *four* equal parts into which it is divided.

In like manner we have the names *fifths, sixths, sevenths, eighths, ninths, tenths, twentieths, thirty-fourths, forty-sixths,* etc.

Halves, thirds, fourths, etc., are expressed by writing the number denoting the name of the parts, as a divisor, under a line, and the number denoting the number of the parts represented, as a dividend, above the line.

Thus, $\frac{1}{2}$ signifies 1 *divided by* 2, or 1 half of 1, and is read, *one half.*

$\frac{2}{3}$ signifies 2 *divided by* 3, or 2 thirds of 1, and is read, *two thirds.*

86. A **Fraction** is a number which represents one or more of the equal parts into which a unit, or one, is divided.

Thus, $\frac{3}{4}$, expressing 3 of the four equal parts of 1, is a fraction.

Principles of Division.

87.—1. *Division is the reverse of multiplication.*

2. *The quotient must be an abstract number when the divisor and dividend are similar numbers.*

3. *The quotient must be a concrete number and the divisor an abstract number when the divisor and dividend are dissimilar numbers.*

4. *The remainder and dividend must be similar numbers.*

5. *The dividend is equal to the product of the integer of the quotient multiplied by the divisor, plus the remainder.*

CASE I.

Short Division.

88.—Ex. 1. Divide 4313 by 4.

$$\begin{array}{rl} \textit{Divisor, } 4)\underline{4313} & \textit{Dividend.} \\ 1078\frac{1}{4} & \textit{Quotient.} \\ \underline{4} & \\ 4313 & \textit{Proof.} \end{array}$$

SOLUTION.—For convenience we first divide the highest order of units. 4 is contained in 4 thousands 1 thousand times. Write 1 in the thousands' order in the quotient.

4 is not contained in 3 hundreds any number of hundred times.

Write 0 in the hundreds' order in the quotient, and unite the 3 hundreds with the 1 ten, making 31 tens.

4 is contained in 31 tens 7 tens times, with a remainder 3 tens. Write 7 in the tens' order in the quotient, and unite the 3 tens with the 3 ones, making 33 ones.

4 is contained in 33 ones 8 times, with a remainder 1. Write 8 in the ones' order in the quotient, and the remainder 1, with 4, the divisor, under it as a part of the quotient. The required quotient is $1078\frac{1}{4}$.

Prove the correctness of the solution by multiplying the integer of the quotient by the divisor, and adding the remainder. For, (Art. 87—5) the dividend must be equal to the integer of the quotient multiplied by the divisor, plus the remainder.

The explanation of the solution may be abridged thus: 4 in 4, 1; 4 in 3, 0; 4 in 31, 7; 4 in 33, 8, with 1 as a remainder. *Ans.* $1078\frac{1}{4}$.

Division is called **Short Division** when in the solution only the divisor, dividend and quotient are written.

Solve and prove—

(2.)	(3.)	(4.)	(5.)	(6.)
$4\overline{)940}$	$6\overline{)672}$	$5\overline{)6570}$	$7\overline{)847}$	$8\overline{)968}$
(7.)	(8.)	(9.)	(10.)	(11.)
$3\overline{)4162}$	$2\overline{)1931}$	$6\overline{)6753}$	$5\overline{)47235}$	$9\overline{)817}$

12. If 6 boys share equally 1386 apples, how many will each boy have? *Ans.* 231.

CASE II.

Long Division.

89.—Ex. 1. Divide 16013 by 5, or find one fifth of 16013.

Divisor. Dividend. Quotient.

$$
\begin{array}{l}
5)16013(3202\frac{3}{5} \\
\underline{15} \\
\;\;10 \\
\;\;\underline{10} \\
\;\;\;\;13 \\
\;\;\;\;\underline{10} \\
\;\;\;\;\;3 \text{ Rem.}
\end{array}
$$

SOLUTION.—5 is not contained in 1 ten-thousand any number of ten-thousand times; hence, we unite the 1 ten-thousand with the 6 thousands, making 16 thousands.

5 is contained in 16 thousands 3 thousands times, with a remainder. Write 3 in the thousands' order of the quotient. 5×3 thousands $=15$ thousands, which, written under the 16 thousands and subtracted, leaves 1 thousand. Unite the 1 thousand with the 0 hundreds, making 10 hundreds.

5 is contained in 10 hundreds 2 hundred times. Write 2 in the hundreds' order of the quotient. 5×2 hundreds = 10 hundreds, which, written under the 10 hundreds and subtracted, leaves no remainder.

5 is not contained in 1 ten any number of tens times. Write 0 in the tens' order of the quotient, and unite with the 1 ten the 3 ones, making 13 ones.

5 is contained in 13 ones 2 ones times, with a remainder. Write 2 in the ones' order in the quotient. 5×2 ones = 10 ones, which, written under the 13 ones and subtracted, leaves 3 remainder.

Write the remainder 3 over the divisor 5 as a part of the quotient, which gives $3202\frac{3}{5}$, the result required.

$$\begin{array}{r} 3202\frac{3}{5} \\ 5 \\ \hline 16010 \\ 3 \\ \hline 16013 \end{array}$$

PROOF.—Prove the solution by multiplying the integer of the quotient by the divisor, and adding the remainder, which gives the dividend; hence, the work is correct.

Division is called **Long Division** when each process of the solution is written.

Solve and prove—

(2.)	(3.)	(4.)	(5.)
8)3368(	11)2354(	9)3706(	12)4004(
(6.)	(7.)	(8.)	(9.)
21)640(	15)915(	25)806(	7)1803(

10. Find one eighth of 1765. *Ans.* $220\frac{5}{8}$.

11. When a steamer sails 3018 miles in 14 days, what is her progress per day? *Ans.* $215\frac{8}{14}$ miles.

12. At the rate of 27 miles per hour, in how many hours will a train of cars run 2563 miles? *Ans.* $94\frac{25}{27}$.

CASE III.

When the Divisor is 1, with One or More Ciphers on the Right.

90. It has been shown that a number is multiplied by 10 by removing each figure one order to the left, which is done by annexing *one* cipher; by 100, by removing each figure *two* orders to the left, or annexing *two* ciphers, etc. (Art. 74.)

Hence, since division is the reverse of multiplication (Art. 87),

To divide by 10, *remove each figure one order to the right by removing the decimal point one order to the left.*

To divide by 100, *remove each figure two orders to the right by removing the decimal point two orders to the left,* etc. Thus,

2563 ÷ 10 is 256.3, or $256\frac{3}{10}$.
2563 ÷ 100 is 25.63, or $25\frac{63}{100}$.
2563 ÷ 1000 is 2.563, or $2\frac{563}{1000}$.

The decimal point in the result separates the *integer* of the quotient from the *fractional part.*

The first order on the right of the decimal point is tenths; the second order, hundredths; the third, thousandths.

Ex. 1. Divide 67325 by 1000.

$67325 \div 1000 =$
$67.325 = 67\frac{325}{1000}$

SOLUTION.—Remove the decimal point in the dividend three orders to the left, giving 67.325, or $67\frac{325}{1000}$, the quotient required.

2. Divide 36540 by 100. *Ans.* $365\frac{40}{100}$.
3. Divide 4632 by 10. *Ans.* $463\frac{2}{10}$.
4. How many thousands in 5328? *Ans.* $5\frac{328}{1000}$.
5. How many thousand feet, and how many feet over exact thousands, are 1634500 feet of boards?

Ans. 1634 thousand feet, and 500 feet over.

6. What is the quotient of 31638954 divided by 100000?

91. Rule for Division.—*Divide the least number of the left-hand orders of the dividend that will contain the divisor, and place the quotient at the right of the dividend in long division, and beneath the dividend in short division.*

Multiply the divisor by this quotient; subtract the result from that part of the dividend which was used; to the remainder annex the next order of the dividend, and divide the number thus formed. Proceed in like manner until all the dividend has been used.

If there be at last a remainder, write it, with the divisor under it, as a fractional part of the quotient.

When the divisor is 1, with one or more ciphers on the right, remove the decimal point in the dividend as many orders to the left as there are ciphers on the right of the divisor. The orders on the left of the point will be the integer of the quotient, and the orders on the right, the fractional part of it.

PROOF.—Multiply the integer of the quotient by the divisor, and add to the product the remainder, if any. If the work is correct, this result will equal the dividend.

PROBLEMS.

Divide—

1. 74317 by 6. *Ans.* $12386\frac{1}{6}$.
2. 10570 by 35. *Ans.* 302.
3. 40161 by 100.
4. 18312 by 24. *Ans.* 763.
5. 99031 by 62.
6. 13505 by 37. *Ans.* 365.
7. 47256 by 6.
8. 31990 by 10. *Ans.* 3199.
9. 71142 by 71. *Ans.* 1002.
10. 33443 by 8.
11. 2815 by 29. *Ans.* $97\frac{2}{29}$.
12. 4449 by 101.
13. 68750 by 25. *Ans.* 2750.
14. 42122 by 103.

How many times—

15. 21 in 1223? *Ans.* $58\frac{5}{21}$.
16. 43 in 34165?
17. 18 in 1499? *Ans.* $83\frac{5}{18}$.
18. 15 in 75850?
19. 16 in 3251? *Ans.* $203\frac{3}{16}$.

How much is—

20. One fifth of 95340?
21. One tenth of 9534?
22. One eighty-first of 973?
23. One nineteenth of 3640?
24. One thousandth of 31673?

25. When 112 muskets are worth 1344 dollars, what is the value of each? *Ans.* 12 dollars.

26. How many tons of coal, at 10 dollars per ton, can be bought with 13670 dollars?

27. $65120 \div 37 =$ what number? *Ans.* 1760.

28. $1651 \div 127 =$ what number?

29. $14150 \div 115 =$ what number? *Ans.* $123\frac{5}{115}$.

30. $13200 \div 48 =$ what number?

31. $208126 \div 345 =$ what number? *Ans.* $603\frac{91}{345}$.

32. How many acres of land at 100 dollars each can be bought for 25605 dollars?

33. A field of 101 acres yields 2125 bushels of grain. How much is the yield per acre? *Ans.* $21\frac{4}{101}$ bushels.

34. A man has 14250 dollars, which he wishes to invest in horses at 250 dollars each. How many horses can he purchase?

35. It is proposed to divide a tract of land containing 72000 acres into farms of 320 acres each. How many farms will it make? *Ans.* 225.

36. In an orchard there are 44520 trees in 212 equal rows. How many trees are there in each row?

37. If the dividend is 126072 and the divisor 612, what is the quotient?

38. The height of a mountain in Asia is 28176 feet, and the height of Mount Washington is 6634 feet. How many times as high as the latter is the former? *Ans.* $4\frac{1640}{6634}$.

TEST QUESTIONS.

92.—1. What is DIVISION? The dividend? The divisor? The quotient? What in division corresponds to the factors of the product in multiplication? What is the product? What, then, in division may be regarded as the factors of the dividend?

2. What is the SIGN of division? When written between two numbers what does it denote? In what other ways may division be denoted?

3. Recite the PRINCIPLES of division. Show that division is the reverse of multiplication. Show when the quotient will be an abstract number. When the quotient will be a concrete number.

4. What is a REMAINDER in division? Why are the remainder and dividend similar numbers? How may the remainder be changed to a fractional part of the quotient?

5. Recite the RULE for division. How does long division differ from short division? What is the reason for removing the decimal point to the left in dividing by 10, 100, etc.?

6. What is the PROOF of division? The reason for it? Since multiplication and division are the reverse of each other, how may multiplication be proved? Show that the product divided by the multiplier gives the multiplicand.

SECTION VIII.

REVIEW PROBLEMS.

MENTAL EXERCISES.

93.—Ex. 1. The factors of a product are 11 and 12. What is the product?

2. The product of 15 by 9 is what number?

3. If a boy can earn 18 dollars in one month, how many dollars can he earn in 10 months?

4. At 14 dollars a ton, what will 13 tons of hay cost?

SOLUTION.—If one ton cost 14 dollars, 13 tons will cost 13 times 14 dollars, or 182 dollars. The multiplication may be conveniently performed thus: 13 times 14 = 3 times 14 + 10 times 14; 3 times 14 = 42; 10 times 14 = 140; 42 + 140 = 182.

5. At 16 dollars each, what will 12 garments cost?

6. If 14 men can do a piece of work in 21 days, in how many days can one man do it?

7. John lives 5 miles from a certain place; Andrew, 7 miles farther away; and Benjamin, 8 times as far as Andrew. How far from the place does Benjamin live?

8. How many lengths of 12 feet each are there in a fence which is 132 feet long?

9. A and B start from points 121 miles apart, and travel toward each other, A at the rate of 5 miles per hour, and B at the rate of 6 miles per hour. In how many hours will they meet?

10. When flour is worth 8 dollars a barrel, how many barrels of flour will pay for 24 tons of coal at 5 dollars a ton?

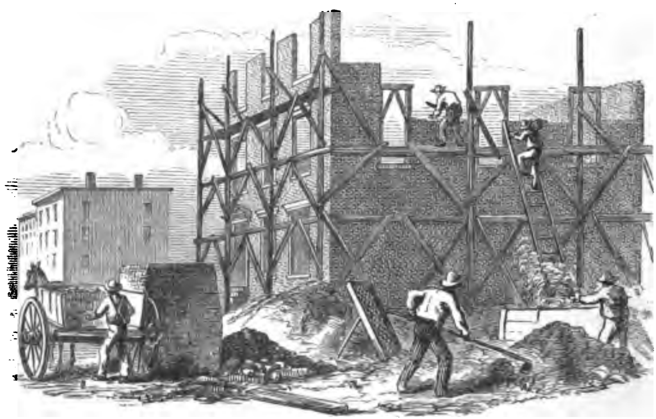
11. If 10 men can do a piece of work in 6 days, in how many days can 4 men do it?

SOLUTION.—If 10 men can do a piece of work in 6 days, 1 man can do it in 10 times 6 days, which are 60 days; and 4 men can do it in one fourth of 60 days, which is 15 days.

12. When 12 tons of coal at 7 dollars a ton pay for 21 pairs of boots, how much are the boots worth a pair?

13. If 8 cords of wood will buy 32 pairs of shoes, how many cords will buy 40 pairs of shoes?

14. I sold 5 dozen of eggs at the rate of 4 for 7 cents, and received 5 cents in money, and the balance in coffee at 25 cents a pound. How many pounds of coffee did I receive?



WRITTEN EXERCISES.

94.—Ex. 1. If in making mortar, 6 bushels of sand are required for each cask of lime used, how many bushels of sand will be required for 97 casks of lime? *Ans.* 582.

2. How many loads, each containing 1345 bricks, are there in a pile containing 286485 bricks?

3. If the front and rear walls of a house each require for their construction 31650 bricks, and the other two walls each require 43400 bricks, how many bricks are required for the four walls? *Ans.* 150100.

4. The factors of a product are 3043 and 405. What is that product? *Ans.* 1232415.

5. The product of two factors is 9225; one of the factors is 45. What is the other factor?

```
45)9225(205
   90
   ---
    225
    225
    ---
```

SOLUTION.—Since a product is the result obtained by multiplying one of two factors by the other, the quotient obtained by dividing the product by one of the factors must be the other factor. $9225 \div 45 = 205$, the factor required.

6. If a product is 9225 and the multiplicand 205, what is the multiplier?

7. If a man has 15000 dollars, and should spend enough of it to pay for a farm of 80 acres at 78 dollars per acre, how much would he have left?

8. If a merchant should purchase 1011 barrels of flour at 12 dollars a barrel, and pay down 7919 dollars, how much would he then owe for the flour? *Ans.* 4213 dollars.

9. The dividend is 15750 and the divisor 25. What is the quotient, or the other factor of the dividend? *Ans.* 630.

10. What is the average of 16, 22 and 28?

SOLUTION.—The *average* of *two* numbers is *one half* of the sum of those numbers; the average of *three* numbers is *one third* of their sum, etc. The sum of 16, 22 and 28 is 66, and one third of 66 is 22.

11. The elevation of the northern lakes above the sea is as follows: Superior, 627 feet; Michigan, 587 feet; Huron, 574 feet; and Ontario, 282 feet. What is their average elevation above the sea? *Ans.* $517\frac{1}{2}$ feet.

12. What is the value of $(14 + 6) + 16 \times 2 - (63 - \overline{19 + 4}) + 45 \div 9 - 3$?

$$(14 + 6) + 16 \times 2 - (63 - \overline{19 + 4}) + 45 \div 9 - 3$$
$$= 20 + 16 \times 2 - 40 + 45 \div 9 - 3$$
$$= 20 + 32 - 40 + 5 - 3; = 57 - 43; = 14$$

SOLUTION.—Combining first the numbers in the parentheses, we have 20 for the value of $(14 + 6)$, and 40 for the value of $(63 - \overline{19 + 4})$. Combining the numbers affected by the signs of multiplication and those by the sign of division, we have 32 for the value of 16×2, and 5 for $45 \div 9$. Then, combining the numbers as indicated by the signs of addition and subtraction, we have 14 as the value required.

13. What is the value of $4 + 6 \times 5 - 16 \div 8 - (4 \times 2)$?

14. Henry has 7375 dollars, which is 7 times as much as Daniel has, lacking 780 dollars. How many dollars has Daniel? *Ans.* 1165.

15. Two candidates at an election received in the aggregate 15653 votes. If one of them received 783 votes more than the other, how many votes did the other receive?

SECTION IX.

FACTORS AND DIVISORS.

95.—Ex. 1. Of what two integers is 6 the product?

2. What two integers multiplied together produce 15? What two produce 21?

3. What integers are factors of 6? Of 15? Of 21?

4. Of what three integers is 30 the continued product?

5. Of what sets of two integers is 30 the product?

$30 = 2 \times 15$, or 3×10, or 5×6.

6. Of what sets of integers greater than 1 is 24 the product?

7. Name some numbers that are the product of integers greater than 1.

8. What are the smallest integers greater than 1 that will divide 21 without a remainder? 30 without a remainder?

9. Give the sets of integers greater than 1 which, when multiplied together, will produce 30.

10. Of what number are 3 and 5 the factors? 2, 3 and 5 the factors?

DEFINITIONS.

96. The **Factors** of a number are the integers which being multiplied together will produce that number.

Thus, 2, 3 and 5 are the factors of 30; for $2 \times 3 \times 5 = 30$.

97. A **Prime Number** is an integer that has no factor except itself and 1.

Thus, 1, 3, 5, 7 and 11 are prime numbers.

98. A **Composite Number** is an integer that has other factors besides itself and 1.

Thus, 4 and 6 are composite numbers, since $4 = 2 \times 2$; and $6 = 2 \times 3$.

99. A **Prime Factor** is a factor which is a prime number.

The prime factor 1 is not commonly mentioned, since it is a factor of every integer.

Numbers are said to be *mutually prime*, or *prime to each other*, when they have no common factor except 1.

100. Factoring is the process of finding the factors of composite numbers.

The number of times a number is taken as a factor may be denoted by writing a small figure, called an *Exponent*, at the right and above the figure or figures of the factor.

Thus, $3^2 = 3 \times 3$, and denotes that 3 is taken twice as a factor.

$11^3 = 11 \times 11 \times 11$, and denotes that 11 is taken 3 times as a factor.

101. An **Exact Divisor** of a number is any integer which will divide the number without a remainder.

Thus, 1, 2, 3, 4, 6 and 12 are each exact divisors of 12.

The Exact Divisors of a number are called, also, *Divisors*, or *Measures*, of that number, and must be factors of it.

A number is said to be *divisible* by its exact divisors.

Thus, 12 is divisible by its exact divisors 1, 2, 3, 4, 6 and 12.

102. Any number is divisible by 2 when its right-hand figure is 0, 2, 4, 6, or 8.

For such numbers are composed of some exact number of twos.

Numbers divisible by 2 are *Even Numbers*, and all others are *Odd Numbers*.

103. A number is divisible by 4 if its tens and ones are divisible by 4.

For 4 is an exact divisor of 100, and of any number of hundreds; hence, if the tens and ones of a number are divisible by 4, the number itself must be.

Thus, 648 and 7312 are each divisible by 4.

104. A number is divisible by 5 if its right-hand figure is 0 or 5.

A number whose right-hand figure is 0 is an exact number of tens; a number whose right-hand figure is 5 is an exact number of tens plus 5. 5 is an exact divisor of 5 or 10; hence, any exact number of tens, or any exact number of tens plus 5, is divisible by 5.

Thus, 70 and 75 are each divisible by 5.

105. A number is divisible by 3 or 9 when the sum of the ones represented by its figures is divisible by 3 or 9.

Take, for example, the number 7542; $7542 = 7000 + 500 + 40 + 2$; and $7000 = 7$ times $999 + 7$; $500 = 5$ times $99 + 5$; $40 = 4$ times $9 + 4$; and $2 = 2$; where the figures expressing the number of each order plus the exact number of times 9 are the figures of the given number. Now, 7 times 999, 5 times 99 and 4 times 9, being each divisible by 9 and by 3, if the sum of the ones represented by the figures of the number are divisible by 9 or by 3, the number itself is thus divisible.

Thus, 7542 and 9765 are each divisible by 9 and by 3.

106. Principles.—1. *Every number is equal to the product of all its prime factors.*

2. *A number is divisible by all its prime factors, and by all the products of two or more of them, and is divisible by no other numbers.*

WRITTEN EXERCISES.

107.—Ex. 1. What are the prime factors of 70?

$$\begin{array}{r} 2\underline{)70} \\ 5\underline{)35} \\ 7 \end{array}$$

Proof, $2 \times 5 \times 7 = 70$

SOLUTION. — Since the right-hand figure is 0, we can divide by the prime numbers 2 and 5. (Arts. 102 and 104.)

Dividing by these prime numbers gives for a quotient 7, which is also prime. Hence, the prime factors of 70 are 2, 5 and 7.

2. What are all the factors or divisors of 66?

$$\begin{array}{r} 2\underline{)66} \\ 3\underline{)33} \\ 11 \end{array} \qquad \begin{array}{r} 2,\ 3,\ 11 \\ 3 \times 2 = 6 \\ 11 \times 2 = 22 \\ 11 \times 3 = 33 \\ 11 \times 3 \times 2 = 66 \end{array}$$

SOLUTION.—Since every prime factor of a number, and every product of two or more of these prime factors, is an exact divisor of the number, and no other numbers can be exact divisors of that number (Art. 106—2), the prime factors 2, 3 and 11, and the products of 3 by 2, 11 by 2, 11 by 3, and 11 by 2 times 3, must be all the factors or exact divisors of 66.

Hence, 2, 3, 11, 6, 22, 33 and 66 are the factors and divisors required.

3. What are the prime factors of 84? *Ans.* 2^2, 3 and 7.

4. What are all the factors or divisors of 56?

Ans. 2^3, 7, 4, 8, 14, 28, 56.

108. Rule for Factoring.—*Divide the given number by any of its prime factors greater than 1. Divide the quotient, if composite, in like manner, and so proceed* *until the last quotient is a prime number. The last quotient and the several divisors will be the prime factors of the number.*

The prime factors of a given number, and the various products of these factors, by taking two together, three together, etc., are all the different factors or exact divisors of that number.

PROBLEMS.

What are the prime factors of—

1. 75?	*Ans.* 3, 5, 5.	7. 99?	*Ans.* 3^2, 11.
2. 144?	*Ans.* 2^4, 3^2.	8. 231?	*Ans.* 3, 7, 11.
3. 116?		9. 875?	
4. 340?	*Ans.* 2^2, 5, 17.	10. 1110?	*Ans.* 2, 3, 5, 37.
5. 180?		11. 4004?	
6. 3809?	*Ans.* 13, 293.	12. 6783?	*Ans.* 3, 7, 17, 19.

What are all the different factors or divisors of—

13. 70?	*Ans.* 2, 5, 7, 10, 14, 35 and 70.	16. 42?	*Ans.* 2, 3, 6, 7, 14, 21 and 42.
14. 30?		17. 63?	
15. 56?	*Ans.* 2, 4, 7, 8, 14, 28 and 56.	18. 105?	*Ans.* 3, 5, 7, 15, 21, 35 and 105.

19. How many of the different factors or divisors of 100 are prime, and how many are composite?

Ans. Three are prime and six are composite.

20. How many of the different factors or divisors of 210 are prime, and how many are composite?

TEST QUESTIONS.

109.—1. What are the FACTORS of a number? What is a prime number? Name some number that is the product of two prime numbers. What is a composite number? Give an example of a composite number.

2. What is a PRIME FACTOR? Name a prime factor of 30. A composite factor of 30. When are two numbers prime to each other?

3. What is an EXACT DIVISOR of a number? How do even numbers differ from odd numbers? Of what are divisors or measures of a number factors?

4. By what is a number said to be DIVISIBLE? How can you know that a number is divisible by 2? That a number is divisible by 4? By 5? By 9 or 3?

5. What is FACTORING? How do you denote the number of times a factor is taken? How do you find all the factors of a number?

6. What are PRINCIPLES of factoring? Show that 18 is the product of all its prime factors. Show that 18 is divisible by the various products of its prime factors.

7. What is the RULE for factoring? In factoring an even number why do you divide by 2? In factoring numbers why do you divide by their prime factors in succession?

SECTION X.

COMMON DIVISORS.

110.—Ex. 1. What numbers are exact divisors of 15? Of 20?

2. What number is an exact divisor of both 15 and 20?

3. What is the unit of 15? Of 20? Of the exact divisor of both 15 and 20?

4. What numbers are exact divisors of 33 dollars?

5. What number is an exact divisor of both 22 dollars and 33 dollars?

6. What is the unit of 22 dollars? Of 33 dollars? Of the exact divisors of both 22 dollars and 33 dollars?

7. What are the exact divisors common to 12 and 18?

8. What prime factors have 12 and 18 in common? What is the product of those factors?

9. What is the greatest exact divisor common to 12 and 18? To 8 and 20?

10. What exact divisor is common to 5 and 3 times 5?

11. What exact divisors are common to 6 and 7 times 6?

12. What exact divisors are common to 6 and 42? To 6 and 42 and their sum? To 6 and 42 and their difference?

DEFINITIONS.

111. A **Common Divisor,** or **Common Measure,** of two or more numbers is any exact divisor (Art. 101) of each of those numbers.

Thus, 2 is a common divisor of 8, 10 and 18.

Only similar numbers can have a common divisor, since one number to measure another must have the same unit.

112. The **Greatest Common Divisor,** or **Greatest Common Measure,** of two or more numbers is the greatest exact divisor of each of them.

Thus, 6 is the greatest common divisor of 12, 24 and 42.

Numbers that are prime to each other have no common divisor, for they can have no common factor greater than 1.

113. Principles.—1. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

2. *A divisor of a number is a divisor of any integral number of times that number.*

3. *A common divisor of two or more numbers is also a divisor of their sum and of their difference.*

CASE I.

Common Divisor.

114.—Ex. 1. What is a common divisor of 14 and 20.

$$\begin{array}{r|l} 2 & 14,\ 20 \\ \hline & 7,\ 10 \end{array}$$

SOLUTION.—2 is an exact divisor of any number whose right-hand figure is 4 or 0 (Art. 102); hence 2 is an exact divisor of both 14 and 20, and therefore a common divisor of the given numbers.

2. What are all the common divisors of 45 and 90?

$$45 = 3 \times 3 \times 5$$
$$90 = 3 \times 3 \times 5 \times 2$$

or,

$$\begin{array}{r|l} 3 & 45,\ 90 \\ \hline 3 & 15,\ 30 \\ \hline 5 & 5,\ 10 \\ \hline & 1,\ 2 \end{array}$$

$$3 \times 3 = 9$$
$$3 \times 5 = 15$$
$$3 \times 3 \times 5 = 45$$

SOLUTION.—By factoring we find the common prime factors of 45 and 90 to be 3, 3 and 5; hence 3, 3 and 5 are common divisors of 45 and 90.

Since every product of two or more of the prime factors must be a divisor, the various products that can be formed by 3, 3 and 5 must likewise be common divisors of the given numbers. These products are 3×3, or 9; 5×3, or 15; and $5 \times 3 \times 3$, or 45. Hence, 3, 5, 9, 15 and 45 are the common divisors required.

3. What is a common divisor of 27 and 39? *Ans.* 3.

4. What are all the common divisors of 45 and 75? *Ans.* 3, 5 and 15.

115. Rule for finding Common Divisors.—*Find the prime factors of the given numbers. All common factors are common divisors, and all the various products of those factors are all the common divisors.*

PROBLEMS.

1. What is a common divisor of 25 and 35? *Ans.* 5.

2. What are all the different common divisors of 27 and 81?

3. What are the common composite factors or divisors of 24, 72 and 84? *Ans.* 4, 6 and 12.

4. What are all the common divisors of 105, 210, 315. *Ans.* 3, 5, 7, 15, 21, 35 and 105.

CASE II.

Greatest Common Divisors.

116.—Ex. 1. What is the greatest common divisor of 12, 18 and 78?

$$12 = 2 \times 3 \times 2$$
$$18 = 2 \times 3 \times 3$$
$$78 = 2 \times 3 \times 13$$
$$\textit{Greatest Com. Div.} = 2 \times 3 = 6$$

Or,

$$\begin{array}{r|l} 2) & 12,\ 18,\ 78 \\ \hline 3) & 6,\ 9,\ 39 \\ \hline & 2,\ 3,\ 13 \end{array}$$

SOLUTION.—By factoring we find the prime factors common to 12, 18 and 78 are 2 and 3.

Since the product of all the common prime factors is the greatest common divisor (Art. 113), 2×3, or 6, must be that divisor.

Or, dividing by 2, we take out that common factor. Dividing the resulting quotients by 3, we take out that common factor and obtain quotients that are prime to each other. Hence 2 and 3 are all the factors common to the given numbers, and their product, 2×3, or 6, must be the greatest common divisor required.

2. What is the greatest common divisor of 91 and 133?

```
91)133(1
   91
   --
   42)91(2
      84
      --
       7)42(6
         42
         --
```

SOLUTION.—Since any number is the greatest divisor of itself, if 91 be a divisor of 133, it must be the greatest common divisor of 91 and 133. We find that it is not a divisor of 133, since on trial 42 remains.

If 42 be a divisor of 91, it is also a divisor of 133, which is once 91, plus 42. (Art. 113—2.) It is not a divisor of 91, since on trial 7 remains.

If 7 is a divisor of 42, it must be also a divisor of 91, which is twice 42, plus 7. On trial it is found to be a divisor of 42.

Now, 7 is the greatest divisor of 7 and 42; hence, 7 is the greatest common divisor of 42 and 91, and therefore 7 is the greatest common divisor of 91 and 133.

3. What is the greatest common divisor of 84 and 132?

117. Rules for finding the Greatest Common Divisor.—*1. Find the prime factors common to the given numbers, and the product of those factors will be the greatest common divisor required. Or,*

2. Divide the greater number by the less, and if there be a remainder, divide the divisor by it, and so continue to divide the last divisor by the last remainder till an exact divisor is found. That divisor will be the greatest common divisor of the two numbers.

3. If more than two numbers are given, find first the greatest common divisor of two of them, and then the greatest common divisor of that divisor and another of the numbers, and so on till all the numbers have been used. The last common divisor will be the greatest common divisor of all the numbers.

PROBLEMS.

What is the greatest common divisor of—

1. 32, 48 and 80? *Ans.* 16.
2. 75 and 165?
3. 72 and 168? *Ans.* 24.
4. 308 and 630? *Ans.* 14.
5. 91 and 117?
6. 21, 30, 39 and 81? *Ans.* 3.

7. I have three rooms, the first of which is 16 feet wide; the second, 20 feet; and the third, 24 feet. What must be the width of carpeting which will exactly fit each room?

Ans. 4 feet.

8. There is a garden 84 feet wide and 1068 feet long. What must be the length of the longest rails that will, without cutting, exactly enclose it? *Ans.* 12 feet.

9. Find the greatest common divisor of 99 bushels, 261 bushels and 504 bushels. *Ans.* 9 bushels.

10. James has 66 dollars, Edward has 77 dollars, and Arthur has 264 dollars. If they should purchase flour at the highest price per barrel that would allow each to exactly use his money, how much would the flour cost per barrel?

Ans. 11 dollars.

SECTION XI.

MULTIPLES.

118.—Ex. 1. What number is 7 times 6? 5 times 6?

2. What number is some integral number of times 6?

3. What prime factors have 6 and 7 times 6 in common? 6 and 5 times 6?

4. What prime factors have 6 and 42 in common? 6 and 30?

5. What two numbers are contained in 6 an exact number of times? Of what two numbers is 6 an exact number of times?

6. Of what two numbers is 15 an exact number of times? Of what number are those two numbers prime factors?

7. What numbers from 5 to 30 contain 5 an integral number of times?

8. What numbers from 6 to 24 contain both 2 and 3 an integral number of times?

9. What is the least number that contains both 2 and 3 an integral number of times?

10. What is the least number that is an integral number of times 3 and 4? What are the prime factors of 3 and 4? Of 12?

11. What is the least number that is an integral number of times 10 and 6? What are the prime factors of 10 and 6 which when taken the least number of times will form those numbers? What are the prime factors of 30?

DEFINITIONS.

119. A **Multiple** of a number is any integral number of times that number.

Thus, 10, which is twice 5, is a multiple of 5.

120. A **Common Multiple** of two or more numbers is any number which is an integral number of times each of them.

Thus, 12, which is 3 times 4, is a common multiple of 3 and 4.

121. The **Least Common Multiple** of two or more numbers is the least number which is an integral number of times each of them.

Thus, 6 is the least common multiple of 2 and 3.

Only similar numbers can have a common multiple, since numbers must be similar to be factors of the same product.

122. Principles.—1. *A multiple of a number contains all the prime factors of that number.*

2. *A common multiple of two or more numbers contains all the prime factors of those numbers.*

3. *The least common multiple of two or more numbers is the least number that contains all the prime factors of those numbers.*

CASE I.

Common Multiples.

123. Ex. 1.—Find a common multiple of 9 and 11.

$11 \times 9 = 99$ SOLUTION.—Since a common multiple of the given numbers must be a number which contains those numbers as factors, their product, or 99, is a common multiple.

2. Find a common multiple of 5, 7 and 9. *Ans.* 315.

124. Rule for finding a Common Multiple.—*Multiply the given numbers together, and the product will be a common multiple of those numbers.*

6

PROBLEMS.

1. What is a common multiple of 3, 5 and 6? *Ans.* 90.
2. Find a common multiple of 11 and 13.
3. What is a common multiple of 7, 10 and 25? *Ans.* 1750.

CASE II.

Least Common Multiples.

125. Ex. 1.—Find the least common multiple of 4, 12 and 30.

$$4 = 2 \times 2$$
$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$2 \times 2 \times 3 \times 5 = 60$$

SOLUTION.—A multiple of 4 must contain its prime factors 2 and 2; a multiple of 12 must contain the additional prime factor 3; and a multiple of 30 must contain the additional prime factor 5.

2, 2, 3 and 5 are all the prime factors of the numbers; hence, the product of these factors, or 60, is their least common multiple.

2. Find the least common multiple of 42, 49 and 70.

$$\begin{array}{r|l} 7 & 42,\ 49,\ 70 \\ \hline 2 & 6,\ \ 7,\ 10 \\ \hline & 3,\ \ 7,\ \ 5 \end{array}$$
$$7 \times 2 \times 3 \times 7 \times 5 = 1470$$

SOLUTION.—By division we take out the prime factor 7, common to the given numbers, and have left the factors 6, 7 and 10.

We take out the prime factor 2, common to 6 and 10, and have left the factors 3, 7 and 5, which have no factor common to any two of them.

Hence, 7, 2, 3, 7 and 5 are all the prime factors of the numbers, and their product, 1470, is the least common multiple required.

3. Find the least common multiple of 8, 11 and 15. *Ans.* 1320.

126. Rules for finding the Least Common Multiple.—*1. Find the prime factors of the given numbers, and the product of the different prime factors; each factor being taken the greatest number of times it occurs in any of the numbers, will be the least common multiple. Or,*

2. Place the given numbers in a horizontal line;

divide by any prime number that is a factor of two or more of them, and write the quotients and undivided numbers below. Divide these, if possible, in like manner, and so continue until the quotients and undivided numbers are prime to each other. The product of the divisors and the numbers remaining in the last horizontal line will be the least common multiple.

PROBLEMS.

What is the least common multiple of—

1. 24, 15 and 16? *Ans.* 240.	4. 13 and 29? *Ans.* 377.
2. 42 and 56? *Ans.* 168.	5. 18, 27 and 30?
3. 9, 11 and 48?	6. 60, 50 and 35? *Ans.* 2100.

7. Find the least common multiple of 40, 36, 32, 30, 28, 24, 20 and 18.

40, 36, 32, 30, 28, 24, 20, 18

$40 \times 9 \times 4 \times 7 = 10080$

SOLUTION.—Write 40 as a factor of the answer. The largest factor common to 40 and 36 is 4; write 9, the remaining factor of 36, as a factor of the answer.

The largest factor common to 40 and 32 is 8; write 4, the remaining factor of 32, as a factor of the answer.

The largest factor common to 40 and 30 is 10; and 3, the remaining factor of 30, is found in 9.

The largest factor common to 40 and 28 is 4; write 7, the remaining factor of 28, as a factor of the answer.

The largest factor common to 40 and 24 is 8; and 3, the remaining factor of 24, is found in 9.

The factors of 20 are found in 40, and the factors of 18 in 36. The continued product of the factors of the answer is 10080.

8. Find the least common multiple of 8, 10, 11, 90 and 132.

9. What is the least common multiple of 7, 16, 21 and 28?

10. Find the smallest number that will exactly contain 9, 15, 18 and 20. *Ans.* 180.

11. What is the least number of cents with which you may purchase either slates at 18 cents or arithmetics at 63 cents each? *Ans.* 126.

12. What is the smallest sum of money for which I can purchase calves at 17 dollars each, yearlings at 34 dollars each, or cows at 68 dollars each? *Ans.* 68 dollars.

13. What is the least number of acres that can be exactly divided into lots of 12 acres, 15 acres or 16 acres each? *Ans.* 240.

TEST QUESTIONS.

127.—1. What is a COMMON DIVISOR of two or more numbers? Why cannot 4 dollars and 6 yards have a common divisor?

What is the greatest common divisor of two or more numbers? What numbers have no common divisor?

2. What is the PRINCIPLE in relation to the greatest common divisor of numbers? Of what is the divisor of a number a divisor? Of what is a common divisor of two or more numbers also a divisor?

3. What is the RULE for finding the common divisor of two or more numbers? For finding the greatest common divisor of two or more numbers?

4. What is a MULTIPLE of a number? A common multiple of two or more numbers? The least common multiple of two or more numbers?

5. What is the PRINCIPLE in relation to a multiple of a number? In relation to a multiple of two or more numbers? In relation to the least common multiple of two or more numbers?

6. What is the RULE for finding a common multiple? For finding the least common multiple of two or more numbers? When numbers are prime to each other, how is their least common multiple found?

SECTION XII.

FACTORS IN DIVISION.

128. The **Value** of a quotient in division depends upon the relative values of dividend and divisor. Hence,

Any change in the factors of the dividend or divisor must affect the value of the quotient.

Thus, $24 \div 6 = 4$;

and $(24 \times 2) \div 6 = 8$, or, $24 \div (6 \div 2) = 8$;

also, $(24 \div 2) \div 6 = 2$, or, $24 \div (6 \times 2) = 2$.

The same change in the factors of both dividend and divisor does not affect the value of the quotient.

Thus, $(24 \div 2) \div (6 \div 2) = 4$; or, $(24 \times 2) \div (6 \times 2) = 4$.

129. General Principles of Division.—1. *Multiplying the dividend, or dividing the divisor, multiplies the quotient.*

2. *Dividing the dividend, or multiplying the divisor, divides the quotient.*

3. *Dividing or multiplying both dividend and divisor by the same number does not change the quotient.*

CASE I.

Division by Factors.

130. Ex. 1. Divide 9702 by 21, using the factors of 21.

3)9702
7)3234
462

SOLUTION.—The factors of 21 are 3 and 7. Since 21 times a number is 7 times 3 times the number, one twenty-first of a number must be one seventh of one third of that number.

One third of 9702, the dividend, is 3234, and one seventh of 3234 is 462.

2. Divide 4677 by 45, using factors.

5)4677
9) 935, 2 ones = 2
103, 8 fives = 40
True Remainder, 42

SOLUTION.—45 is equal to 5×9.

Dividing 4677 by 5, we have 935 *fives*, and 2 *ones* as a remainder.

Dividing by 9, we have 103 forty-fives, and 8 fives as a remainder.

The first partial remainder is 2 ones, or 2, and the second partial remainder 8 fives, or 40; hence, $2 + 40$, or 42, is the whole or true remainder, and $103\frac{42}{45}$ is the quotient required.

The factors 5, 3 and 3 could have been used with the same result.

3. Divide 825 by 36, using factors. *Ans.* $22\frac{33}{36}$.

131. Rule for Dividing by Factors.—*Find any convenient set of factors of the divisor; divide the dividend by one of these factors, and the quotient thus obtained by another, and so on, till all the factors are used.*

If there be one or more remainders, multiply each by the divisors preceding the one that produced it, and add the products and the remainder, if any, from the first division. The sum will be the true remainder.

PROBLEMS.

Divide, using factors—

1. 2954 by 14. *Ans.* 211.
2. 3728 by 28. *Ans.* $133\frac{1}{7}$.
3. 8316 by 27.
4. 88763 by 32. *Ans.* $2773\frac{27}{32}$.
5. 47839 by 42. *Ans.* $1139\frac{1}{42}$.
6. 11630 by 81.
7. 2520 by 105. *Ans.* 24.
8. 196473 by 72. *Ans.* $2728\frac{57}{72}$.

9. Divide 94596 by 2300, using the factors 23 and 100.

$$
\begin{array}{l}
23|\,00)945|\,96(41\frac{296}{2300} \\
\quad\;\; \underline{92} \\
\quad\;\;\; 25 \\
\quad\;\;\; \underline{23} \\
\quad\;\;\;\; 2
\end{array}
$$

SOLUTION.—Dividing by 100, we have 945 *hundreds*, and 96 ones as a remainder.

Dividing by 23, we have 41 *twenty-three hundreds*, and 2 hundreds as a remainder.

2 hundreds + 96 ones = 296, the true remainder, and $41\frac{296}{2300}$ is the quotient required.

In the computation we denote, for brevity, the factoring of the divisor by cutting off the two ciphers by a mark, and in the division of the dividend by 100 we set off the remainder by the same mark.

10. Divide 782967 by 3700, using the factors 37 and 100. *Ans.* $211\frac{2267}{3700}$.

11. Divide 46370 by 90, using the factors 9 and 10.

12. What is the quotient of 345600 divided by 5000? *Ans.* $69\frac{600}{5000}$.

13. Divide 16632 by 5148, using the factors 11, 18 and 26.

14. If 700 barrels of apples cost 2100 dollars, how much will one barrel cost? *Ans.* 3 dollars.

15. If 63 bushels of wheat make one load, how many full loads can be made from 1937 bushels, and how many bushels will remain? *Ans.* 30 full loads, and 47 bushels over.

CASE II.

Cancellation.

132. Cancellation is the process of shortening computations by striking out equal factors from the dividend and divisor, and using only the remaining factors.

133.—Ex. 1. Divide 11 × 3 × 2 by 11 × 2.

$$\frac{\overset{1}{\cancel{11}} \times 3 \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{11}} \times \underset{1}{\cancel{2}}} = \frac{1 \times 3 \times 1}{1 \times 1} = 3$$

Solution. — Indicate the division by writing the dividend over the divisor.

Divide both dividend and divisor by the factors 11 and 2, by cancelling those common factors in both, which does not change the quotient. (Art. 129—3.)

When a factor is cancelled, 1 remains, and if not written is understood.

2. Divide 15 × 6 × 7 by 10 × 18.

$$\frac{\overset{3}{\cancel{15}} \times \cancel{6} \times 7}{\underset{2}{\cancel{10}} \times \cancel{18}} = 3\frac{1}{2}$$

Solution.—Cancelling the factor 5, common to both dividend and divisor, we have in the dividend 3 in place of 15, and in the divisor 2 in place of 10. We next cancel 3 × 6, or 18, common to both dividend and divisor, and have left $\frac{7}{2}$, or $3\frac{1}{2}$.

3. Divide 11 × 9 × 8 by 11 × 3 × 2. *Ans.* 12.

134. Rule for Cancellation.—*Cancel in the dividend and divisor all factors common to both, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

PROBLEMS.

1. Divide 35 × 8 × 3 by 12 × 7 × 5.
2. Divide 42 × 15 × 80 by 75 × 24. *Ans.* 28.
3. Divide 96 × 63 × 5 by 72 × 35.
4. Divide 108 × 77 × 2 by 18 × 26 × 11. *Ans.* $3\frac{3}{13}$.
5. Divide 65 × 16 × 33 by 26 × 22 × 8.
6. How many are (300 × 45 × 6) ÷ (150 × 30 × 18)?
7. How many barrels of beef, at 21 dollars a barrel, are worth as much as 14 tons of coal, at 6 dollars a ton? *Ans.* 4.
8. How many bushels of corn, at 90 cents a bushel, will pay for 120 yards of cloth, at 15 cents a yard?
9. I exchanged 90 bushels of potatoes, at 75 cents a bushel, for tubs of butter containing 54 pounds each, at 25 cents a pound. How many tubs of butter did I receive? *Ans.* 5.

SECTION XIII.

ANALYSIS.

135.—Ex. 1. If 24 bushels of wheat cost 72 dollars, what will 5 bushels cost?

SOLUTION.—If 24 bushels cost 72 dollars, 1 bushel will cost one twenty-fourth of 72 dollars, which is 3 dollars. If 1 bushel cost 3 dollars, 5 bushels will cost 5 times 3 dollars, which are 15 dollars.

2. If 15 men can earn 75 dollars in a given time, how much can 7 men earn in the same time?

3. When 7 hats cost 35 dollars, how much will 15 hats cost?

4. If 7 hats cost 35 dollars, how many hats will cost 75 dollars?

SOLUTION.—If 7 hats cost 35 dollars, 1 hat will cost $\frac{1}{7}$ of 35 dollars, which is 5 dollars, and as many hats will cost 75 dollars as 5 dollars are contained times in 75 dollars, which are 15.

5. If 4 persons require 48 dollars' worth of provisions in a certain time, how many persons will require 108 dollars' worth in the same time?

6. When 13 bushels of corn are worth as much as 39 bushels of oats, how many bushels of corn are worth as much as 27 bushels of oats?

7. If 17 men can earn as much in one day as 51 boys, who are each paid 50 cents a day, how much can 1 man earn in a day?

8. If 9 barrels of flour are worth as much as 36 cords of wood, how many cords of wood are worth as much as 7 barrels of flour?

9. When 28 cords of wood are worth as much as 7 barrels of flour, how many cords of wood are worth as much as 9 barrels of flour?

DEFINITION.

136. Analysis, in Arithmetic, is the process of stating, in regular order, the reasons for each step in the solution of a problem whose conditions require several computations.

WRITTEN EXERCISES.

Ex. 1. If 12 horses cost 2100 dollars, how much will 21 horses cost at the same rate?

$$\begin{array}{r} 12)\underline{2100}\text{ dollars.} \\ 175 \\ \underline{21} \\ 175 \\ \underline{350} \\ 3675\text{ dollars.} \end{array}$$

Or,

$$\frac{\overset{525}{\cancel{2100}} \times \overset{7}{\cancel{21}}}{\underset{4}{\cancel{12}}} = 3675$$

SOLUTION BY ANALYSIS.—If 12 horses cost 2100 dollars, 1 horse will cost one twelfth of 2100 dollars, which is 175 dollars.

If 1 horse cost 175 dollars, 21 horses will cost 21 times 175 dollars, which are 3675 dollars.

SOLUTION BY CANCELLATION.—If 12 horses cost 2100 dollars, 1 horse will cost one twelfth of 2100 dollars, and 21 horses will cost 21 times as much. Indicating the work, and cancelling, we have 3675, the same result.

2. When 45 tons of coal cost 270 dollars, what will 60 tons cost? *Ans.* 360 dollars.

3. If 180 quarts of oats be sufficient for 30 horses for a certain time, how many quarts will be sufficient for 63 horses for the same time?

4. If 15 men can do a piece of work in 54 days, in what time can 9 men do the same?

5. What time will 48 men require to mow a field that 10 men can mow in 32 days? *Ans.* $6\frac{2}{3}$ days.

6. A cistern can be filled in 265 minutes by 5 equal pipes running into it. In what time could it be filled by 25 such pipes? *Ans.* 53 minutes.

TEST QUESTIONS.

137.—1. Upon what does the VALUE of a quotient depend? What effect upon the quotient is produced by multiplying the dividend or dividing the divisor? What effect by dividing the dividend or multiplying the divisor? What changes in both dividend and divisor do not affect the value of the quotient?

2. How do you DIVIDE BY USING FACTORS? If there be one or more remainders, how do you find the true remainder?

3. What is CANCELLATION? The rule for cancellation? Analysis?

SECTION XIV.

REVIEW PROBLEMS.

MENTAL EXERCISES.

138.—Ex. 1. Of what numbers are 2, 3 and 11 the prime factors?

2. What are all the exact divisors of 66?

3. Name the composite numbers from 2 to 30. From 30 to 45.

4. Five 20's are how many 4's? Seven 12's are how many 14's?

5. I sold 9 oranges at 8 cents apiece, and spent the money for melons at 18 cents each. How many melons did I purchase?

6. What common factors have 18 and 42?

7. What is the greatest number that will exactly divide 27 and 63?

8. When cheese is 15 cents a pound, and butter 35 cents, what is the least number of pounds of cheese that can be exchanged for an exact number of pounds of butter?

9. When butter is 45 cents a pound, how many pounds of sugar, at 18 cents a pound, will cost as much as 2 pounds of butter?

10. If 5 men can perform a piece of work in 14 days, in what time can 7 men do it?

11. If 8 men can do a piece of work in 15 days, how many men can do it in 12 days?

12. When coffee is 35 cents a pound, how many pounds of sugar, at 14 cents a pound, will cost as much as 2 pounds of coffee?

13. If 6 men can do a piece of work in 14 days, how many men can do it in 7 days?

14. I sold 11 pears at 6 cents each, and bought with the money, oranges at 3 cents each. How many oranges did I buy?

WRITTEN EXERCISES.

139.—Ex. 1. What are the prime factors of 56?

2. Which of the numbers 31, 98 and 101 are prime numbers?

3. Find the greatest common divisor of 4165 and 686.

4. Which of the numbers 700, 575 and 335 have 25 for an exact divisor?

5. Find all the exact divisors of 90.

6. What is the least common multiple of 8, 11 and 13?
Ans. 1144.

7. What exponent will denote the number of times that 5 is taken as a factor in 3125?

8. I have 14 bushels of oats, 22 bushels of rye and 24 bushels of corn. What is the capacity of the largest sacks, of equal size, into which the whole may be put without mixing?
Ans. 2 bushels.

9. Three men start at the same time and place to walk in the same direction round a circle. A can make the circuit in 5 hours, B in 8 hours, and C in 10 hours. In what time after they start will they all be together again at the point of starting? *Ans.* 40 hours.

10. If 1350 men can build a bridge in 30 days, in what time can 1500 men build it?

11. When the cost of 42 pounds of rice is 294 cents, what is the cost of 28 pounds? *Ans.* 196 cents

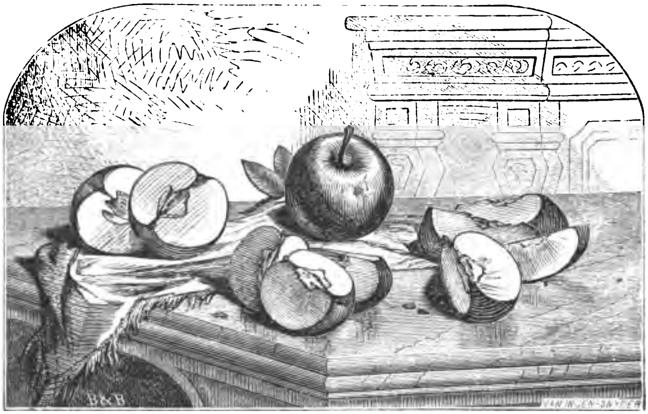
12. How many tons of iron can be bought for 95285 dollars, if 50 tons can be bought for 4250 dollars? *Ans.* 1121.

13. If 27 men can earn 2187 dollars in one month, how many men can earn 5832 dollars in the same time?
Ans. 72.

14. If 18 men can sow a field of oats in 12 days, how long will it take 48 men to sow a field of the same size?

15. How many pounds of tea, at 80 cents a pound, are equal in value to 6 bushels of wheat, worth 280 cents a bushel?

16. I exchanged 15 pieces of cloth, each containing 30 yards worth 14 cents per yard, for 21 barrels of apples, each containing 3 bushels. How much did the apples cost me per bushel? *Ans.* 100 cents.



SECTION XV.

COMMON FRACTIONS.

140.—Ex. 1. If an apple be cut into two equal parts, what part of the apple will one of the pieces be?

2. What is one half of an apple, or one half of anything? One third of an apple, or one third of anything?

3. One of the three equal parts of an apple is what part of the whole? Two of the three equal parts are what part of the whole?

4. What is one fourth of anything? Two fourths? Three fourths?

5. How many halves in a single thing or unit? How many thirds? How many fourths?

6. What is one fifth of a unit? One sixth? One seventh? One eighth?

7. What are two fifths of anything? Three fifths? Four fifths?

8. Which is the greater—a half or a third? A third or a fourth? A third or a fifth?

9. How are halves expressed by figures? Thirds? Fourths? Fifths?

10. What does $\frac{1}{3}$ signify? What does $\frac{2}{3}$ signify?

11. In $\frac{3}{4}$, what expresses the number of parts into which a unit has been divided? What expresses the number of parts taken?

DEFINITIONS.

141. A **Fraction** is a number which expresses one or more of the equal parts into which a unit is divided.

Thus, one half, two thirds, five fourths, etc., are fractions.

142. The **Unit of the Fraction** is the unit, or whole thing, which is considered as divided into parts.

Thus, the unit of the fraction of an apple is one apple; the unit of the fraction of a dollar is one dollar, etc.

When no particular unit is named, the abstract unit 1 is understood.

Thus, halves, thirds, etc., are understood to be halves, thirds, etc., of 1.

143. A **Fractional Unit** is one of the equal parts of the unit of the fraction.

Thus, one half, one third, etc., is the fractional unit of halves, thirds, etc.

144. The **Denominator** of a fraction is the number which denominates or names the parts of the unit.

Thus, four is the denominator of $\frac{3}{4}$.

145. The **Numerator** of a fraction is the number which numerates or numbers the fractional units taken.

Thus, three is the numerator of $\frac{3}{4}$.

146. The **Terms** of a fraction are its numerator and denominator.

147. A **Common Fraction** is an expression of any number of parts of a unit, written by placing the numerator above the denominator, with a line between them.

Thus, $\frac{2}{5}$, which is an expression of two fifths, is a common fraction.

148. A **Proper Fraction** is one whose numerator is less than its denominator; and an **Improper Fraction** is one whose numerator is not less than its denominator.

Thus, $\frac{3}{4}$, $\frac{4}{5}$, etc., are proper fractions, and $\frac{4}{4}$, $\frac{5}{4}$, etc., are improper fractions.

149. A **Mixed Number** is a number expressed by an integer and a fraction; as, $5\frac{2}{3}$.

150. **Similar Fractions** are such as have the same denominator; as, $\frac{5}{8}$, $\frac{7}{8}$, etc.

151. **Dissimilar Fractions** are such as have different denominators; as, $\frac{5}{7}$, $\frac{4}{9}$, etc.

152. An **Integer** may be expressed fractionally by writing it as a numerator, with 1 as its denominator.

Thus, 3 may be expressed $\frac{3}{1}$, and be read *three ones*, or *three.*

153. Fractions are read by pronouncing the number in the numerator, and then naming the parts denoted by the denominator.

Thus, $\frac{1}{2}$ is read *one half;* $\frac{2}{3}$, *two thirds;* $\frac{3}{21}$, *three twenty-firsts*, etc.

154. Fractions may not only be regarded as a number of parts of a unit, but as another method of indicating division.

Thus, $\frac{3}{5}$ may be regarded as $\frac{3}{5}$ of 1; $\frac{1}{5}$ of 3; or 3 divided by 5.

155. Principles.—1. *The value of a fraction is the quotient obtained by dividing the numerator by the denominator.*

2. *The value of a fraction is less than* 1 *when the numerator is less than the denominator.*

3. *The value of a fraction equals or exceeds* 1 *when the numerator equals or exceeds the denominator.*

EXERCISES.

156. Name the kind of fraction, and read—

1. $\frac{3}{7}$.	4. $\frac{8}{9}$.	7. $\frac{20}{21}$.	10. $\frac{18}{18}$.
2. $\frac{6}{5}$.	5. $\frac{13}{14}$.	8. $\frac{17}{42}$.	11. $\frac{11}{27}$.
3. $2\frac{1}{2}$.	6. $\frac{17}{100}$.	9. $19\frac{5}{9}$.	12. $36\frac{53}{281}$.

Write in figures—

13. Nine tenths.
14. Seven eighths.
15. Eleven thirteenths.
16. 21 seventy-seconds.
17. Three one-hundredths.
18. 17 two-hundred-twentieths.
19. Six thousand one nineteenths.
20. How may 3 be expressed fractionally?
21. Is the value of $\frac{9}{8}$ greater or less than 1? Of $\frac{3}{19}$?

SECTION XVI.

REDUCTION OF FRACTIONS.

157. Reduction of Fractions is the process of changing their form of expression without changing their value.

CASE I.

Fractions Reduced to Larger or Smaller Terms.

158.—Ex. 1. One third of an apple is how many sixths of an apple?

2. What is $\frac{1}{3}$ expressed in terms twice as large?

3. Express $\frac{1}{4}$ in terms twice as large. $\frac{3}{4}$ in terms 3 times as large.

4. Two sixths of an apple are how many thirds of an apple? What is $\frac{2}{6}$ expressed in terms one half as large?

5. Two eighths are how many fourths? Express $\frac{9}{12}$ in terms one third as large.

6. Ten fifteenths are how many fifths? Express $\frac{12}{36}$ in terms one sixth as large.

DEFINITIONS.

159. A fraction is reduced to **Larger, or Higher, Terms** when expressed in an equivalent fraction with larger terms.

160. A fraction is reduced to **Smaller, or Lower, Terms** when expressed in an equivalent fraction with smaller terms.

161. A fraction is in its **Lowest Terms** when expressed in terms which are prime to each other.

162. Principle.—*Multiplying or dividing both terms of a fraction by the same number does not change its value.*

For, in the one case, as the number of parts is increased, their size is diminished; and, in the other case, as the number of parts is diminished, their size is increased.

Thus, $\frac{6}{3}=2$; $\frac{6\times3}{3\times3}=\frac{18}{9}=2$; or $\frac{6\div3}{3\div3}=\frac{2}{1}=2$.

WRITTEN EXERCISES.

163.—Ex. 1. Reduce $\frac{4}{7}$ to forty-seconds.

$\frac{4\times 6}{7\times 6}=\frac{24}{42}$ Solution.—Since 42, the required denominator, is 6 times as large as 7, the given denominator, and, since multiplying both terms of a fraction by the same number does not change its value, (Art. 162,) we multiply both terms of $\frac{4}{7}$ by 6, which gives $\frac{24}{42}$, the fraction required.

2. Change $\frac{4}{5}$ to an equivalent fraction whose numerator is 20.
3. Reduce $\frac{24}{42}$ to its lowest terms.

$\frac{24\div 2}{42\div 2}=\frac{12}{21}$; $\frac{12\div 3}{21\div 3}=\frac{4}{7}$ Solution.—Since dividing both terms of a fraction by the same number does not change the value of the fraction, we divide both terms of $\frac{24}{42}$ by 2, or cancel that factor in each term, which gives, as the fraction in lower terms, $\frac{12}{21}$. Dividing both terms of $\frac{12}{21}$ by 3 gives $\frac{4}{7}$, which, since the terms are prime to each other, is the result required.

$\frac{24\div 6}{42\div 6}=\frac{4}{7}$ Second Solution.—Since 6 is the greatest common divisor of the terms of $\frac{24}{42}$, we can obtain the lowest terms of the fraction by dividing both terms by that divisor, which gives $\frac{4}{7}$, the same result as at first obtained.

4. Reduce $\frac{42}{68}$ to its lowest terms.
5. Change $\frac{20}{25}$ to its lowest terms.

164. Rules for Reduction of Fractions to Higher or the Lowest Terms.—*1. To reduce a fraction to higher terms, multiply both terms of the fraction by such a number as will give the required term.*

2. To reduce a fraction to its lowest terms, cancel in both terms all common factors, or divide both terms by their greatest common divisor.

PROBLEMS.

1. Reduce $\frac{5}{7}$ to forty-ninths. *Ans.* $\frac{35}{49}$.
2. Reduce $\frac{3}{8}$ and $\frac{3}{4}$ to sixteenths. *Ans.* $\frac{6}{16}$, $\frac{12}{16}$.
3. Reduce $\frac{6}{9}$ and $\frac{2}{3}$ to twenty-sevenths.
4. Reduce $\frac{4}{35}$, $\frac{5}{7}$ and $\frac{11}{21}$ to one-hundred-fifths.

Reduce to lowest terms—

5. $\frac{35}{49}$. *Ans.* $\frac{5}{7}$.
6. $\frac{48}{56}$.
7. $\frac{182}{196}$. *Ans.* $\frac{13}{14}$.
8. $\frac{84}{170}$. *Ans.* $\frac{42}{85}$.
9. $\frac{171}{189}$. *Ans.* $\frac{19}{21}$.
10. $\frac{1702}{1886}$.

11. In what lower terms can $\frac{24}{60}$ be expressed?

Ans. $\frac{12}{30}$, $\frac{8}{20}$, $\frac{6}{15}$, $\frac{4}{10}$ and $\frac{2}{5}$.

CASE II.

Integers or Mixed Numbers Reduced to Improper Fractions.

165.—Ex. 1. How many fourths in 1 orange? In 3 oranges? In 6 oranges?

2. How many thirds in 1 apple? In 4 apples? In 7 apples?

3. How many fifths in 1? In 3? In 6?

4. How many sixths of a cake are 2 cakes? Are $2\frac{1}{6}$ cakes? $3\frac{1}{6}$ cakes? $3\frac{5}{6}$ cakes?

5. How many sevenths in 1? In $1\frac{1}{7}$? In $5\frac{2}{7}$?

WRITTEN EXERCISES.

166.—Ex. 1. Reduce 29 to sixths.

$$\begin{array}{l} 29 \\ \underline{6 \text{ sixths}} \\ 174 \text{ sixths} = \frac{174}{6} \end{array}$$

SOLUTION.—Since in 1 there are 6 sixths, in 29 there must be 29 times 6 sixths, which are $\frac{174}{6}$.

2. Reduce $18\frac{5}{9}$ to an equivalent improper fraction.

$$18\frac{5}{9} = 18 + \frac{5}{9};$$

$$18 = \frac{162}{9};\ \frac{162}{9} + \frac{5}{9} = \frac{167}{9}$$

SOLUTION.—Since in 1 there are $\frac{9}{9}$, in 18 there must be 18 times $\frac{9}{9}$, or $\frac{162}{9}$; $\frac{162}{9}$ and $\frac{5}{9}$ are $\frac{167}{9}$, the fraction required.

3. Reduce 19 to fifths. *Ans.* $\frac{95}{5}$.

4. Change $41\frac{3}{8}$ to an improper fraction. *Ans.* $\frac{331}{8}$.

167. Rule for Reduction of Integers or Mixed Numbers to Improper Fractions.—*Multiply the integer by the given denominator, and, if there be a fractional part, add its numerator to the product. The result written over the given denominator will be the required fraction.*

PROBLEMS.

Reduce to equivalent improper fractions—

1. $104\frac{3}{7}$.	*Ans.* $\frac{731}{7}$.	5. $42\frac{3}{5}$.	
2. $28\frac{7}{9}$.		6. $23\frac{19}{23}$.	*Ans.* $\frac{548}{23}$.
3. $19\frac{8}{11}$.	*Ans.* $\frac{217}{11}$.	7. $28\frac{11}{12}$.	
4. $144\frac{5}{9}$.		8. $115\frac{3}{125}$.	*Ans.* $\frac{14378}{125}$.

9. Change 36 to thirteenths and 41 to fourteenths.
Ans. $\frac{468}{13}$, $\frac{574}{14}$.

CASE III.

Improper Fractions Reduced to Integers or Mixed Numbers.

168.—Ex. 1. In four fourths of an orange, how many oranges? In twelve fourths of an orange?

2. How many apples in three thirds of an apple? In twelve thirds of an apple?

3. How many ones in $\frac{3}{3}$? In $\frac{4}{4}$? In $\frac{5}{5}$? In $\frac{12}{4}$? In $\frac{25}{5}$?

4. How many cakes in $\frac{13}{6}$ of a cake? In $\frac{23}{6}$ of a cake?

WRITTEN EXERCISES.

169.—Ex. 1. Reduce $\frac{167}{9}$ to an equivalent integer or mixed number.

$\frac{167}{9} = 167 \div 9 = 18\frac{5}{9}$ Solution.—Since 9 ninths equal one, 167 ninths must equal as many ones as 9 is contained times in 167, or $18\frac{5}{9}$ times. Hence, $\frac{167}{9} = 18\frac{5}{9}$.

2. Reduce $\frac{95}{8}$ to an equivalent integer or mixed number.

170. Rule for Reduction of Improper Fractions to Integers or Mixed Numbers.—*Divide the numerator by the denominator.*

PROBLEMS.

Reduce to an integer or mixed number—

1. $\frac{387}{3}$.	*Ans.* 129.	4. $\frac{151}{1}$.	*Ans.* 151.
2. $\frac{114}{6}$.		5. $\frac{218}{5}$.	
3. $\frac{218}{95}$.	*Ans.* $2\frac{28}{95}$.	6. $\frac{347}{12}$.	*Ans.* $28\frac{11}{12}$.

7. What is the value of $\frac{248}{8}$ dollars? *Ans.* 31 dollars.

8. How many miles are $\frac{6263}{29}$ miles?

CASE IV.

Dissimilar Fractions Reduced to Similar Fractions.

171.—Ex. 1. How many sixths of an apple is 1 third of an apple? Are 2 thirds of an apple?

2. Express $\frac{2}{3}$ and $\frac{1}{2}$ each as sixths.

3. Express $\frac{1}{4}$ and $\frac{3}{8}$ each as sixteenths.

4. What is a common multiple of the denominators of $\frac{1}{4}$ and $\frac{3}{8}$?

5. Express $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{6}$ each as twelfths. What is the least common multiple of the denominators of $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{6}$?

DEFINITIONS.

172. Fractions have a **Common Denominator** when their denominators are alike.

173. Fractions have the **Least Common Denominator** when their denominators are the smallest that they can have in common.

174. Fractions are said to be reduced to a common denominator when they are changed to equivalent fractions with denominators alike.

175. Principles.—1. *A common denominator of two or more fractions is a common multiple of their denominators.*

2. *The least common denominator of two or more fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

176.—Ex. 1. Reduce $\frac{5}{9}$ and $\frac{7}{27}$ to equivalent fractions having a common denominator.

$$\frac{5}{9} = \frac{15}{27}$$
$$\frac{7}{27}$$

SOLUTION.—Since the denominator of $\frac{7}{27}$ is 3 times the denominator of $\frac{5}{9}$, we multiply both terms of $\frac{5}{9}$ by 3, which gives as its equivalent $\frac{15}{27}$. Hence, $\frac{15}{27}$ and $\frac{7}{27}$ are the fractions required.

2. Reduce $\frac{14}{4}$, $\frac{5}{6}$ and $\frac{7}{12}$ to similar fractions.

Ans. $\frac{42}{12}$, $\frac{10}{12}$ and $\frac{7}{12}$.

3. Reduce $\frac{3}{4}$ and $\frac{1}{5}$ to similar fractions.

$\frac{3}{4}=\frac{15}{20}$

$\frac{1}{5}=\frac{4}{20}$

SOLUTION.—Multiplying both terms of $\frac{3}{4}$ by 5, the denominator of $\frac{1}{5}$, we have $\frac{15}{20}$; and multiplying both terms of $\frac{1}{5}$ by 4, the denominator of $\frac{3}{4}$, we have $\frac{4}{20}$. Hence, $\frac{3}{4}$ and $\frac{1}{5}=\frac{15}{20}$ and $\frac{4}{20}$, which are similar fractions.

4. Reduce $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{11}{12}$ to equivalent fractions having the least common denominator.

$\frac{1\times 12}{2\times 12}=\frac{12}{24}$

$\frac{5\times 3}{8\times 3}=\frac{15}{24}$

$\frac{11\times 2}{12\times 2}=\frac{22}{24}$

SOLUTION.—The least common multiple of the denominators 2, 8 and 12 is 24; hence, 24 is the least common denominator.

$\frac{1}{2}$ reduced to twenty-fourths is $\frac{12}{24}$; $\frac{5}{8}$ is $\frac{15}{24}$; and $\frac{11}{12}$ is $\frac{22}{24}$; hence, $\frac{12}{24}$, $\frac{15}{24}$ and $\frac{22}{24}$ are the fractions required.

5. Reduce $\frac{5}{12}$, $\frac{11}{15}$ and $\frac{11}{24}$ to the least common denominator.

Ans. $\frac{50}{120}$, $\frac{88}{120}$, $\frac{55}{120}$.

177. Rules for Reducing Fractions to a Common Denominator.— *1. Multiply both terms of one or more of the fractions by any number that will make the denominators alike. Or,*

2. Multiply both terms of each fraction by the denominators of the other fractions.

178. Rule for Reducing Fractions to the Least Common Denominator.—*Find the least common multiple of all the denominators for the least common denominator, and multiply both terms of each fraction by such a number as will reduce it to that denominator.*

PROBLEMS.

Reduce to equivalent fractions having a common denominator—

1. $\frac{2}{9}$, $\frac{6}{7}$, $\frac{2}{5}$. *Ans.* $\frac{70}{315}$, $\frac{270}{315}$, $\frac{126}{315}$.
2. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{9}$.
3. $\frac{3}{8}$ and $\frac{3}{48}$. *Ans.* $\frac{18}{48}$, $\frac{3}{48}$.
4. Reduce $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{8}{11}$ to similar fractions.
5. Reduce $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{12}$ and $\frac{4}{18}$ to similar fractions.

Reduce to similar fractions having the least common denominator—

6. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{12}$ and $\frac{4}{18}$. *Ans.* $\frac{27}{36}$, $\frac{20}{36}$, $\frac{21}{36}$, $\frac{8}{36}$.

7. $\frac{7}{8}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.

8. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{4}$ and $\frac{15}{16}$.

9. $\frac{3}{7}$, $\frac{2}{8}$, $\frac{1}{9}$, $1\frac{11}{12}$. *Ans.* $\frac{216}{504}$, $\frac{126}{504}$, $\frac{56}{504}$, $\frac{966}{504}$.

10. Express $7\frac{1}{2}$, $5\frac{1}{4}$ and $1\frac{1}{3}$ as similar fractions, with the least common denominator. *Ans.* $\frac{90}{12}$, $\frac{63}{12}$, $\frac{16}{12}$.

TEST QUESTIONS.

179.—1. How does a FRACTION differ from an integer? How does the unit of a fraction differ from a fractional unit? What is the unit and what is the fractional unit of $\frac{3}{4}$?

2. What are the TERMS of a fraction? Which term names the parts? Which term numbers the parts? What is the numerator and what is the denominator of $\frac{3}{4}$?

3. How is a COMMON FRACTION written? When is a fraction called a proper fraction? When an improper fraction? What is a mixed number?

4. How is a common fraction expressed? How may an integer be expressed fractionally? What are similar fractions? Dissimilar fractions?

5. How is a fraction read? In what three ways may $\frac{3}{4}$ be regarded?

6. Upon what does the VALUE of one of the parts of a fraction depend? What is the value of a fraction? When is the value of a fraction 1? When less than 1? When more than 1?

7. What is REDUCTION of fractions? When is a fraction reduced to higher terms? When to lower terms?

8. When is a fraction in its LOWEST TERMS? How may a fraction be reduced to higher terms? How to its lowest terms? Upon what principle does the rule for the reduction of fractions to higher or to lower terms depend?

9. How is an INTEGER or MIXED NUMBER reduced to an improper fraction? How is an improper fraction reduced to an integer or mixed number?

10. When have several fractions a COMMON DENOMINATOR? When the least common denominator? What is the least common denominator of several fractions? What is the rule for finding common denominators?

SECTION XVII.

ADDITION OF FRACTIONS.

180.—Ex. 1. James gave his brother 3 eighths of a melon and his sister 4 eighths. What part of the melon did he give both?

2. John had $\frac{1}{2}$ of a dollar, and his father gave him $\frac{2}{5}$ of a dollar. What part of a dollar had he then?

3. How many eighths are $\frac{4}{8}$ and $\frac{3}{8}$? How many tenths are $\frac{1}{2}$ and $\frac{2}{5}$?

4. What is the sum of $3\frac{1}{4}$ and $2\frac{3}{8}$? Of $\frac{3}{8}+\frac{1}{4}+2+3$?

181. Principle.—*Only fractions having a common denominator can be directly added.*

For, fractions having a common denominator are similar, and only similar numbers can be added. (Art. 46—1.)

WRITTEN EXERCISES.

182.—Ex. 1. What is the sum of $\frac{1}{2}$, $\frac{4}{5}$ and $\frac{3}{8}$?

$$\frac{1}{2}+\frac{4}{5}+\frac{3}{8}=\frac{20}{40}+\frac{32}{40}+\frac{15}{40}=\frac{20+32+15}{40}=\frac{67}{40}=1\frac{27}{40}$$

SOLUTION. — Reducing the given fractions to their least common denominator, we have $\frac{20}{40}+\frac{32}{40}+\frac{15}{40}=\frac{67}{40}$, or $1\frac{27}{40}$, which is the sum required.

2. What is the sum of $\frac{8}{19}$, $\frac{6}{19}$ and $\frac{17}{19}$? *Ans.* $1\frac{7}{19}$.

3. What is the sum of $3\frac{5}{8}$, $\frac{2}{3}$ and 15?

$$\begin{aligned} 3\frac{5}{8} &= 3\frac{15}{24} \\ \frac{2}{3} &= \frac{16}{24} \\ 15 &= 15 \\ \text{Sum}, &\ \ 19\frac{7}{24} \end{aligned}$$

SOLUTION.—Reducing the fractions to similar fractions, we have $3\frac{5}{8}=3\frac{15}{24}$, and $\frac{2}{3}=\frac{16}{24}$.

The sum of these fractions is $\frac{31}{24}$, which equals $1\frac{7}{24}$; and $1\frac{7}{24}$ added to 18, the sum of the integers, gives $19\frac{7}{24}$, the sum required.

4. What is the sum of $14\frac{3}{4}$ and $16\frac{7}{8}$? *Ans.* $31\frac{5}{8}$.

183. Rule for Addition of Fractions.—*1. Reduce the fractions, if necessary, to similar fractions, add their numerators, and write under the sum the common denominator.*

If there be mixed numbers or integers, add the fractions and integers separately, and unite the results.

PROBLEMS.

What is the sum of—

1. $\frac{7}{16}$ and $\frac{19}{30}$? *Ans.* $1\frac{17}{240}$.
2. $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{3}{4}$?
3. $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{9}{16}$? *Ans.* $2\frac{7}{48}$.
4. $\frac{5}{11}$, $\frac{7}{11}$ and $\frac{9}{11}$?
5. $\frac{7}{10}$, $\frac{11}{12}$ and $\frac{4}{9}$? *Ans.* $2\frac{11}{180}$.
6. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$? *Ans.* $3\frac{1}{8}$.
7. $15\frac{3}{8}$, $2\frac{11}{12}$ and $27\frac{1}{4}$? *Ans.* $45\frac{13}{24}$.
8. $12\frac{1}{2}$, $24\frac{3}{4}$ and $17\frac{5}{8}$?
9. $6\frac{63}{80}$, $\frac{2}{7}$ and $7\frac{1}{2}$? *Ans.* $14\frac{321}{560}$.
10. $120\frac{51}{52}$, 11 and $2\frac{5}{26}$?

11. What is the sum of $6 + 5\frac{1}{5} + 3\frac{1}{10} + 15\frac{3}{4}$?

12. A farmer has in one field $41\frac{3}{16}$ acres; in a second, $30\frac{5}{8}$ acres; and in a third, $60\frac{3}{4}$ acres. How many acres are there in the three fields? *Ans.* $132\frac{9}{16}$.

SECTION XVIII.

SUBTRACTION OF FRACTIONS.

184.—Ex. 1. If you should have 7 eighths of a dollar, and should spend 6 eighths of a dollar, what part of a dollar would you have left?

2. John had $\frac{9}{10}$ of a melon, and gave $\frac{1}{2}$ of the melon to a classmate. What part of the melon had he left?

3. How many eighths are $\frac{7}{8}$ less $\frac{6}{8}$? $\frac{5}{8}$ less $\frac{1}{2}$?

4. How much is $1\frac{3}{5}$ less $\frac{3}{4}$? How much is $\frac{20}{20} + \frac{16}{20}$ less $\frac{15}{20}$?

185. Principle.—*Only fractions having a common denominator can be subtracted.*

For, fractions having a common denominator are similar, and only similar numbers can be subtracted. (Art. 57—1.)

WRITTEN EXERCISES.

186.—Ex. 1. What is the difference between $\frac{4}{9}$ and $\frac{4}{27}$?

$$\frac{4}{9} - \frac{4}{27} = \frac{12}{27} - \frac{4}{27} = \frac{12-4}{27} = \frac{8}{27}$$

SOLUTION. — Reducing the given fractions to similar fractions, and finding the difference between their numerators, we have $\frac{8}{27}$, which is the difference required.

2. What is the difference between $\frac{31}{37}$ and $\frac{3}{10}$? *Ans.* $\frac{199}{370}$.

3. What is the difference between $8\frac{1}{4}$ and $5\frac{2}{3}$?

$$8\frac{1}{4} = 8\frac{3}{12} = 7\frac{15}{12}$$
$$5\frac{2}{3} = \quad\quad 5\frac{8}{12}$$
$$\textit{Difference,}\quad 2\frac{7}{12}$$

Solution.—The given fractions reduced to similar fractions give $8\frac{1}{4} = 8\frac{3}{12}$, and $5\frac{2}{3} = 5\frac{8}{12}$.

Since $\frac{8}{12}$ cannot be taken from $\frac{3}{12}$, we take 1 one, or $\frac{12}{12}$, from 8 ones, leaving 7 ones, and adding the $\frac{12}{12}$ to the $\frac{3}{12}$ have $\frac{15}{12}$; $7\frac{15}{12} - 5\frac{8}{12} = 2\frac{7}{12}$, the difference required.

4. What is the difference between $9\frac{1}{7}$ and $8\frac{1}{4}$? *Ans.* $\frac{25}{28}$.

187. Rule for Subtraction of Fractions.—*Reduce the fractions, if necessary, to similar fractions, and write the difference of the numerators over the common denominator.*

If there be mixed numbers, subtract first the fractional part of the subtrahend, and then the integral part, and unite the results.

PROBLEMS.

What is the difference between—

1. $\frac{5}{12}$ and $\frac{4}{15}$? *Ans.* $\frac{3}{20}$.
2. $\frac{27}{29}$ and $\frac{13}{29}$?
3. $\frac{17}{18}$ and $\frac{5}{6}$? *Ans.* $\frac{1}{9}$.
4. $\frac{97}{100}$ and $\frac{3}{7}$?
5. $\frac{8}{9}$ and $\frac{4}{13}$? *Ans.* $\frac{68}{117}$.
6. $\frac{111}{112}$ and $\frac{3}{4}$?
7. $14\frac{3}{7}$ and $96\frac{1}{3}$? *Ans.* $81\frac{19}{21}$.
8. $71\frac{1}{2}$ and $\frac{17}{19}$?

9. A boy had $\frac{29}{32}$ of a bushel of chestnuts, and sold $\frac{3}{4}$ of a bushel. What part of a bushel had he left? *Ans.* $\frac{5}{32}$.

10. A merchant owned $\frac{7}{8}$ of a ship, and sold $\frac{4}{5}$ of the ship. What part of the ship had he left?

11. If $4\frac{5}{6}$ be taken from 13, what will be left?

$$13 = 12\frac{6}{6}$$
$$4\frac{5}{6}$$
$$8\frac{1}{6}$$

Solution.—Taking one of the 13 ones and calling it $\frac{6}{6}$, we have $13 = 12\frac{6}{6}$.

From $12\frac{6}{6}$ taking $4\frac{5}{6}$, we have $8\frac{1}{6}$, the result required.

12. How much less than 800 is $61\frac{77}{111}$? *Ans.* $738\frac{34}{111}$.

13. If you should buy a horse for $200\frac{7}{12}$ dollars, and should pay down $149\frac{5}{8}$ dollars, how much would you owe for him?

SECTION XIX.

MULTIPLICATION OF FRACTIONS.

CASE I.

Fractions Multiplied by Integers.

188.—Ex. 1. At 3 tenths of a dollar each, what will 3 slates cost?

2. If a man can cut $\frac{3}{16}$ of a cord of wood in 1 hour, how much can he cut in 5 hours?

3. At $\frac{2}{3}$ of a cent each, what will 6 apples cost?

4. How much is 3 times $\frac{3}{10}$ of a dollar? 5 times $\frac{3}{16}$ of a cord?

5. How many thirds are 6 times $\frac{2}{3}$? How many ones are 6 times $\frac{2}{3}$?

6. How much will 8 men earn at $3\frac{1}{2}$ dollars a day? How much at $2\frac{3}{4}$ dollars a day?

189. Principle.—*Multiplying the numerator or dividing the denominator of a fraction by any number multiplies the fraction by that number.*

For, in the one case the number of parts is increased, while their size remains unchanged; and in the other case the size of the parts is increased, while their number remains unchanged.

Thus, $\frac{6}{3} = 2$; but $\frac{6 \times 3}{3} = \frac{18}{3} = 6$; and $\frac{6}{3 \div 3} = \frac{6}{1} = 6$.

WRITTEN EXERCISES.

190.—Ex. 1. Multiply $\frac{11}{16}$ by 8.

$$\frac{11}{16} \times 8 = \frac{11 \times 8}{16} = \frac{88}{16} = 5\frac{8}{16} = 5\frac{1}{2}$$

Or,

$$\frac{11}{16} \times 8 = \frac{11}{16 \div 8} = \frac{11}{2} = 5\frac{1}{2}$$

Solution.—8 times $\frac{11}{16}$ are $\frac{88}{16}$, which, reduced, equals $5\frac{1}{2}$. Or, since dividing the denominator of a fraction also multiplies the fraction, 8 times $\frac{11}{16}$ are $\frac{11}{2}$, or $5\frac{1}{2}$, the same result as before.

2. Multiply $\frac{16}{18}$ by 5. *Ans.* $4\frac{8}{18}$.

3. What is the product of $11\frac{3}{4}$ multiplied by 8?

$$\begin{array}{l} 11 \times 8 = 88 \\ \frac{3}{4} \times 8 = \ \ 6 \\ \hline 11\frac{3}{4} \times 8 = 94 \end{array}$$

SOLUTION.—Since $11\frac{3}{4}$ equals $11 + \frac{3}{4}$, the product of $11\frac{3}{4}$ multiplied by 8 is the same as 8 times 11 plus 8 times $\frac{3}{4}$.

8 times 11 are 88, and 8 times $\frac{3}{4}$ are $\frac{24}{4}$, or 6. 88 and 6 are 94, the product required.

191. Rule for Multiplying a Fraction by an Integer.—*Multiply the numerator, or divide the denominator, by the integer.*

If the multiplicand be a mixed number, multiply the integer and fraction separately, and add the products.

Multiply—

1. $\frac{11}{24}$ by 12.	*Ans.* $5\frac{1}{2}$.	4. $\frac{15}{19}$ by 13.	*Ans.* $10\frac{5}{19}$.
2. $\frac{10}{18}$ by 71.		5. $\frac{7}{11}$ by 100.	
3. $\frac{5}{9}$ by 63.	*Ans.* 35.	6. $\frac{21}{23}$ by 69.	*Ans.* 63.

7. What is the product of $\frac{9}{16} \times 19$? *Ans.* $10\frac{11}{16}$.

8. What will 9 yards of cloth cost at $5\frac{3}{8}$ dollars a yard?

9. Jason gathered $5\frac{5}{8}$ bushels of apples, and Willie gathered 6 times as many. How many bushels did Willie gather?

10. If it takes $\frac{7}{16}$ of a yard of cloth to make one vest, how many yards will it take to make 24 vests? *Ans.* $10\frac{1}{2}$.

CASE II.

Integers Multiplied by Fractions.

192.—Ex. 1. Henry has 12 apples; how many is 1 third of the number? How many are 2 thirds of the number?

2. John had 20 cents, and gave away 1 fourth of them. How many cents did he give away? What is $\frac{3}{4}$ of 20 cents?

3. How much is $\frac{1}{5}$ of 22 dollars? $\frac{3}{4}$ of 25 dollars?

4. How much is $\frac{5}{6}$ of 28? $\frac{4}{7}$ of 31? $\frac{5}{8}$ of 46?

193. Principle.—*A number is multiplied by a fraction by obtaining such a part of the number as the fraction indicates.*

For, multiplying any integer by 1 is taking it once; multiplying it by $\frac{1}{5}$, is taking 1 *fifth* of it; by $\frac{2}{5}$, is taking 2 *fifths* of it, etc.

WRITTEN EXERCISES.

194.—Ex. 1. Multiply 35 by $\frac{5}{7}$, or find $\frac{5}{7}$ of 35.

$$35\times\frac{5}{7}=\frac{\overset{5}{\cancel{35}}\times 5}{\cancel{7}}=25$$

Or,

$$35\times\frac{5}{7}=\frac{\overset{5}{\cancel{35}}}{\cancel{7}}\times 5=25$$

SOLUTIONS.—$\frac{5}{7}=\frac{1}{7}$ of 5; hence, $\frac{5}{7}$ times 35 = $\frac{1}{7}$ of 5 times 35; or $\frac{35\times 5}{7}=\frac{175}{7}=25$.

Or, since $\frac{5}{7}=5$ times $\frac{1}{7}$, $\frac{5}{7}$ of 35 = 5 times $\frac{1}{7}$ of 35; $\frac{1}{7}$ of 35 is 5, and 5 times 5 are 25.

2. Multiply 45 by $\frac{3}{4}$, or find $\frac{3}{4}$ of 45. *Ans.* $33\frac{3}{4}$.

195. Rules for Multiplying an Integer by a Fraction.—*1. Multiply the integer by the numerator of the multiplier, and divide the product by the denominator. Or,*

2. Divide the integer by the denominator of the multiplier, and multiply the quotient by the numerator.

PROBLEMS.

Multiply—

1. 7 by $\frac{2}{5}$. *Ans.* $2\frac{4}{5}$.
2. 19 by $\frac{3}{11}$.
3. 13 by $\frac{7}{10}$. *Ans.* $9\frac{1}{10}$.
4. 163 by $\frac{3}{7}$.
5. 105 by $\frac{4}{35}$. *Ans.* 12.
6. 121 by $\frac{7}{8}$.
7. 220 by $\frac{3}{20}$. *Ans.* 33.
8. 100 by $\frac{5}{16}$.

9. What is the value of $136\times\frac{33}{100}$. *Ans.* $44\frac{22}{25}$.

10. What will $3\frac{2}{5}$ tons of hay cost at $25 per ton?

$$25\times 3\ =75$$
$$25\times\ \frac{2}{5}=10$$
$$25\times 3\frac{2}{5}=85$$

Or,

$$25\times\frac{17}{5}=85$$

SOLUTIONS.—If 1 ton cost 25 dollars, 3 tons will cost 3 times 25 dollars, or 75 dollars, and $\frac{2}{5}$ of a ton will cost $\frac{2}{5}$ of 25 dollars, or 10 dollars. 75 dollars and 10 dollars are 85 dollars, the result required.

Or, if 1 ton cost 25 dollars, $3\frac{2}{5}$, or $\frac{17}{5}$, tons will cost $\frac{17}{5}$ times 25 dollars, or $\frac{17}{5}$ of 25 dollars, which is 85 dollars, the same result.

11. What will $31\frac{5}{6}$ tons of coal cost at 12 dollars a ton? *Ans.* 382 dollars.

12. What will $\frac{7}{8}$ of an acre of land cost at 120 dollars per acre?

13. How much must be paid for $17\frac{3}{4}$ hundred-weight of sugar, at 17 dollars per hundred-weight?

14. If a train of cars moves at the rate of 25 miles per hour, how far will it move in $24\frac{47}{60}$ hours?

Ans. $619\frac{7}{12}$ miles.

CASE III.

Fractions Multiplied by Fractions.

196.—Ex. 1. How much is 1 third of 6 eighths of a dollar? How much is 2 times 1 third of 6 eighths of a dollar?

2. If you have $\frac{4}{6}$ of a melon, and your brother has $\frac{3}{4}$ as much, what part of a melon has your brother?

3. How much is $\frac{6}{8}$ multiplied by $\frac{2}{3}$? $\frac{4}{6}$ multiplied by $\frac{3}{4}$?

4. What is $\frac{1}{3}$ of $\frac{6}{9}$? What is $\frac{2}{3}$ of $\frac{6}{9}$? How much is $\frac{6}{9}$ multiplied by $\frac{2}{3}$?

DEFINITION.

197. A Compound Fraction is a fraction of a fraction.

Thus, $\frac{2}{3}$ of $\frac{5}{8}$ is a compound fraction. The word *of* in the expression denotes multiplication.

WRITTEN EXERCISES.

198.—Ex. 1. Multiply $\frac{8}{9}$ by $\frac{3}{7}$, or find $\frac{3}{7}$ of $\frac{8}{9}$.

$$\frac{8}{9}\times\frac{3}{7}=\frac{24}{63}=\frac{8}{21}$$

Or,

$$\frac{8}{9}\times\frac{3}{7}=\frac{8\times\cancel{3}}{\underset{3}{\cancel{9}}\times 7}=\frac{8}{21}$$

Solutions.—$\frac{3}{7}$ of $\frac{8}{9}$ is the same as 3 times $\frac{1}{7}$ of $\frac{8}{9}$; $\frac{1}{7}$ of $\frac{8}{9}$ is $\frac{8}{9\times 7}$, or $\frac{8}{63}$, and 3 times $\frac{8}{63}$ are $\frac{8\times 3}{63}$, or $\frac{24}{63}$, which reduced is $\frac{8}{21}$, the result required.

Or, indicating the multiplication and cancelling, we have $\frac{8}{21}$, as before.

2. Multiply $\frac{25}{33}$ by $\frac{4}{27}$, or find $\frac{4}{27}$ of $\frac{25}{33}$.

3. What is the product of $8\frac{2}{3}$ by $4\frac{1}{5}$?

$$8\frac{2}{3}\times 4\frac{1}{5}=\frac{26}{\cancel{3}}\times\frac{\overset{7}{\cancel{21}}}{5}=\frac{182}{5}=36\frac{2}{5}$$

Solution.—$8\frac{2}{3}$ is equal to $\frac{26}{3}$, and $4\frac{1}{5}$ is equal to $\frac{21}{5}$; hence, $8\frac{2}{3}\times 4\frac{1}{5}$ is the same as $\frac{26}{3}\times\frac{21}{5}$. Cancelling and multiplying, we have $\frac{182}{5}$, which reduced is $36\frac{2}{5}$, the result required.

199. Rule for Multiplication of Fractions.—*Multiply the numerators together for the numerator, and the denominators for the denominator, of the product.*

If there be mixed numbers, reduce them to fractions before multiplying.

The value of a *Compound Fraction* is found by performing the multiplication indicated by the expression.

PROBLEMS.

Multiply—

1. $\frac{11}{12}$ by $\frac{18}{35}$. *Ans.* $\frac{33}{70}$.
2. $\frac{2}{3}$ by $\frac{4}{5}$.
3. $\frac{28}{29}$ by $\frac{4}{7}$. *Ans.* $\frac{16}{29}$.
4. $\frac{15}{79}$ by $\frac{18}{23}$.
5. $\frac{9}{10}$ by $\frac{5}{6}$. *Ans.* $\frac{3}{4}$.
6. $\frac{4}{15}$ by $\frac{77}{120}$.
7. $\frac{21}{45}$ by $\frac{8}{21}$. *Ans.* $\frac{8}{45}$.
8. $\frac{17}{2}$ by $\frac{391}{8}$.

9. What is the value of $\frac{5}{12}$ of $\frac{13}{17}$? *Ans.* $\frac{65}{204}$.

10. What is the value of $\frac{4}{5}$ of $\frac{6}{7}$?

11. Multiply $17\frac{3}{4}$ by $14\frac{2}{5}$. *Ans.* $255\frac{3}{5}$.

12. Two factors are $79\frac{8}{16}$ and $9\frac{1}{3}$. What is their product?

13. A boy gathered $\frac{17}{32}$ of a bushel of berries, and his brother gathered $\frac{4}{5}$ as many. What quantity did his brother gather?

14. How many yards are there in $12\frac{3}{4}$ pieces of cloth, each containing $26\frac{1}{2}$ yards?

15. What will $19\frac{7}{20}$ tons of coal cost at $8\frac{3}{4}$ dollars per ton?

16. At $\frac{7}{10}$ of a dollar a yard, what will $\frac{7}{16}$ of a yard of cloth cost?

17. If a family use $1\frac{3}{5}$ barrels of flour in 1 month, how many barrels will it use in $11\frac{1}{2}$ months? *Ans.* $18\frac{2}{5}$.

18. If you should buy a watch for $80\frac{3}{4}$ dollars, and sell it for $\frac{5}{4}$ of the cost, for how much would you sell it?

19. At the rate of $2\frac{3}{8}$ tons per acre, how much hay can be obtained from $21\frac{1}{3}$ acres? *Ans.* $50\frac{2}{3}$ tons.

20. If a man can build $\frac{7}{9}$ of a wall in a day, what part of it can he build in $\frac{6}{7}$ of a day?

21. What is the value of $\frac{5}{6}$ of a bushel of clover-seed at $7\frac{7}{10}$ of a dollar per bushel? *Ans.* $6\frac{5}{12}$ dollars.

22. What is the value of $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{5}{9}$ of $\frac{63}{64}$ of 4? *Ans.* $\frac{5}{16}$.

SECTION XX.

DIVISION OF FRACTIONS.

CASE I.

Fractions Divided by Integers.

200.—Ex. 1. Charles divided 4 fifths of a melon between his two brothers. What part of the melon did he give to each?

2. If $\frac{15}{16}$ of a ship is owned by 5 men in equal shares, what part of the ship is each man's share?

3. If $\frac{4}{5}$ of an acre of land be divided into two equal house-lots, what part of an acre will each lot be?

201. Principle.—*Dividing the numerator, or multiplying the denominator, by any number, divides the fraction by that number.*

For, in the one case the number of parts is diminished, while their size remains unchanged; and in the other case the size of the parts is increased, while their number remains unchanged.

Thus, $\frac{6}{3} = 2$; but $\frac{6 \div 3}{3} = \frac{2}{3}$; and $\frac{6}{3 \times 3} = \frac{6}{9} = \frac{2}{3}$.

WRITTEN EXERCISES.

202.—Ex. 1. Divide $\frac{10}{11}$ by 5.

$$\frac{10}{11} \div 5 = \frac{10 \div 5}{11} = \frac{2}{11}$$

Or,

$$\frac{10}{11} \div 5 = \frac{\overset{2}{\cancel{10}}}{11 \times \cancel{5}} = \frac{2}{11}$$

SOLUTIONS.—Since dividing the numerator of a fraction divides the fraction, $\frac{10}{11}$ divided by 5 gives $\frac{2}{11}$, the result required.

Or, since multiplying the denominator of a fraction divides the fraction, $\frac{10}{11} \div 5 = \frac{10}{11 \times 5}$, or, by reduction, $\frac{2}{11}$, as before.

2. Divide $31\frac{1}{4}$ by 5.

$$31\frac{1}{4} \div 5 = \frac{125}{4} \div 5 = \frac{25}{4} = 6\frac{1}{4}$$

Or,

$$\begin{array}{r} 5)31\frac{1}{4} \\ \hline 6\frac{1}{4} \end{array}$$

SOLUTIONS.—$31\frac{1}{4}$ is equal to $\frac{125}{4}$; hence, $31\frac{1}{4} \div 5$ is the same as $\frac{125}{4} \div 5$; and $\frac{125}{4} \div 5$ is $\frac{25}{4}$, or $6\frac{1}{4}$.

Or, we may divide without reduction. Thus, 5 is contained in $31\frac{1}{4}$, 6 times with a remainder of $1\frac{1}{4}$, which is equal to $\frac{5}{4}$; 5 is contained in $\frac{5}{4}$, $\frac{1}{4}$ time; and 6 plus $\frac{1}{4}$ is $6\frac{1}{4}$, the same result as before.

3. Divide $\frac{72}{98}$ by 12. $\frac{109}{57}$ by 19.

203. Rule for Dividing a Fraction by an Integer.—*Divide the numerator, or multiply the denominator, by the integer.*

If the dividend is a mixed number, reduce it to an improper fraction before dividing; or, divide the integer and fraction separately, and unite the results.

PROBLEMS.

Divide—

1. $\frac{3}{4}$ by 6.	*Ans.* $\frac{1}{8}$.	5. $\frac{9}{14}$ by 12.	*Ans.* $\frac{3}{56}$.
2. $\frac{8}{9}$ by 4.		6. $\frac{17}{4}$ by 51.	
3. $\frac{18}{15}$ by 5.	*Ans.* $\frac{18}{75}$.	7. $\frac{63}{15}$ by 45.	*Ans.* $\frac{7}{75}$.
4. $\frac{144}{11}$ by 12.		8. $\frac{81}{7}$ by 27.	

9. If 11 yards of cloth cost $16\frac{1}{2}$ dollars, how much does 1 yard cost? *Ans.* $1\frac{1}{2}$ dollars.

10. What is the quotient of $\frac{7}{8}$ divided by 9?

11. What is the value of $\frac{135}{8} \div 20$? *Ans.* $\frac{27}{32}$.

12. What is the value of $22\frac{5}{8} \div 7$?

13. Divide $\frac{300}{627}$ by 100. *Ans.* $\frac{1}{209}$.

14. A farmer raised $91\frac{15}{16}$ bushels of wheat upon 5 acres of land. How many bushels was that per acre?

CASE II.

Integers Divided by Fractions.

204.—Ex. 1. How many apples, at $\frac{2}{3}$ of a cent each, can be bought for 6 cents?

2. How many times 2 thirds of a cent in 18 thirds of a cent? How much is $\frac{1}{2}$ of 3 times 6?

3. When coffee is $\frac{3}{8}$ of a dollar a pound, how many pounds can be bought for 6 dollars?

4. How many times is $\frac{3}{8}$ contained in 6? In 12?

5. If meal is $\frac{7}{8}$ of a dollar per bushel, how many bushels can be bought for 14 dollars?

6. How many times is $\frac{7}{8}$ contained in 14? In 21?

WRITTEN EXERCISES.

205.—Ex. 1. Divide 15 by $\frac{3}{4}$.

$$15 \div \frac{3}{4} = \frac{60}{4} \div \frac{3}{4} = 20$$

Or,

$$15 \div \frac{3}{4} = \frac{\overset{5}{\cancel{15}} \times 4}{\cancel{3}} = 20$$

SOLUTIONS.—15 is equal to $\frac{60}{4}$; and 60 fourths divided by 3 fourths gives 20, the result required.

Or, since $15 \div 1$ is 15, $15 \div \frac{1}{4}$ must be 4 times 15, and $15 \div \frac{3}{4}$ must be $\frac{1}{3}$ of 4 times 15, or 20, the same result.

2. Divide 63 by $\frac{3}{5}$. *Ans.* 105.

206. Rule for Dividing an Integer by a Fraction.—*Reduce the dividend to a fraction similar to the divisor, and divide the numerator of the dividend by the numerator of the divisor. Or,*

Multiply the integer by the denominator of the divisor, and divide the result by the numerator.

PROBLEMS.

Divide—

1. 16 by $\frac{8}{9}$.		5. 65 by $\frac{7}{9}$.	
2. 19 by $\frac{2}{3}$.	*Ans.* $28\frac{1}{2}$.	6. 29 by $\frac{3}{4}$.	*Ans.* $38\frac{2}{3}$.
3. 54 by $\frac{6}{7}$.		7. 110 by $\frac{7}{10}$.	
4. 100 by $\frac{31}{40}$.	*Ans.* $129\frac{1}{31}$.	8. 144 by $\frac{12}{13}$.	*Ans.* 156.

9. Divide 40 by $5\frac{2}{5}$.

$$5\frac{2}{5} = \frac{27}{5};\ 40 \div \frac{27}{5} = \frac{40 \times 5}{27} = 7\frac{11}{27}$$

SOLUTION.—$5\frac{2}{5}$ equals $\frac{27}{5}$; $40 \div 5\frac{2}{5}$ is the same as $40 \div \frac{27}{5}$, which gives $7\frac{11}{27}$, the result required.

10. Divide 97 by $5\frac{4}{7}$. *Ans.* $17\frac{16}{39}$.

11. Divide 365 by $365\frac{1}{4}$.

12. At $8\frac{3}{4}$ dollars a ton, how many tons of coal can be bought for 63 dollars? *Ans.* $7\frac{1}{5}$.

13. At $\frac{9}{10}$ of a dollar a yard, how many yards of cloth can be bought for 27 dollars?

14. Smith's farm consists of 133 acres, and is divided into fields of $16\frac{5}{8}$ acres each. How many fields are there?

CASE III.

Fractions Divided by Fractions.

207.—Ex. 1. Into how many parts, of 3 tenths each, can you divide 9 tenths of an orange?

2. How many times is $\frac{4}{12}$ of a melon contained in $\frac{8}{12}$ of a melon?

3. How many sixths are $\frac{2}{3}$? How many times $\frac{1}{6}$ in $\frac{2}{3}$? $\frac{1}{3}$ in $\frac{4}{6}$?

4. How many pounds of coffee, at $\frac{3}{8}$ of a dollar a pound, can be bought for $\frac{3}{4}$ of a dollar?

5. How many times $\frac{2}{9}$ in $\frac{2}{3}$? $\frac{2}{5}$ in $\frac{3}{4}$? $\frac{3}{7}$ in $\frac{9}{14}$?

DEFINITIONS.

208. A **Complex Fraction** is a fraction having a fraction in one or both of its terms.

Thus, $\dfrac{\frac{3}{4}}{\frac{1}{3}}$ is a complex fraction, and is an expression of division of fractions.

209. A fraction is *inverted* when the denominator is taken for the numerator, and the numerator for the denominator.

Thus, $\frac{3}{4}$ inverted is $\frac{4}{3}$; $\frac{2}{7}$ inverted is $\frac{7}{2}$.

WRITTEN EXERCISES.

210.—Ex. 1. Divide $\frac{4}{5}$ by $\frac{2}{9}$.

$$\frac{4}{5} \div \frac{2}{9} = \frac{36}{45} \div \frac{10}{45} = 3\frac{6}{10} = 3\frac{3}{5}$$

Or,

$$\frac{4}{5} \div \frac{2}{9} = \frac{\overset{2}{\cancel{4}} \times 9}{5 \times \cancel{2}} = \frac{18}{5} = 3\frac{3}{5}$$

SOLUTIONS.—Expressed as similar fractions, $\frac{4}{5} \div \frac{2}{9}$ is the same as $\frac{36}{45} \div \frac{10}{45}$; 36 forty-fifths divided by 10 forty-fifths gives $3\frac{6}{10}$, or $3\frac{3}{5}$, the result required.

Or, since $\frac{4}{5} \div 1$ is $\frac{4}{5}$, $\frac{4}{5} \div \frac{1}{9}$ must be 9 times $\frac{4}{5}$, or $\frac{4 \times 9}{5}$, and $\frac{4}{5} \div \frac{2}{9}$ must be $\frac{1}{2}$ of 9 times $\frac{4}{5}$, or $\frac{4 \times 9}{5 \times 2}$, which, reduced, is $3\frac{3}{5}$, the same result.

In the last solution the division is performed by multiplying by the divisor inverted.

2. What is the quotient of $\frac{45}{392}$ divided by $\frac{15}{49}$? *Ans.* $\frac{3}{8}$.

3. What is the quotient of $\frac{155}{360}$ divided by $\frac{5}{36}$?

211. Rules for Dividing a Fraction by a Fraction.—*1. Reduce the divisor and dividend, if necessary, to similar fractions, and divide the numerator of the dividend by the numerator of the divisor. Or,*

2. Multiply the dividend by the denominator of the divisor, and divide the result by the numerator. Or,

3. Multiply the dividend by the divisor inverted.

The value of a *complex fraction* is found by performing the division indicated by the expression.

PROBLEMS.

Divide—

1. $\frac{8}{9}$ by $\frac{2}{3}$.
2. $\frac{7}{9}$ by $\frac{21}{28}$. *Ans.* $\frac{28}{27}$.
3. $\frac{4}{7}$ by $\frac{5}{9}$.
4. $\frac{14}{15}$ by $\frac{7}{10}$. *Ans.* $1\frac{1}{3}$.
5. $\frac{21}{100}$ by $\frac{7}{1000}$.
6. $\frac{17}{21}$ by $\frac{3}{5}$. *Ans.* $1\frac{22}{63}$.
7. $\frac{6}{11}$ by $\frac{2}{7}$.
8. $\frac{3}{7}$ by $\frac{6}{35}$. *Ans.* $2\frac{1}{2}$.
9. $\frac{43}{49}$ by $\frac{5}{49}$.
10. $\frac{13}{19}$ by $\frac{7}{9}$. *Ans.* $\frac{117}{133}$.
11. $\frac{21}{25}$ by $\frac{14}{15}$.
12. $\frac{52}{100000}$ by $\frac{13}{100}$. *Ans.* $\frac{1}{250}$.

13. What is the value of $\frac{13}{15} \div \frac{4}{5}$? *Ans.* $1\frac{1}{12}$.

14. What is the value of $\dfrac{\frac{3}{4}}{\frac{5}{6}}$?

$$\dfrac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3 \times \overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}} \times 5} = \frac{9}{10}$$

SOLUTION.—$\dfrac{\frac{3}{4}}{\frac{5}{6}}$ is the same as $\frac{3}{4} \div \frac{5}{6}$, and $\frac{3}{4} \div \frac{5}{6}$ gives $\frac{9}{10}$, the result required.

15. What is the value of $\dfrac{\frac{7}{8}}{\frac{2}{3}}$? *Ans.* $\frac{15}{16}$.

16. What is the value of $\dfrac{\frac{21}{32}}{\frac{7}{9}}$? *Ans.* $\frac{27}{32}$.

17. Divide $40\frac{1}{2}$ by $\frac{3}{4}$.

$$40\frac{1}{2} = \frac{81}{2}\,;\ \frac{81}{2} \div \frac{3}{4} = \frac{\overset{27}{\cancel{81}} \times \overset{2}{\cancel{4}}}{\cancel{2} \times \cancel{3}} = 54$$

SOLUTION.—$40\frac{1}{2} = \frac{81}{2}$; hence, $40\frac{1}{2} \div \frac{3}{4}$ is the same as $\frac{81}{2} \div \frac{3}{4}$, which gives 54, the result required.

18. How many times is $\frac{9}{10}$ contained in $5\frac{3}{8}$? *Ans.* $5\frac{35}{36}$.

19. What is the value of $17\frac{3}{5} \div 1\frac{14}{15}$?

20. Divide $\frac{11}{12}$ by $7\frac{4}{7}$. *Ans.* $\frac{77}{636}$.

21. $\frac{\frac{3}{8}}{5\frac{1}{2}}$ is equal to what number? *Ans.* $\frac{3}{44}$.

22. If you should spend $\frac{5}{8}$ of a dollar a week, in what time would you spend $7\frac{3}{16}$ dollars? *Ans.* $11\frac{1}{2}$ weeks.

23. If I pay $\frac{9}{10}$ of a dollar for $\frac{3}{4}$ of a bushel of corn, what shall I pay for a bushel?

24. If a man can travel $11\frac{2}{3}$ miles in $1\frac{3}{4}$ hours, in what time can he travel one mile?

25. When $5\frac{1}{4}$ pounds of veal cost $73\frac{4}{5}$ cents, what is the cost per pound? *Ans.* $14\frac{2}{35}$ cents.

SECTION XXI.

RELATION OF NUMBERS.

212.—Ex. 1. What part of 7 cents is 1 cent? Is 2 cents? Is 3 cents?

2. How much is 1 divided by 7? 2 divided by 7? 3 divided by 7?

3. What part of 2 oranges is $\frac{3}{4}$ of an orange?

4. What part of 2 is 1? What part of 2 is $\frac{3}{4}$ of 1?

DEFINITION.

213. The **Relation** of one number to another is the part the former is of the latter, or is the number of times the former contains the latter.

214. Principle.—*Only similar numbers can have relation to each other.*

For, only such numbers as have the same unit can be compared with or measured by each other.

WRITTEN EXERCISES.

215.—Ex. 1. What is the relation of 21 to 7?

$$21 \text{ to } 7 = \frac{21}{7} = 3$$

SOLUTION.—The relation of 21 to 7 is the number of times that 21 contains 7.

21 is 3 times 7; hence 3 is the relation required.

2. What is the relation of $\frac{2}{3}$ to 4, or what part of 4 is $\frac{2}{3}$?

$$\frac{2}{3} \text{ to } 4 = \frac{2}{3} \div 4 = \frac{\overset{1}{\cancel{2}}}{3 \times \underset{2}{\cancel{4}}} = \frac{1}{6}$$

Or,

$$\frac{2}{3 \times 4} = \frac{1}{6}$$

SOLUTIONS.—The relation of $\frac{2}{3}$ to 4 is $\frac{2}{3}$ divided by 4, or $\frac{1}{6}$.

Or, since 1 is $\frac{1}{4}$ of 4, $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{1}{4}$ of 4, and $\frac{2}{3}$ is $\frac{2}{3}$ of $\frac{1}{4}$ of 4, or $\frac{2}{3 \times 4}$ of 4, which equals $\frac{1}{6}$ of 4.

3. What is the relation of $\frac{2}{5}$ to $\frac{3}{4}$, or what part of $\frac{3}{4}$ is $\frac{2}{5}$?

$$\frac{2}{5} \text{ to } \frac{3}{4} = \frac{2}{5} \div \frac{3}{4} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

Or,

$$\frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

SOLUTIONS.—The relation of $\frac{2}{5}$ to $\frac{3}{4}$ is $\frac{2}{5}$ divided by $\frac{3}{4}$, which is $\frac{2 \times 4}{5 \times 3}$, or $\frac{8}{15}$.

Or, since 1 is $\frac{4}{4}$, $\frac{4}{4}$ is $\frac{4}{3}$ of $\frac{3}{4}$, $\frac{1}{5}$ of 1 is $\frac{1}{5}$ of $\frac{4}{3}$ of $\frac{3}{4}$, and $\frac{2}{5}$ of 1 are $\frac{2}{5}$ of $\frac{4}{3}$ of $\frac{3}{4}$, or $\frac{8}{15}$ of $\frac{3}{4}$, as before.

4. What is the relation of $1\frac{4}{5}$ to $\frac{6}{11}$? *Ans.* $3\frac{3}{10}$.

216. Rules for Finding the Relation of Numbers.—*1. Divide the number compared by that with which it is compared. Or,*

2. Divide the number denoting the part by that denoting the whole.

PROBLEMS.

What is the relation of—

1. 31 to 17? *Ans.* $1\frac{14}{17}$.
2. 42 to $\frac{6}{7}$?
3. $\frac{13}{17}$ to 14? *Ans.* $\frac{13}{238}$.
4. $\frac{2}{3}$ to $\frac{27}{28}$?

What part of—

5. 35 is 21? *Ans.* $\frac{3}{5}$.
6. 36 is $\frac{18}{25}$?
7. $\frac{13}{25}$ is $\frac{8}{21}$? *Ans.* $\frac{200}{273}$.
8. $7\frac{1}{7}$ is $\frac{5}{6}$?

9. James had 100 dollars, and gave away $7\frac{1}{2}$ dollars. What part of 100 dollars did he give away? *Ans.* $\frac{3}{40}$.

10. A man gave 125 dollars for a carriage, and sold it for 100 dollars. For what part of the cost did he sell it?

11. If James receives 60 dollars for 15 weeks' work, how much should he receive, at the same rate, for 25 weeks' work?

$$\frac{25}{15} = \frac{5}{3};\quad \frac{\overset{20}{\cancel{60}} \times 5}{\cancel{3}} = 100$$

SOLUTION.—If he receives 60 dollars for 15 weeks, for 25 weeks, which is $\frac{25}{15}$, or $\frac{5}{3}$, as many weeks, he should receive $\frac{5}{3}$ of 60 dollars, or 100 dollars.

12. If a man can hoe a field in $3\frac{3}{4}$ days, what part of it can he hoe in $2\frac{1}{4}$ days? *Ans.* $\frac{3}{5}$.

13. A and B hire a pasture together; A puts in 15 cows, and B puts in 40. If B's share of the rent is 200 dollars, how much is A's?

14. Jason gathered $\frac{5}{8}$ of a bushel of nuts, and Daniel gathered $\frac{3}{5}$ of a bushel. What is the value of Daniel's, if Jason's are worth 150 cents?

15. Jones has 175 sheep, and Reed has $\frac{3}{7}$ as many. How would their numbers compare if Reed were to have 135 more?
Ans. Reed would then have $\frac{6}{5}$ as many as Jones.

TEST QUESTIONS.

217.—1. What kind of fractions only can be ADDED? Why? How are dissimilar fractions prepared for adding? How may mixed numbers be added? Give the rules for addition of fractions.

2. Why is it necessary that fractions should have a common denominator before SUBTRACTING? How may mixed numbers be subtracted? Give the rules for subtraction of fractions.

3. How is a fraction MULTIPLIED by an integer? Why does multiplying the numerator multiply the fraction? Why does dividing the denominator multiply the fraction? How is a number multiplied by a fraction? Why?

4. What is the RULE for multiplying a fraction by an integer? The rules for multiplying an integer by a fraction? When the multiplier is an improper fraction, will the product be greater or less than the multiplicand? Why?

5. What is a COMPOUND FRACTION? How do you find the value of a compound fraction? What are the rules for multiplying a fraction by a fraction?

6. What is a COMPLEX FRACTION? How is a fraction inverted? How is the value of a complex fraction found?

7. In what two ways may a fraction be DIVIDED? Why? What is the rule for dividing a fraction by an integer? What are the rules for dividing an integer by a fraction? The rules for dividing a fraction by a fraction?

8. What is the RELATION of one number to another? What numbers only can have relation to each other? Why? Give the rules for finding the relation of numbers.

SECTION XXII.

REVIEW PROBLEMS.

MENTAL EXERCISES.

218.—Ex. 1. How many apples are there in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$ and $\frac{7}{12}$ of an apple?

2. How much is $\frac{5}{12}$ less than $\frac{3}{4}$? $\frac{11}{12}$ more than $\frac{2}{3}$?

3. Add $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{1}{2}$. $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{3}{16}$.

4. Arthur has $5\frac{3}{4}$ dollars, and John has $9\frac{7}{8}$ dollars. What is the total amount of their money?

5. I bought a hat for $3\frac{3}{10}$ dollars, and gave in payment a five-dollar bill. How much change should I receive?

6. If you should earn in a week $7\frac{1}{2}$ dollars, and spend $5\frac{3}{8}$ dollars, how much would you save?

7. At $5\frac{5}{8}$ dollars each, how much will 11 hats cost?

8. A man owned $\frac{15}{16}$ of a ship, and sold $\frac{2}{3}$ of his share. What part of the ship did he sell?

9. A farmer has 54 sheep, and his son has $\frac{5}{9}$ of that number. How many has his son?

10. I sold a watch for 48 dollars, which was $\frac{4}{5}$ of its value. What was its value?

11. The difference between $\frac{2}{3}$ and $\frac{3}{4}$ of my age is 4 years. What is my age?

12. After spending $\frac{1}{2}$ of $\frac{3}{4}$ of my money I had 22 dollars left. How much did I have at first?

13. If a boy can pick $\frac{15}{16}$ of a bushel of berries in 5 hours, what part of a bushel can he pick in 1 hour?

14. How many yards of cloth, at $\frac{3}{8}$ of a dollar a yard, can be bought for 6 dollars?

15. At $\frac{1}{4}$ of a dollar a peck, how many pecks of apples can be bought for $\frac{7}{8}$ of a dollar?

16. How many months will $5\frac{3}{8}$ barrels of flour last a family which uses $\frac{3}{4}$ of a barrel a month?

17. A boy being asked how much money he had, said that $\frac{2}{5}$ of $\frac{3}{4}$ of it was $4\frac{1}{2}$ dollars. How much did he have?

18. How many times can you pay 10 dollars from your money, if 18 dollars is $\frac{3}{5}$ of it?

19. John bought a horse for 125 dollars, and sold it for $\frac{6}{5}$ of what he gave. How much did he gain by the transaction?

20. If 24 dollars is $\frac{4}{5}$ of your money, how many books at $2\frac{1}{2}$ dollars each can you buy?

21. Mary had 28 cents, and gave $\frac{3}{7}$ of it for 3 oranges. How much did each orange cost?

22. If 3 yards of cloth cost $7\frac{1}{3}$ dollars, what will $\frac{3}{11}$ of a yard cost?

23. If $13\frac{1}{5}$ tons of hay will last 22 horses one month, how many tons will last 15 horses the same time?

24. When butter is $\frac{3}{5}$ of a dollar a pound, how many pounds of butter will pay for 7 pounds of tea, worth $\frac{9}{10}$ of a dollar a pound?

25. When an acre of ground is worth 60 dollars, what part of an acre can be bought for 15 dollars?

26. If I have $\frac{3}{4}$ of $\frac{4}{5}$ of 18 dollars, and you have $\frac{5}{6}$ of 15 dollars, what part of my money is the difference of our money?

27. If $\frac{4}{5}$ of a bushel of oats be given for $\frac{2}{3}$ of a bushel of corn, what is the cost of a bushel of oats when corn is $\frac{3}{4}$ of a dollar a bushel?

28. How many pairs of shoes, at $1\frac{3}{5}$ dollars a pair, may be exchanged for 4 pairs of boots, at $4\frac{4}{5}$ dollars a pair?

WRITTEN EXERCISES.

219.—Ex. 1. Which is the greatest and which the least of the fractions $\frac{2}{13}$, $\frac{7}{45}$ and $\frac{3}{20}$? *Ans.* $\frac{7}{45}$, greatest; $\frac{3}{20}$, least.

2. A owns $\frac{1}{2}$ of a steamboat, B owns $\frac{1}{3}$, C owns $\frac{1}{7}$, and D owns the rest. What part of the steamboat does D own?

3. The sum of two numbers is $13\frac{3}{4}$, and one of the numbers is $5\frac{11}{13}$. What is the other number? *Ans.* $7\frac{47}{52}$.

4. The difference between two numbers is $\frac{1}{12}$, and the smaller of the numbers is $\frac{9}{20}$. What is the larger number?

5. What number divided by $\frac{63}{64}$ will give $\frac{3}{8}$?

6. If $\frac{7}{8}$ of my money is 3612 dollars, how much money have I? *Ans.* 4128 dollars.

7. At $7\frac{2}{9}$ dollars a ton, what part of a ton of coal can be bought for $2\frac{1}{6}$ dollars?

8. What is the greatest common divisor of $1\frac{1}{2}$, $\frac{3}{4}$, and $2\frac{2}{5}$?

$$1\frac{1}{2},\ \frac{3}{4},\ 2\frac{2}{5} = \frac{30}{20},\ \frac{15}{20},\ \frac{48}{20}$$

$$\textit{Greatest Com. Divisor, } \frac{3}{20}$$

SOLUTION.—$1\frac{1}{2}$, $\frac{3}{4}$, $2\frac{2}{5}$, expressed as similar fractions, having the *least* common denominator, are $\frac{30}{20}$, $\frac{15}{20}$, $\frac{48}{20}$.

The greatest common divisor of the numerators of these fractions is 3; and the greatest common divisor of the denominators is 20. Hence, $\frac{3}{20}$ is the greatest common divisor required.

9. What is the width of the largest blocks that will exactly fit across each of two walks, the one $4\frac{1}{2}$ feet wide, and the other $6\frac{3}{4}$ feet? *Ans.* $2\frac{1}{4}$ feet.

10. At the rate of $5\frac{1}{4}$ miles per hour, what time will be required to walk 126 miles?

11. A and B own a lot; A owns $\frac{3}{8}$ of it, and B owns $\frac{5}{8}$. The part that B owns is worth 30 dollars more than that owned by A. How much is each of their parts worth?

Ans. A's, 45 dollars; B's, 75 dollars.

12. What is the least common multiple of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{12}$?

$$\frac{2}{3},\ \frac{3}{4},\ \frac{1}{12} = \frac{8}{12},\ \frac{9}{12},\ \frac{1}{12}$$

$$\textit{Least Com. Mult.} = \frac{72}{12} = 6$$

SOLUTION.—$\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{12}$, expressed as similar fractions, having the *least* common denominator, are $\frac{8}{12}$, $\frac{9}{12}$, $\frac{1}{12}$. The least common multiple of the numerators 8, 9 and 1 is 72; the least common multiple of the denominators is 12. Hence, $\frac{72}{12}$, or 6, is the least common multiple required.

13. What is the least number of cents that will pay for a number of peaches at $\frac{3}{4}$ of a cent each, a number of apples at $\frac{2}{3}$ of a cent each, and a number of pears at $\frac{5}{6}$ of a cent each?

14. When $\frac{3}{5}$ of a ton of coal costs $6\frac{2}{5}$ dollars, how much does $\frac{5}{8}$ of a ton cost? *Ans.* $6\frac{2}{3}$ dollars.

15. A pole was broken into two pieces. One of the pieces, which measured $31\frac{1}{5}$ feet, was $\frac{3}{8}$ of the length of the pole. How long was the pole?

16. I have three rooms, $13\frac{1}{2}$ feet, $15\frac{3}{4}$ feet, and 18 feet wide, respectively. What is the widest carpeting that will cover each with whole breadths?

17. John has 164 dollars, which is $\frac{8}{11}$ as much as Willard has. How much has Willard? *Ans.* $225\frac{1}{2}$ dollars.

18. If $\frac{3}{8}$ of an acre of land cost 48 dollars, what will $\frac{7}{16}$ of an acre cost?

$$\frac{3}{8}=\frac{6}{16};\ \frac{\overset{8}{\cancel{48}}\times 7}{\cancel{6}}=56$$

SOLUTION.—$\frac{3}{8}$ of an acre is $\frac{6}{16}$ of an acre. If $\frac{6}{16}$ of an acre cost 48 dollars, $\frac{7}{16}$ of an acre, or $\frac{7}{6}$ as much, will cost $\frac{7}{6}$ of 48 dollars, or 56 dollars.

19. $26\frac{2}{3}$ is $\frac{4}{9}$ of what number?

20. If $\frac{2}{3}$ of a sum of money is 220 dollars, how much is $\frac{3}{11}$ of it? *Ans.* 90 dollars.

21. When $3\frac{4}{7}$ cords of wood can be bought for 25 dollars, how much can be bought for $10\frac{1}{2}$ dollars?

22. If a man inherits $\frac{3}{7}$ of an estate of 2000 acres, and sells $\frac{2}{3}$ of his share, how many acres does he retain? *Ans.* $285\frac{5}{7}$.

23. James bought a horse and carriage for 400 dollars. The horse cost $\frac{3}{5}$ of the price of the carriage. What was the price of each?

24. If 5 be added to each term of the fraction $\frac{5}{3}$, how much will the value of the fraction be diminished? *Ans.* $\frac{5}{12}$.

25. If 8 men can do a piece of work in $\frac{1}{2}$ of $\frac{6}{7}$ of $\frac{21}{4}$ of $3\frac{1}{2}$ days, how many men could do it in 1 day? *Ans.* 63.

26. An estate worth \$6000 has been divided as follows: The original owner had 5 children, each of his children had 4 children, and his grandchildren now own the estate in equal shares. What fraction represents the share of each grandchild, and what is the value of 15 of the shares?

Ans. $\frac{1}{5}$ of $\frac{1}{4}$ of \$6000; \$4500.

27. A man, whose estate was worth 5000 dollars, bequeathed to his widow, $\frac{1}{3}$ of it; to his son, $\frac{3}{10}$ of it; and the remainder in equal shares to his two daughters. What was the value of the share of each?

Ans. Widow, $1666\frac{2}{3}$ dollars; the son, 1500 dollars; and each daughter, $916\frac{2}{3}$ dollars.

SECTION XXIII.

DECIMAL FRACTIONS.

220.—Ex. 1. If a melon be divided into ten equal parts, what part of the whole will one of the pieces be? Two of the pieces?

2. What is the fractional unit of a number of tenths?

3. If a tenth of a melon be divided into ten equal parts, what part of the melon will one of the pieces be? Two of the pieces?

4. In a number of hundredths, what is the fractional unit?

5. If a hundredth of a unit be divided into ten equal parts, what part of the unit will one of the parts be?

6. In a number of thousandths, what is the fractional unit?

7. How many thousandths is a hundredth? How many hundredths is a tenth? How many tenths is 1?

8. How is one tenth written as a common fraction? One hundredth? One thousandth?

9. How are two tenths written as a common fraction? Four hundredths? Five thousandths?

DEFINITIONS.

221. A **Decimal Fraction** is a number of tenths, hundredths or thousandths, etc.

Thus, 1 tenth, 3 tenths, 7 thousandths, are decimal fractions.

Decimal fractions are so called because they are decimal divisions of a unit.

Thus, if 1 be divided into 10 equal parts, each part is 1 tenth.

"	$\frac{1}{10}$	"	" 10	"	"	1 hundredth.
"	$\frac{1}{100}$	"	" 10	"	"	1 thousandth.
"	$\frac{1}{1000}$	"	" 10	"	"	1 ten-thousandth.

And so on.

222. Since, in the decimal system of notation, a figure of any order expresses $\frac{1}{10}$ the value expressed by the same figure written in the next order on the left, each figure written at the right of the decimal point must express a value $\frac{1}{10}$ as great as that expressed by the same figure written in the next order on the left.

Hence, $\frac{1}{10}$	may	be written,	*0.1*,	or simply,	*.1*.
$\frac{1}{100}$	"	"	*0.01*,	"	*.01*.
$\frac{1}{1000}$	"	"	*0.001*,	"	*.001*.
$\frac{1}{10000}$	"	"	*0.0001*,	"	*.0001*.

And so on, in continuation of the notation of integers, to the right of ones, since decimal fractions and integers have the same scale.

223. The **Decimal Point** (.) distinguishes the decimal fraction from an integer.

224. A **Decimal** is a decimal fraction expressed without its denominator.

225. A **Pure Decimal** consists of a decimal only.

Thus, .35, which is read 35 hundredths, is a pure decimal.

226. A **Mixed Decimal** consists of an integer and a decimal.

Thus, 15.7, which is read 15 ones and 7 tenths, or 15 and 7 tenths, is a mixed decimal.

227. A **Complex Decimal** consists of a decimal with a common fraction annexed.

Thus, .005$\frac{4}{7}$, which is read 5$\frac{4}{7}$ thousandths, is a complex decimal.

228. The **Names** of decimal orders are derived from the names of the orders of integers.

Thus, the name of the first order of decimals below ones is *tenths*, corresponding to *tens*, the name of the first order higher than ones; the second order below ones is *hundredths*, corresponding to *hundreds*, the second order higher than ones; and so on. That is, the names of the orders below and above ones correspond to each other, with the exception of the decimal termination *ths*.

TABLE OF ORDERS.

Millions.	Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Ones.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
7	6	5	4	3	2	1	.	2	3	4	5	6	7
INTEGERS.								DECIMALS.					

The number in the table is read: seven million six hundred fifty-four thousand three hundred twenty-one, and two hundred thirty-four thousand five hundred sixty-seven millionths.

229. Principles.—1. *Ten of any order of decimals are equal to one of the order next higher.*

For, decimals are a continuation of the notation of integers, below units or ones, by orders of tenths, hundredths, etc., on a scale of ten.

2. *The denominator of a decimal is* 1 *with as many ciphers annexed as there are orders in the decimal.*

For, .1 is equal to $\frac{1}{10}$; .03 is equal to $\frac{3}{100}$, etc.

3. *Integers and decimals may form one expression.*

For, both have the same scale, and the value of the unit denoted by a figure in the expression of each is determined by its order with reference to the decimal point.

READING DECIMALS.

230.—Ex. 1. Read .308.

SOLUTION.—.308 = 3 tenths 0 hundredths 8 thousandths; or, .3 = 30 hundredths = 300 thousandths, and 300 thousandths + 8 thousandths = 308 thousandths. Hence, .308 may be read *308 thousandths.*

2. Read 61.25.

SOLUTION.—61.25 = 61 ones and decimal .25. .25 = 2 tenths 5 hundredths, or, since .2 = 20 hundredths, 20 hundredths + 5 hundredths = 25 hundredths. Hence, 61.25 may be read *61 ones and 25 hundredths;* or, *61 and 25 hundredths.* It may also be read *6125 hundredths.*

3. Read .1573; read 4.59; read 13.1342; read 1.0001.

231. Rule for Reading Decimals.—*Read the expression, if a pure decimal, as an integer, giving it the name of its right-hand order. Or,*

Read the expression, if a mixed decimal, as a mixed number, or as its equivalent improper fraction.

PROBLEMS.

Copy and read—

1. .1256.	4. .09.	7. .3106.
2. .304$\frac{1}{3}$.	5. .0505.	8. .60135.
3. .00372.	6. .77777.	9. .000003.

10. Copy and read as a mixed decimal, 5.19.

11. Copy and read as an improper fraction, 11.5.

Copy and read as mixed numbers—

12. 6.171.	16. 45.1145.	20. 100.000001.
13. 10.001.	17. 73.00956.	21. 1005.63004.
14. 91.57$\frac{1}{2}$.	18. 99.00003.	22. 1131.40056.
15. 1.5072.	19. 7.00031$\frac{1}{3}$.	23. 134971.023.

WRITING DECIMALS.

232.—Ex. 1. Write three hundred five thousandths.

SOLUTION.—Three hundred five thousandths = 305 thousandths = 30 hundredths and 5 thousandths; and 30 hundredths = 3 tenths and 0 hundredths. Hence, 305 thousandths = 3 tenths 0 hundredths 5 thousandths = .305.

2. Write four, and fifty-nine hundredths.

3. Write thirteen, and one thousand three hundred forty-two ten-thousandths.

233. Rule for Writing Decimals.—*Write the decimal as in integers, and place the decimal point so that each figure shall stand in its proper order, marking the absence of units of any order, if necessary, by a cipher.*

PROBLEMS.

Write in figures—

1. Five hundredths. *Ans.* .05.

2. Sixty-seven hundredths.

3. One hundred fifteen thousandths.
4. Thirty-five ten-thousandths. *Ans.* .0035.
5. Seventy, and seventy-five thousandths. *Ans.* 70.075.
6. Five hundred fifty-three hundred-thousandths.
7. Six hundred, and six and one-fourth hundredths. *Ans.* 600.06$\frac{1}{4}$.
8. One thousand four hundred one millionths.
9. One hundred thousand, and 563 hundred-thousandths.
10. Fourteen millions, and 15003 ten-millionths.
11. One hundred sixty-three thousand, and one hundred sixty-three thousandths. *Ans.* 163000.163.
12. Seven million, and seventy-three hundred millionths.
13. One billion one million one, and five hundred thirty thousand eleven billionths. *Ans.* 1001000001.000530011.

Write in the decimal form—

14. $\frac{12}{100}$. *Ans.* .12.
15. $\frac{9}{10}$.
16. $\frac{45}{1000}$.
17. $18\frac{103}{100000}$. *Ans.* 18.00103.
18. $\frac{2002}{10000}$. *Ans.* .2002.
19. $65\frac{81}{100}$.
20. $131\frac{7}{10}$. *Ans.* 131.7.
21. $1000\frac{8}{1000000}$.

SECTION XXIV.

REDUCTION OF DECIMALS.

CASE I.

Decimals Reduced to a Common Denominator.

234.—Ex. 1. In 3 tenths of a melon, how many hundredths of a melon?

2. In 4 tenths, how many hundredths? In 40 hundredths, how many thousandths?

3. Express 4 as tenths. .5 as hundredths. .25 as thousandths. 4.0 as hundredths.

4. Express 5 and .2 each in hundredths. Express .4 and .15 each in thousandths.

235. Principle.—*Annexing ciphers to a decimal, or removing ciphers from the right of a decimal, does not change the value expressed.*

For annexing ciphers does not change the order of the other figures with reference to the decimal point.

Thus, $.5 = .50 = .500$, or $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$

WRITTEN EXERCISES.

236.—Ex. 1. Reduce .9, .104 and 3.1945 to equivalent decimals having the least common denominator.

$.9 = .9000$
$.104 = .1040$
3.1945

SOLUTION.—The smallest decimal order of any of the decimals is that of ten-thousandths.

Since annexing ciphers to decimals does not change their values, we reduce .9 to ten-thousandths by annexing three ciphers and .104 to ten-thousandths by annexing one cipher, which gives as the decimals required, .9000, .1040 and 3.1945.

2. Reduce .006, 7.45 and .0005 to a common denominator.

237. Rule for Reduction of Decimals to a Common Denominator.—*Make the decimals have the same number of decimal orders, by annexing ciphers.*

PROBLEMS.

1. Reduce .15 and .0016 to equivalent decimals having a common denominator.

2. Change .5, .007 and 8 to similar decimals.

Ans. .500, .007 and 8.000.

3. Reduce 3.4, .00324 and .65 to equivalent decimals having the least common denominator.

CASE II.

Decimals Reduced to Common Fractions.

238.—Ex. 1. In 5 tenths of a melon there are how many halves of a melon?

2. How many halves in $\frac{5}{10}$? In $\frac{50}{100}$? $\frac{500}{1000}$?

3. How many fourths in $\frac{25}{100}$? In $\frac{50}{100}$? $\frac{75}{100}$? $\frac{250}{1000}$?

4. How many fifths in $\frac{2}{10}$? In $\frac{6}{10}$? $\frac{20}{100}$? $\frac{200}{1000}$?

5. How many halves in .5? Fourths in .75? Fifths in .8?

WRITTEN EXERCISES.

239.—Ex. 1. Reduce .055 to an equivalent common fraction.

$.055=\frac{55}{1000}=\frac{11}{200}$ SOLUTION.—Omitting the decimal point and writing the denominator changes .055 to the equivalent common fraction $\frac{55}{1000}$, which, reduced to its lowest terms, is $\frac{11}{200}$.

2. Reduce .35 to an equivalent common fraction.

240. Rule for Reduction of Decimals to Common Fractions.—*Omit the decimal point, write the denominator under the given numerator, and, if necessary, reduce the fraction to its lowest terms.*

PROBLEMS.

Reduce to common fractions in their lowest terms—

1. .0075.	*Ans.* $\frac{3}{400}$.	5. .08.	*Ans.* $\frac{2}{25}$.
2. .375.		6. 1.06.	
3. .03125.	*Ans.* $\frac{1}{32}$.	7. .096.	*Ans.* $\frac{12}{125}$.
4. .3216.	*Ans.* $\frac{201}{625}$.	8. .006943.	

9. Reduce 1.06 to a mixed number. *Ans.* $1\frac{3}{50}$.

10. Reduce 503.1875 to a mixed number. *Ans.* $503\frac{3}{16}$.

11. Reduce $.16\frac{2}{3}$ to an equivalent common fraction.

$$.16\frac{2}{3}=\frac{16\frac{2}{3}}{100}=\frac{\frac{50}{3}}{100}=\frac{50}{300}=\frac{1}{6}$$

SOLUTION.—$.16\frac{2}{3}$ is equal to $\frac{16\frac{2}{3}}{100}$, which, reduced (Art. 211), is $\frac{1}{6}$, the fraction required.

12. Reduce $.10\frac{5}{11}$ to an equivalent common fraction.

13. Reduce $5.3\frac{3}{4}$ to an equivalent mixed number.

14. Reduce $.43\frac{52}{99}$ to an equivalent common fraction. *Ans.* $\frac{4309}{9900}$.

CASE III.

Common Fractions Reduced to Decimals.

241.—Ex. 1. In 1 half of a melon, how many tenths of a melon? In 1 fifth of a melon?

2. How many tenths in $\frac{3}{5}$? Hundredths in $\frac{3}{5}$?

3. How many hundredths in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{5}$?

WRITTEN EXERCISES.

242.—Ex. 1. Reduce $\frac{3}{8}$ to an equivalent decimal.

$$\begin{array}{r} 8\overline{)3.000} \\ \hline .375 \end{array}$$

SOLUTION.—$\frac{3}{8}$ is $\frac{1}{8}$ of 3. 3 is equal to 30 tenths; $\frac{1}{8}$ of 30 tenths is 3 tenths, with 6 tenths remaining.

6 tenths are 60 hundredths; $\frac{1}{8}$ of 60 hundredths is 7 hundredths, with 4 hundredths remaining.

4 hundredths are 40 thousandths; $\frac{1}{8}$ of 40 thousandths is 5 thousandths. 3 tenths + 7 hundredths + 5 thousandths = .375. Hence, $\frac{3}{8}$ is equivalent to .375.

2. Reduce $\frac{11}{200}$ to an equivalent decimal.

243. Rule for Reduction of Common Fractions to Decimals.—*Reduce the numerator to tenths, hundredths, etc., by annexing ciphers; divide the result by the denominator, and point off as many orders for decimals in the quotient, as there were ciphers annexed.*

PROBLEMS.

Reduce to decimals—

1. $\frac{9}{25}$.	*Ans.* .36.	5. $\frac{19}{20}$.	*Ans.* .95.
2. $\frac{3}{4}$.		6. $\frac{7}{8}$.	
3. $\frac{5}{2}$.	*Ans.* 2.5.	7. $\frac{201}{625}$.	*Ans.* .3216.
4. $\frac{3}{32}$.	*Ans.* .09375.	8. $\frac{1}{128}$.	*Ans.* .0078125.

9. Reduce $3\frac{3}{4}$ to a mixed decimal.

$$3\frac{3}{4} = 3 + \frac{3}{4} = 3.75$$

SOLUTION.—$3\frac{3}{4} = 3$ and $\frac{3}{4}$. $\frac{3}{4}$ is .75; hence, $3\frac{3}{4}$ is 3 and .75, or 3.75.

10. Reduce $47\frac{3}{16}$ to a mixed decimal. *Ans.* 57.1875.
11. Reduce $\frac{412}{125}$ to a decimal.
12. Reduce $100\frac{24}{25}$ to a mixed decimal. *Ans.* 100.96.
13. Reduce $.15\frac{1}{4}$ to a pure decimal.
14. Reduce $1.00\frac{4}{5}$ to a mixed decimal.
15. Reduce $503\frac{3}{16}$ to a mixed decimal.
16. Reduce $\frac{2}{3}$ to a complex decimal of two orders.

$$\begin{array}{r} 3\overline{)2.00} \\ \hline .66\frac{2}{3} \end{array}$$

SOLUTION.—$\frac{2}{3}$ is $\frac{1}{3}$ of 2. 2 is equal to 20 tenths; $\frac{1}{3}$ of 20 tenths is 6 tenths, with 2 tenths remaining.

2 tenths are 20 hundredths; $\frac{1}{3}$ of 20 hundredths is 6 hundredths, with 2 hundredths remaining.

Here the continual recurring of the same figure in the result, with a like remainder, shows that $\frac{2}{3}$ has no equivalent pure decimal. In such cases, when the reduction has been carried to any desirable extent, we may express the remainder in the form of a common fraction, and the result will be a complex decimal; or we may use the sign + to indicate the incompleteness of the result, as .66 + for $.66\frac{2}{3}$.

17. Reduce $\frac{1}{7}$ to a complex decimal of four orders.
18. Reduce $\frac{1}{9}$ to a decimal of four orders. *Ans.* .1111 +.
19. Reduce $\frac{8}{11}$ to a decimal of four orders.
20. Reduce $\frac{7}{12}$ to a complex decimal of five orders.
21. Reduce $\frac{4}{9000}$ to a decimal of six orders. *Ans.* .000444 +.
22. Reduce $\frac{413}{165}$ to a mixed decimal of five orders.

TEST QUESTIONS.

244.—1. What is a DECIMAL FRACTION? Why is it called decimal? How is a decimal, when written, distinguished from an integer? What is the denominator of a decimal?

2. What do the figures at the right of the DECIMAL POINT express? From what are the names of decimal orders derived? To what do the orders tenths, hundredths, thousandths, etc., correspond? Of what does a pure decimal consist? A mixed decimal? A complex decimal?

3. How many units of any order in a decimal are equal to one of the order next higher? Why may integers and decimals form one expression? What is the rule for reading decimals? For writing decimals?

4. How are decimals REDUCED to a common denominator? Why does annexing a cipher to a decimal not change its value? What is the rule for reducing decimals to common fractions?

5. What is the RULE for reduction of common fractions to decimals? How do you proceed when the common fraction has no equivalent pure decimal?

6. What is a NUMBER? What are figures? What is numeration? Notation? What is the scale of numbers? What is the scale of the ordinary system? What is termed the decimal system? (Art. 33.)

7. What is an INTEGER? A unit of an integer? (Arts. 3, 4.) A fraction? A fractional unit? The unit of a fraction? (Arts. 141, 143.) A decimal fraction? How is a common fraction expressed by figures? How is a decimal expressed by figures? How do integers and decimals correspond in expression? (Art. 228.)

SECTION XXV.

ADDITION AND SUBTRACTION OF DECIMALS.

245.—Ex. 1. How many tenths are 5 tenths and 4 tenths?

2. How many tenths are $\frac{5}{10}$ and $\frac{2}{10}$? .4 and .3?

3. How many hundredths are $\frac{25}{100}$ and $\frac{7}{100}$? .43 and .8?

4. How many hundredths are $\frac{32}{100}$ less $\frac{7}{100}$? .51 less .8?

246. Principle.—*Decimals which are similar may be added or subtracted like integers.*

WRITTEN EXERCISES.

247.—Ex. 1. Add 13.634, 35.423 and 8.56.

$$\begin{array}{r} 13.634 \\ 35.423 \\ 8.560 \\ \hline 57.617 \end{array}$$

Solution.—Writing the numbers so that all the figures of the same order stand in the same column, and adding as in addition of integers, gives 57.617, the sum required.

The 8.56 is made similar to the other decimals by annexing a cipher.

2. From 963.75 subtract 585.125.

$$\begin{array}{r} 963.750 \\ 585.125 \\ \hline 378.625 \end{array}$$

Solution.—Writing the numbers so that figures of the same order stand in the same column, and subtracting as in subtraction of integers, gives 378.625, the difference required.

The cipher annexed to the minuend is usually understood, and the subtraction performed in the same manner as if it were written.

3. What is the sum of 145.07, 3.476 and 11.05?

4. What is the difference between 56.77 and 7.899?

248. Rule for Addition and Subtraction of Decimals.— *Write the numbers so that figures of the same order shall be in the same column.*

Add or subtract in the same manner as if the numbers were integers, and place the decimal point at the left of the order of tenths in the result.

PROBLEMS.

1. What is the sum of .89, .269, 15.2 and .2? *Ans.* 16.559.
2. What is the sum of 11.35, 19, 3.41 and 100.678?
3. What is the difference between .9173 and .2138? *Ans.* .7035.
4. What is the difference between 407 and 91.713? *Ans.* 315.287.
5. Required the value of 270.2 less 75.4075.
6. $450 + 376.004 + 1.08 + .76 + .05 =$ what number? *Ans.* 827.894.
7. $.001 - .00099 =$ what number? *Ans.* .00001.
8. $100 - .10101 =$ what number? *Ans.* 99.89899.
9. What is the sum of ten thousand one hundred one thousandths, ninety-nine, and eighty-nine thousand eight hundred ninety-nine hundred thousandths?
10. What is the sum of 98.75 miles, 100.3655 miles and 15.7875 miles?
11. If 41.674 cubic feet of oak wood, or 64.693 cubic feet of white pine wood, will weigh a ton, how many more cubic feet are there in a ton of pine than in a ton of oak? *Ans.* 23.019 cubic feet.
12. Add seventy-five hundredths; eight, and sixty-seven hundredths; seven, and three hundred fifty-five thousandths; and thirty-one, and seven hundred thirty-five thousandths. *Ans.* 48.51.
13. From three millions three, take three, and three millionths.
14. In one field there are 31.175 acres; in another, 9.1825 acres; and in a third, 25.75 acres. How many acres in the three?
15. What part of 1 must be added to 9.999999 to make 10? *Ans.* One millionth part.
16. From the sum of one and five hundredths, eighteen and thirty-five ten-thousandths, and four hundred four millionths, subtract eleven and ninety-nine ten-millionths. *Ans.* 8.0538941.

SECTION XXVI.

MULTIPLICATION OF DECIMALS.

249.—Ex. 1. How many tenths of a dollar are 3 times 3 tenths of a dollar?

2. How many tenths are 3 times $\frac{2}{10}$? 4 times $\frac{2}{10}$?

3. How many hundredths of a dollar are 7 times 6 hundredths of a dollar?

4. How many hundredths are 7 times $\frac{6}{100}$? 6 times .08?

5. How many hundredths in $\frac{3}{10} \times \frac{2}{10}$? $.6 \times .4$?

6. How many thousandths in $\frac{11}{1000} \times 8$? $\frac{11}{100} \times \frac{8}{10}$?

7. How many ones are 10 times $\frac{6}{10}$? 10 times .6?

8. How many tenths are 10 times $\frac{8}{100}$? 10 times .05?

9. How many hundredths are 9 times $\frac{8}{100}$? How many tenths are 3 times .9?

250. Principles.—1. *Each removal of the decimal point one order to the right makes the value of an expression ten-fold.*

For, by the removal, each figure is made to express units of the next higher order.

Thus, 16.4 = sixteen and four tenths; and 164. = one hundred sixty-four, or ten times 16.4.

2. *Each removal of the decimal point one order to the left makes the value of an expression one-tenth as large as before.*

For, by the removal, each figure is made to express units of the next lower order.

Thus, 164. = one hundred sixty-four; and 16.4 = sixteen and four-tenths, which is one tenth as much as 164.

WRITTEN EXERCISES.

251.—Ex. 1. Multiply .49 by 6.

$$\begin{array}{r} .49 \\ 6 \\ \hline 2.94 \end{array}$$

SOLUTION.—6 times 9 hundredths are 54 hundredths, or 5 tenths and 4 hundredths.

Write 4 for the hundredths of the result, and reserve the 5 tenths. 6 times 4 tenths are 24 tenths; 24 tenths and 5 tenths are 29 tenths, or 2 ones and 9 tenths. Write 2 ones 9 tenths in the result, which gives 2.94, the product required.

2. Multiply 7.6 by .06.

$$\begin{array}{r} 7.6 \\ .06 \\ \hline .456 \end{array}$$

SOLUTION.—.06 is the same as $\frac{1}{100}$ of 6; hence, .06 times 7.6 is the same as $\frac{1}{100}$ of 6 times 7.6.

6 times 7.6 are 45.6, and $\frac{1}{100}$ of 6 times 7.6 is $\frac{1}{100}$ of 45.6, which, found by removing the decimal point two orders to the left, is .456. Hence, 7.6 multiplied by .06 is .456.

Or, the solution may be explained as follows:

6 hundredths times 6 tenths is 36 thousandths, or 3 hundredths and 6 thousandths. Write 6 in the thousandths' order in the result, and reserve the 3 hundredths.

6 hundredths times 7 is 42 hundredths; 42 hundredths and 3 hundredths are 45 hundredths, or 4 tenths and 5 hundredths, which we write in the result, and have, as before, .456.

By the first explanation it appears that in decimals, as in common fractions (Art. 195),

Multiplying by a fraction is the same as multiplying by its numerator and dividing the result by its denominator.

By that process, when the factors are decimals,

The number of decimal orders in the product is made as many as there are in both factors.

3. Multiply 1.704 by .35. *Ans.* .5964.
4. Multiply .0051 by 51. *Ans.* .2601.

252. Rules for Multiplication of Decimals.—*1. Multiply as in integers, and place the decimal point in the product, so that it shall have as many decimal orders as are contained in both factors. Or,*

2. If the multiplier is a decimal, multiply by its numerator and divide by its denominator.

PROBLEMS.

Multiply—

1. .125 by .025. *Ans.* .003125.
2. 8.25 by 4.5.
3. 958 by .34. *Ans.* 325.72.
4. 500.83 by 121. *Ans.* 60600.43.
5. 1.007 by .0041.
6. 4.6337 by 100. *Ans.* 463.37.
7. 4.6337 by 1000.
8. 3.007 by .36. *Ans.* 1.08252.
9. .285 by .003.
10. 3.84062 by 70000. *Ans.* 268843.4.

11. What is the product of 9.688 by $.2\frac{3}{4}$? *Ans.* 2.6642.

12. What will 56 pounds of coffee cost, at $.37\frac{1}{2}$ of a dollar per pound?

13. If a box must have a capacity of 24.958 cubic feet to contain a ton of anthracite coal, what must be the capacity of a box that will contain 4.5 tons?

14. What is the weight of 128 cubic feet of common soil, if the weight of a cubic foot is 137.125 pounds?

Ans. 17552 pounds.

SECTION XXVII.

DIVISION OF DECIMALS.

253.—Ex. 1. In 9 tenths of a dollar how many times 3 tenths?

2. How many times 3 tenths is $\frac{6}{10}$? Is .9?

3. In 42 hundredths of a dollar how many times 6 hundredths of a dollar?

4. How many times is .8 contained in .48? .3 in .06?

5. What is $6 \div 10$? $.6 \div 10$? $.6 \div .6$? $.06 \div .6$?

WRITTEN EXERCISES.

254.—Ex. 1. Divide 1.345 by 5.

$$\begin{array}{r} 5)\underline{1.345} \\ .269 \end{array}$$

SOLUTION.—1.345 is 1345 thousandths; $\frac{1}{5}$ of 1345 thousandths is 269 thousandths, or .269.

2. Divide 1.46 by .25.

$$\begin{array}{l} 25)146.00(5.84 \\ \quad\underline{125} \\ \quad\;\; 210 \\ \quad\;\; \underline{200} \\ \quad\quad 100 \\ \quad\quad \underline{100} \end{array}$$

SOLUTION.—.25 is the same as $\frac{25}{100}$, or $\frac{1}{100} \times 25$. First, divide 1.46 by $\frac{1}{100}$, by multiplying by 100, which is done by removing the decimal point in the dividend two orders to the right, making the dividend 146. 146 divided by 25 is 5, with 21 ones remaining. 21 ones are 210 tenths, which divided by 25 is 8 tenths, with 10 tenths remaining. 10 tenths are 100 hundredths, which, divided by 25, is 4 hundredths. Hence, 1.46 divided by $.25 = 5.84$. Or,

Multiplying both the divisor and dividend by 100, the denominator of the divisor, making the divisor a whole number, and then dividing, we have the quotient 5.84, as before.

By the first explanation it appears that in decimals, as in common fractions (Art. 206),

Dividing by a fraction is performed by multiplying by its denominator and dividing the result by its numerator.

By that process in the division of decimals,

The number of decimal orders in the quotient is made as many as there are in the dividend, less the number in the divisor.

3. Divide .5964 by .35. *Ans.* 1.704.
4. Divide .2601 by 51. *Ans.* .0051.

255. Rules for Division of Decimals.—*1. If the divisor is an integer, divide as in integers, and point off as many decimal orders in the quotient as there are decimal orders in the dividend.*

2. If the divisor is a decimal, make it an integer by moving the decimal point to the right, and move the decimal point in the dividend as many orders to the right, and then divide. Or,

3. If the divisor is a decimal, multiply the dividend by its denominator and divide the result by the numerator.

When the divisor is 10, 100, etc., the division may be performed simply by removing the decimal point in the dividend as many orders to the left as there are ciphers in the divisor.

PROBLEMS.

Divide—

1. .456 by .06. *Ans.* 7.6.	6. 463.37 by 1000.
2. 463.37 by 100.	7. 21 by 56. *Ans.* .375.
3. 1.606 by 44. *Ans.* .0365.	8. .1606 by 44. *Ans.* .00365.
4. 1.08252 by .36.	9. 6 by .006. *Ans.* 1000.
5. 325.72 by 958. *Ans.* .34.	10. 172.8 by .0144.

11. What is the value of $21.17 \div .0073$?
12. What is the value of $.015625 \div 25$? *Ans.* .000625.
13. What is the value of $2.15565 \div 1.05$?
14. What is the quotient of 3.672 by .81? *Ans.* 4.533+.

When, as in the last problem, the division will not terminate, the sign + may be annexed, to indicate that the quotient is not complete. The quotient is then called an *approximate* quotient.

15. What is the approximate quotient of 45.5 divided by 2100? *Ans.* .0216+.

16. If you should travel 787.5 miles in 210 hours, at what rate per hour would you travel?

17. If .0001 is a dividénd and 1.25 a divisor, what is the quotient? *Ans.* .00008.

18. Divide three thousand one hundred twenty-five millionths, by one hundred twenty-five thousandths.

19. If 375 bushels of potatoes be worth as much as 7.5 tons of hay, how many bushels are worth as much as 1 ton?

Ans. 50.

20. A tract of land containing 125.4 acres was sold for 7586.7 dollars; what was the price per acre?

21. How many casks, each containing 31.5 gallons, can be filled from a vat containing 368.25 gallons?

Ans. 11; and 21.75 gallons remain.

22. If 16.284 cubic feet of fire-bricks weigh one ton, how many loads, of a ton each, will a pile of such bricks, containing 333.822 cubic feet, make?

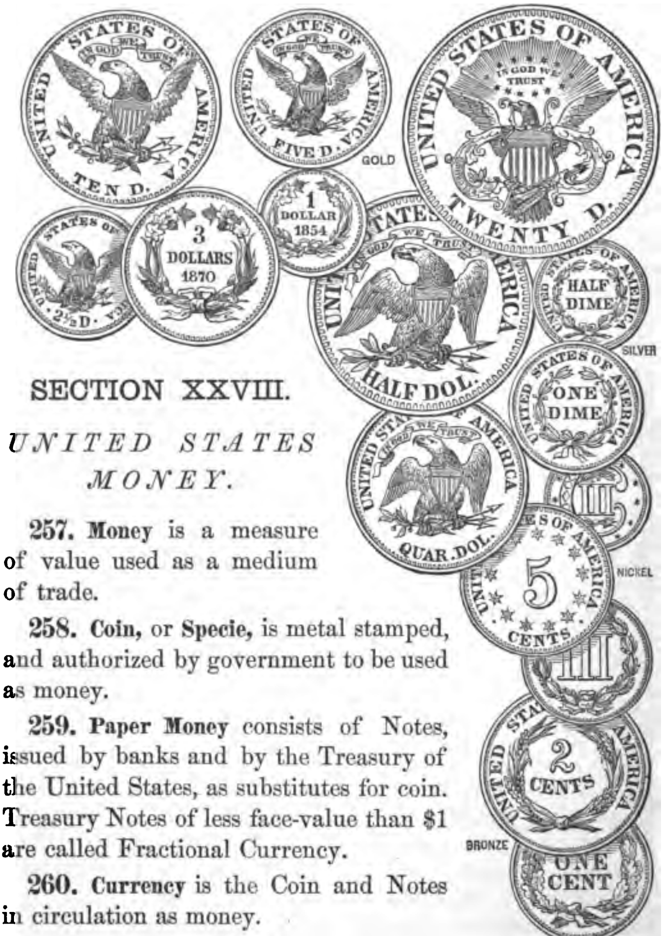
Ans. 20; and 8.142 cubic feet remain.

TEST QUESTIONS.

256.—1. What kind of decimals can be ADDED or SUBTRACTED? What is the rule for addition or subtraction of decimals? What does the decimal point in the result of addition or subtraction mark?

2. In what simple way may the value of a decimal expression be MULTIPLIED by 10? Why does the removal of the decimal point in an expression one order to the right make the value expressed tenfold? What are the rules for multiplication of decimals?

3. In what simple way may the value of a decimal expression be DIVIDED by 10? Why does the removal of the decimal point in an expression, one order to the left, make the value expressed one tenth as large? What are the rules for division of decimals? When is a quotient called an approximate quotient?



SECTION XXVIII.

UNITED STATES MONEY.

257. Money is a measure of value used as a medium of trade.

258. Coin, or **Specie,** is metal stamped, and authorized by government to be used as money.

259. Paper Money consists of Notes, issued by banks and by the Treasury of the United States, as substitutes for coin. Treasury Notes of less face-value than $1 are called Fractional Currency.

260. Currency is the Coin and Notes in circulation as money.

Bullion is uncoined gold or silver. An *Alloy* is a baser metal mixed with a finer.

A *Token* is a coin whose intrinsic value is less than that assigned it by law.

261. United States Money is the currency of the United States.

TABLE.

10 mills (m.)	*are*	*1 cent . . . ct. or c.*
10 cents	"	*1 dime . . . . . . d.*
10 dimes, or 100 cents,	"	*1 dollar . . . . . . $.*

262. The **Coins** of the United States are made of gold, silver, nickel and bronze.

The *Gold Coins* are the fifty-dollar piece, double eagle or twenty-dollar piece, eagle or ten-dollar piece, half-eagle, quarter-eagle, three-dollar piece and dollar.

The *Silver Coins* are the dollar, half-dollar, quarter-dollar, dime, half-dime and three-cent piece.

The *Nickel Coins* are the five-cent piece and three-cent piece.

The *Bronze Coins* are the two-cent piece and cent.

The *gold coins* are made of 9 parts of pure gold and 1 part of an alloy consisting of silver and copper. The *silver coins* are made of 9 parts of pure silver and 1 part of copper. The *nickel coins* are made of 75 parts of copper and 25 parts of nickel. The *bronze coins* are made of 95 parts of copper and 5 parts of zinc and tin.

263. Canada Money, or the money of the Dominion of Canada, consists, like United States money, of dollars and cents. Of this money, 100 cents are 1 dollar.

The *Canada coins* are the twenty-cent, ten-cent and five-cent pieces, made of silver, and the cent, made of bronze.

264. The **Dollar** is the principal unit of United States money.

Dimes, cents and mills may be written respectively as tenths, hundredths and thousandths of dollars, and, when decimally expressed, may be separated from dollars by the decimal point.

265. Dimes, or tens of cents, are commonly regarded as a number of cents.

Thus, 15 dollars 3 dimes 6 cents 7 mills are written $15.367, and read *fifteen dollars thirty-six cents seven mills.*

266. Any decimal of a dollar less than a cent may be read as a decimal of a cent.

Thus, $42.5025 may be read *forty-two dollars fifty and twenty-five hundredths cents.*

WRITTEN EXERCISES.

267. Write and read—

1. $2.45; $17.17; $43.47; $.95; $60.
2. $.05; $.005; $.555; $.606; $6.06.
3. $.0556; $.7585; $.7008; $70.08756.
4. $300.30; $45.303; $90.909; $40.0025.

Write—

5. Fifty cents; five cents five mills.
6. Two dollars twenty-five cents; thirty dollars four cents.
7. 11 dollars 37 cents; 213 dollars 73 cents.
8. 7 mills; 31 cents 5 mills; 14 cents 6 mills.
9. 3 dollars $\frac{40}{100}$; 15 cents 5 mills; 7 dollars 5 mills.
10. 38 and 5 tenths cents; 17 dollars 17 cents; 7 tenths mills.
11. 3 and 3 tenths cents; 15 and 75 hundredths dollars.
12. 63 dollars 63 cents; 6 and 5 tenths mills.
13. 600 dollars 6 and 6 tenths cents.

SECTION XXIX.

REDUCTION OF UNITED STATES MONEY.

268.—Ex. 1. How many mills are 9 cents? Are 11 cents?

2. How many cents are $3? Are $5? Are $6?

3. In $1.15, how many cents? In $2.25?

4. In 90 mills, how many cents? In 110 mills?

5. How many mills are 5 dimes? Are $5?

6. How many dollars are 300 cents? Are 115 cents? Are 500 cents? Are 5000 mills?

DEFINITIONS.

269. Denomination is the name of the unit expressing a number.

Of two denominations, the *higher* is that which expresses the greater value, and the *lower* is that which expresses the less value.

270. Reduction is the process of changing a number to an equivalent number of a different denomination.

WRITTEN EXERCISES.

271.—Ex. 1. Reduce 56 cents to mills.

$56 \times 10 = 560$ SOLUTION.—Since 1 cent is 10 mills, 56 cents must be 56 times 10 mills, or 560 mills.

2. Reduce $43 to cents.

$43 \times 100 = 4300$ SOLUTION.—Since $1 is 100 cents, $43 must be 43 times 100 cents, or 4300 cents.

3. Reduce $25.46 to cents.

$25.46 \times 100 = 2546$ SOLUTION.—Since $1 is 100 cents, $25.46 must be 25.46 times 100 cents, or 2546 cents.

4. Reduce $43 to mills.

$43 \times 1000 = 43000$ SOLUTION.—Since $1 is 1000 mills, $43 must be 43 times 1000 mills, or 43000 mills.

5. Reduce 560 mills to cents.

$560 \div 10 = 56$ SOLUTION.—Since 10 mills are 1 cent, 560 mills must be as many cents as 10 mills are contained times in 560 mills, which are 56 times. Hence, 560 mills are 56 cents.

6. Reduce 2546 cents to dollars.

$2546 \div 100 = 25.46$ SOLUTION.—Since 100 cents are $1, 2546 cents must be as many dollars as 100 cents are contained times in 2546 cents, which are 25.46 times. Hence, 2546 cents are $25.46.

7. Reduce 43000 mills to dollars.

$43000 \div 1000 = 43$ SOLUTION.—Since 1000 mills are $1, 43000 mills must be as many dollars as 1000 mills are contained times in 43000 mills, which are 43 times. Hence, 43000 mills are $43.

8. How many mills are 75 cents?
9. How many cents are 750 mills?
10. How many mills are $17?
11. How many dollars are 17000 mills?

272. Rules for Reduction of United States Money.—*1. To reduce cents to mills, multiply by 10; dollars to cents, multiply by 100; and dollars to mills, multiply by 1000; and give the sign of the denomination as required.*

2. To reduce dollars and cents to cents, or dollars, cents and mills to mills, simply remove the dollar-sign and the decimal point.

3. To reduce mills to cents, divide by 10; cents to dollars, divide by 100; and mills to dollars, divide by 1000; and give the sign of the denomination as required.

PROBLEMS.

Reduce to mills—		Reduce to cents—	
1. 62½ cents.	*Ans.* 625m.	4. 875 mills.	*Ans.* 87½ cts.
2. $13.		5. $93.50.	
3. $6.755.		6. 6790 mills.	*Ans.* 679 cts.

7. How many cents are $14.50? *Ans.* 1450.
8. How many dollars are 1560 cents? *Ans.* 15.60.
9. How many dollars are 14444 mills?
10. Reduce 2089 mills to dollars.
11. Reduce 45395 mills to dollars. *Ans.* $45.395.

SECTION XXX.

COMPUTATIONS IN UNITED STATES MONEY.

273. Computations in United States Money, since in it the dollar is regarded as the unit in a decimal system of notation, are performed as in integers and decimals.

In final results of computations such parts of a cent as are halves, fourths or eighths are generally expressed as common fractions.

WRITTEN EXERCISES.

274.—Ex. 1. What is the sum of $5.87½, $31, $7.50 and $4.37½? *Ans.* $48.75.

2. Add $13.625, $92.50, $60 and $31¼.

3. I paid $5.75 for a hat, $7 for a pair of boots, $1.25 for a handkerchief, and $43.75 for a suit of clothes. How much did the whole cost me? *Ans.* $57.75.

4. What is the sum of \$100.50, \$79.62$\frac{1}{2}$, \$1.67 and \$19.06$\frac{1}{4}$?

5. A merchant has on deposit in one bank \$1634.55; in another, \$37007; in a third, \$3106.75; and has on hand \$563.79. How much money has he in all? *Ans.* \$42312.09.

6. A gentleman has a house in the city worth \$21360, which is \$9836.75 more than his farm cost him. What did his farm cost him?

7. I bought goods to the amount of \$33767.50 on credit, and to the amount of \$9000 for cash. How much does the former amount exceed the latter?

8. If a man's property is valued at \$25360.50, and his debts are \$13675.875, what is he worth? *Ans.* \$11684.62$\frac{1}{2}$.

9. Wilson has dry-goods worth \$931.45, groceries worth \$833.97$\frac{1}{2}$, hardware worth \$363.31$\frac{1}{4}$, a store worth \$9000, and land worth \$1000.75. How much is he worth?

10. If you should save 31 cents a day for 313 working days, what would be the amount saved? *Ans.* \$97.03.

11. A father divided equally among his 9 children \$28369.80. What was the share of each?

12. How much must you save a day to lay by \$97.03 in 313 days?

13. How much must be paid for transporting 5673 bushels of wheat at 4$\frac{1}{2}$ cents a bushel? *Ans.* \$255.28$\frac{1}{2}$.

14. If a man spend 12 cents a day for cigars, and 25 cents for drink, how much will he spend in one year, or 365 days?

15. How much is the rent of a farm of 245$\frac{1}{2}$ acres at \$3.40 per acre? *Ans.* \$834.70.

16. If it requires \$7.50 to give a certain number of boys 12$\frac{1}{2}$ cents each, what is the number of boys?

17. A drover who had \$276.25, bought 5 cows of equal value, which took all the money he had, except \$50. What was the cost of each cow? *Ans.* \$45.25.

18. How many bushels of wheat, at a freight of 4$\frac{1}{2}$ cents a bushel, must be transported for \$255.28$\frac{1}{2}$?

19. I bought 37 hogsheads of molasses, but the price falling, I was obliged to sell it at a loss of \$173.53. What was the loss on each hogshead? *Ans.* \$4.69.

275.—Ex. 1. How much must be paid for 8 barrels of flour, at $9 per barrel?

2. When $72 is paid for 8 barrels of flour, how much is paid for 1 barrel?

3. How many barrels of flour, at $9 a barrel, can be bought for $72?

4. What part of a dollar is $12\frac{1}{2}$ cents? Is 25 cents? Is 50 cents?

5. At $12\frac{1}{2}$ cents per pound, how many pounds of sugar can be bought for a dollar?

6. At 25 cents a yard, how many yards of cambric can be bought for a dollar?

7. At $12\frac{1}{2}$ cents per pound, how much must be paid for 8 pounds of sugar? For 24 pounds?

8. At 25 cents a yard, how much must be paid for 4 yards of cambric? For 40 yards?

9. At $37\frac{1}{2}$ cents, or $\$\frac{3}{8}$ per yard, how much must be paid for 16 yards of cloth? For 48 yards?

10. At 75 cents, or $\$\frac{3}{4}$ per pound, what will 8 pounds of tea cost? 24 pounds?

DEFINITIONS.

276. **Price** is the money-value assigned to the measuring unit of any commodity.

277. **Quantity,** in the sale of property, is the amount of any commodity, and is expressed by the number of times it contains the measuring unit.

278. **Cost** is the value assigned to an entire quantity.

279. An **Aliquot Part** of a number is an exact half, third, fourth, etc., or any exact fractional part of that number. Hence, the aliquot parts of a number are found by successive divisions of the number by 2, 3, 4, 5, etc.

In *Business*, frequent use is made of the convenient aliquot parts of a dollar given in the following

TABLE.

5 cts. $= \frac{1}{20}$ *of $1.*	$6\frac{1}{4}$ *cts.* $= \frac{1}{16}$ *of $1.*
10 cts. $= \frac{1}{10}$ *of $1.*	$8\frac{1}{3}$ *cts.* $= \frac{1}{12}$ *of $1.*
20 cts. $= \frac{1}{5}$ *of $1.*	$12\frac{1}{2}$ *cts.* $= \frac{1}{8}$ *of $1.*
25 cts. $= \frac{1}{4}$ *of $1.*	$16\frac{2}{3}$ *cts.* $= \frac{1}{6}$ *of $1.*
50 cts. $= \frac{1}{2}$ *of $1.*	$33\frac{1}{3}$ *cts.* $= \frac{1}{3}$ *of $1.*

CASE I.

To Find the Cost of a Given Quantity when the Price is an Aliquot Part of a Dollar.

280.—Ex. 1. At $.25 a bushel, what will 856 bushels of apples cost?

4)856 = No. bushels.
214 = No. dollars.

Solution.—At $1 a bushel, 856 bushels will cost $856; and at $.25, or $\frac{1}{4}$ of a dollar, a bushel, 856 bushels will cost $\frac{1}{4}$ of $856, or $214.

2. At $.12½ a pound, what will 960 pounds of sugar cost?

281. Rule for Finding the Cost when the given Price is an Aliquot Part of a Dollar.—*First find the cost at $1, and then take the aliquot part of this amount.*

PROBLEMS.

1. How much will 796 bushels of oats cost, at $.50 per bushel?

2. At $.33⅓ per yard, how much must be paid for 6 pieces of dress goods, each piece containing 31 yards? *Ans.* $62.

3. How much must be paid for 1986 pounds of granulated sugar, at $.16⅔ a pound?

4. What must be paid for 9684 pounds of cheese, at $.12½ per pound? *Ans.* $1210.50.

CASE II.

To Find the Cost when the Price of 100 or 1000, and the Quantity, are given.

282.—Ex. 1. At $225 per hundred, what will 950 choice pear trees cost?

```
  $225
  9.50
 11250
 2025
$2137.50
```

SOLUTION.—950 is 9.50 hundreds. If 1 hundred cost $225, 9.50 hundreds must cost 9.50 times $225, or $2137.50.

2. At $14 a thousand, what will 5545 bricks cost?

```
 5.545
    14
 22180
 5545
$77.630
```

SOLUTION.—5545 is 5.545 thousands. At $14 a thousand, 5.545 thousands will cost as many dollars as the product of 5.545 by 14, or $77.63.

3. How much must be paid for 11125 hoop-poles, at $40 per thousand? *Ans.* $445.

283. Rule for Finding the Cost when the Price of 100 or 1000 is given.—*Point off the number expressing the quantity into hundreds or thousands, as may be required, and find the product of the price and this number.*

PROBLEMS.

1. What will be the cost of 11750 feet of pine boards, at $55 per thousand feet? *Ans.* $646.25.
2. What will 5635 melons cost, at $14 per hundred?
3. What is the freight on 19362 pounds of merchandise, at $.45 per thousand? *Ans.* $8.71 +.
4. What will 7536 fish cost, at $3.13 per hundred?
5. How much will 15500 laths cost, at $.90 per hundred, and 17960 feet of timber, at $35 per thousand feet?

CASE III.

To Find the Cost when the Price of a Ton of 2000 pounds, and the Quantity, are given.

284.—Ex. 1. What will 4800 pounds of coal cost, at \$7.55 per ton?

$$\begin{array}{lr} 2)\underline{4.800} & \$7.55 \\ \quad 2.400 = & \underline{2.4} \\ & 3020 \\ & \underline{1510} \\ & \$18.120 \end{array}$$

Solution.—4800 pounds are 4.800 thousand pounds.

Since 2000 pounds are 1 ton, there must be half as many tons as there are thousand pounds, or 2.4 tons.

At \$7.55 per ton, 2.4 tons will cost 2.4 times \$7.55, or \$18.12.

2. What will 9756 pounds of hay cost, at \$25 per ton? *Ans.* \$121.95.

3. What will 11500 pounds of coal cost, at \$9 per ton? *Ans.* \$52.02.

285. Rule for finding the Cost when the Quantity and the Price of a Ton of 2000 pounds are given.—*Multiply the price by half the number of thousands in the quantity.*

PROBLEMS.

1. How much must be paid for transporting 96140 pounds of coal, at \$1.40 per ton? *Ans.* \$67.29$\frac{4}{5}$.

2. At \$10 per ton, what will 5154 pounds of ground plaster cost?

3. A farmer bought 50 bags of superphosphate of lime, each containing 125 pounds, at \$55 per ton. What did it cost him? *Ans.* \$171.87$\frac{1}{2}$.

4. I bought one load of hay weighing 23456 pounds, and another weighing 31640 pounds. What was the cost of both, at \$31.50 per ton?

5. What will be the freight on 27340 pounds of coal, at \$2 per ton, and on 65400 pounds of merchandise, at \$1.10 per ton? *Ans.* \$63.31.

6. How much must be paid for 14 loads of coal, each weighing 1750 pounds, at \$6.50 per ton? *Ans.* \$79.62$\frac{1}{2}$.

SECTION XXXII.

BILLS AND ACCOUNTS.

286. A **Bill of Goods** is a written statement of articles sold, the quantity and price of each article, and the entire cost of the whole, together with the date of the transaction and the names of the purchaser and seller.

287. A **Bill of Services** is a written statement of labor performed, and the time, kind and value of such services.

288. An **Invoice** is a full statement of goods or merchandise forwarded to the purchasers, with marks, numbers and contents of each package, and the charges for cartage, insurance, etc.

289. The **Footing** of a bill is the total cost of the items.

290. A **Debtor** is the party who owes a debt, and a **Creditor** is the party to whom the debt is owed.

291. A bill is **Receipted** when the creditor, or some one acting for him, acknowledges its payment in writing.

292. An **Account** is an entry or record of items of debt and credit between parties.

293. A **Statement of an Account** is a bill of the items of an account.

In bills and accounts the character @ signifies *at*. C. is often used for *hundreds*, and M. for *thousands*.

WRITTEN EXERCISES.

294. *Bill Unreceipted.*

New York, *Jan.* 5, 1871.

Mr. George W. Lane,

Bought of James Ross & Co.

20 yd.	*Calico,*	@	*$ .17*	*$ 3*	*40*
12 "	*Gingham,*	@	*.40*	*4*	*80*
14 "	*Cambric,*	@	*.25*	*3*	*50*

Find the footing of this bill.

Bill Receipted.

PHILADELPHIA, *March* 4, 1871.

DR. CHARLES N. THAYER,

Bought of T. B. GRIMSHAW & Co.

15 lb. Japan Tea,	@	*$1.20*	$
10 " Rio Coffee,	@	*.35*	
20 gal. Molasses,	@	*.80*	
2 bush. Corn Meal,	@	*1.00*	

Received payment,

T. B. GRIMSHAW & Co.

What is the footing of this bill?

Bill Receipted by Clerk.

CHICAGO, *Dec.* 27, 1871.

MESSRS. REED & FULLER,

To WILLIAM OGDEN, *Dr.*

1870.					
Oct.	*4*	*To 6 pr. Calf Boots,*	@	*$6.00*	$
"	*20*	*" 8 " Kip Boots,*	@	*3.50*	
Nov.	*6*	*" 20 " Ladies' Kid Shoes,*	@	*1.60*	
"	"	*" 3 doz. Men's Hats,*	@	*18.00*	
Dec.	*29*	*" 6 pr. Misses' Kid Boots,*	@	*2.25*	
					$

Received payment,

WILLIAM OGDEN,

per L. T. SMITH.

What is the footing of this bill?

Bill of Services.

BOSTON, *Jan.* 6, 1871.

MAJ. JOHN STONE,

To HENRY L. DOTEN, *Dr.*

1870.				
Nov.	*11*	*For Labor of myself and apprentice, on House,*	*$7*	*50*
Dec.	*3*	*" " on Barn, 10 days, @ $3.50*	*35*	*00*
			$42	*50*

Received payment by Note,

HENRY L. DOTEN.

What is the footing of this bill?

Statement of Account.

NEW ORLEANS, *Nov.* 1, 1871.

COL. LOUIS LAMERT,

To JOHN DEGAN & Co., *Dr.*

1871.				
Aug.	*12*	*To 15650 ft. Pine Boards, No. 1, @ $80.00 per M.,*	*$*	
"	*16*	*" 4600 ft. Clapboards, extra, @ $60.00 per M.,*		
Sept.	*13*	*" 6350 ft. Pine Lumber, No. 3, @ $65.00 per M.,*		
			$	
		Cr.		
Aug.	*15*	*By Merchandise, as by his bill, $750.00*		
Oct.	*9*	*" Cash, 950.00*	*1700*	
		Balance due J. D. & Co.,	*$*	

Received payment,

JOHN DEGAN & CO.

What is the balance of the account due J. D. & Co.?

6. James Riley & Co. bought, July 7, 1870, of Joseph Herr, Trenton, N. J., 15 tons of coal, at $6.50 per ton; 19 tons of coal, at $5.25 per ton; and 14¼ cords of wood, at $7.25 per cord. Make a bill of the purchase, find the footing, and receipt the bill for Joseph Herr. *Ans.* Footing, $302.37½.

7. Feb. 17, 1870, Patrick Mahoney bought of James Burrows, Harrisburg, Pa., 3 barrels of flour, at $9.50 a barrel; 20 pounds of coffee, at 30 cents a pound; and 25 pounds of sugar, at 17½ cents a pound. Feb. 18, 1870, he paid $20; and March 3, 1870, he rendered a bill for 3 days' work, at $1.75 a day. Put this in the form of an account, and find the balance due James Burrows. *Ans.* $13.62½.

TEST QUESTIONS.

295.—1. What is MONEY? How does paper money differ from coin or specie? What is the money in common circulation called? What is United States money? Recite the table of United States money.

2. Of what are the COINS of the United States made? What coins are of gold? Of silver? Of nickel? Of bronze?

3. What is the money of the Dominion of Canada called? Of what does it consist? What are the Canada coins?

4. What is the UNIT of United States money? How may dimes, cents and mills be decimally expressed? Why is United States money a decimal currency? How are dimes commonly regarded? How may a decimal of a dollar less than a cent be read?

5. What is DENOMINATION? Of two denominations which is the higher? What is reduction? How are cents reduced to mills? Dollars to mills? Dollars and cents to cents? Dollars, cents and mills to mills? Mills to cents? Mills to dollars?

6. How are COMPUTATIONS in United States money performed? What is quantity in the sale of property? What is price? What is cost? What are aliquot parts of a number? What aliquot parts of a dollar are frequently used in business?

7. How is the COST found when the price at an aliquot part of a dollar and the quantity are given? When the price of 100 or 1000, and the quantity are given? When the price of a ton of 2000 pounds, and the quantity are given?

8. What is a BILL OF GOODS? A bill of services? An invoice? The footing of a bill? When is a bill receipted? What is an account? A statement of an account?

SECTION XXXIII.

REVIEW PROBLEMS.

WRITTEN EXERCISES.

296.—Ex. 1. I gave .15 of a sum of money in charity, .375 of it for payment of debts, and .125 of it for a library. What fractional part of the sum had I left? *Ans.* $\frac{7}{20}$.

2. What common fraction is equivalent to .41$\frac{2}{3}$? *Ans.* $\frac{5}{12}$.

3. What will be the cost of 610 yards of cloth, at \$1.37$\frac{1}{2}$ per yard?

$$\begin{array}{r} 4)\$610.00 \\ 2)152.50 \\ 76.25 \\ \hline \$838.75 \end{array}$$

SOLUTION.—\$1.37$\frac{1}{2}$ is equal to \$1 + \$.25 + \$.12$\frac{1}{2}$. 610 yards at \$1 per yard will cost \$610; at \$.25, or \$$\frac{1}{4}$, will cost $\frac{1}{4}$ of \$610, or \$152.50; and at \$.12$\frac{1}{2}$, or $\frac{1}{2}$ of \$$\frac{1}{4}$, will cost $\frac{1}{2}$ of \$152.50, or \$76.25. Hence, 610 yards at \$1.37$\frac{1}{2}$ per yard will cost \$610 + \$152.50 + \$76.25, or \$838.75.

4. How much must be paid for 9870 bushels of wheat, at \$.87$\frac{1}{2}$ per bushel? *Ans.* \$8636.25.

5. If you should pay \$73.50 for a wagon, and 3 times as much for a horse, what would be the cost of both?

6. The annual fall of rain in Boston is 39.23 inches; in Providence, 36.74 inches; in New York, 36 inches; and in Washington, 34.62 inches. What is the average?

7. How much can you save in a month of 26 working days, if you earn \$1.75 and spend 62$\frac{1}{2}$ cents each day?

8. What is the cost of 4565 feet of joist at \$23 per M., and 13640 feet of boards at \$535.50 per M.?

9. The annual fall of rain in Savannah is 55 inches, and for each inch the volume of rain is 17.3387 million gallons per square mile. What is the volume of rain in Savannah for a square mile? *Ans.* 953.6285 million gallons.

10. A merchant bought molasses at \$55.45 per cask, and sold it at \$63.70 per cask, thereby gaining \$858. How many casks did he buy? *Ans.* 104.

11. A fire destroyed .375 of a quantity of goods worth \$2000. What sum did a man lose who owned .25 of the whole?

12. What is the net profit of a garden, according to the following items?—

Expenses.—Paid for plowing and carting, $31; for fertilizers and seeds, $77.15; for labor and cultivating, $31; and for cost of harvesting and marketing, $43.18.

Income.—Sold 56 bushels of potatoes, at $1.50; 13 bushels of peas, at $3; 70 dozen cabbages, at $1.12½; 50 dozen green corn, at $.37½; and 170 boxes of strawberries, at 30 cents.

Ans. Net profit, $89.17.

13. Henry McIntire of Nashville has worked for Jacob Sangster 5 days, at $4.25 per day; and he has furnished 5000 bricks, at $13.25 per thousand, and 3 casks of lime, at $1.80 per cask. Make out the statement of the account, and find the amount due.

SECTION XXXIV.

DENOMINATE NUMBERS.

297. Quantity, in general, is that which admits of measurement or computation.

Thus, distance, space, etc., which can be measured, and numbers, money, etc., which can be computed, are quantities.

298. A **Measure** is a unit used in estimating or determining quantity.

299. *A quantity is measured* by finding how many times the quantity contains the unit.

Thus, the length of a table is measured or determined by applying a foot-rule, and thus finding how many times the length of the rule is contained in the length of the table.

MEASURES OF EXTENSION.

300. Extension is that which has one or more of the dimensions, length, breadth and depth, or thickness.

301. A **Line** is that which has only length.

302. A **Surface** is that which has only length and breadth.

303. A **Solid,** or **Volume,** is that which has length, breadth and thickness, or depth.

LINEAR MEASURES.

304. **Linear** or **Long Measures** are those used in ascertaining lengths and distances.

TABLE.

12 inches (in.)	*are*	*1 foot*	*ft.*
3 feet	*"*	*1 yard*	*yd.*
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.	*"*	*1 rod*	*rd.*
320 rods	*"*	*1 mile*	*mi.*

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

305. In **Cloth Measure** the linear yard is divided into *halves, quarters, eighths* and *sixteenths*, or *nails.*

2 sixteenths, or 2 nails,	*are*	*1 eighth*	*8th.*
2 eighths, or 4 nails	*"*	*1 quarter*	*qr.*
4 quarters	*"*	*1 yard*	*yd.*

306. The **Surveyor's Chain,** called *Gunter's chain,* used in measuring roads and boundaries of land, is 4 rods in length, and is subdivided as follows:

7.92 inches	*are*	*1 link*	*li.*
100 links, or 4 rods,	*"*	*1 chain*	*ch.*
80 chains	*"*	*1 mile*	*mi.*

Engineers' Measuring Tape, **used in measuring railroads and canals, is 100 feet in length, with each foot divided into tenths.**

In measuring *ropes*, *cables* and *short sea-distances*, 6 feet are *1 Fathom*, and 120 fathoms are *1 Cable-Length*. In measuring *longer sea-distances*, 1.15 + miles, or $\frac{1}{60}$ of the average length of degrees measured on a meridian of the earth, is a *Geographic* or *Nautical Mile*, or *Knot*, and 3 nautical miles, or 3.45 + common miles, are *1 Nautical League*.

A *Furlong* is 40 rods, but this term is becoming obsolete.

A *Meter* is 39.37 inches; a *Kilometer*, or 1000 meters, is .62137 of a mile; a *Centimeter*, or $\frac{1}{100}$ of a meter, is .3937 of an inch.

307.—Ex. 1. How many inches are 10 feet? Are 12 feet?

2. In 48 inches how many feet? In 96 inches how many feet?

3. How many feet in 2 rods? In 3 rods?

4. What part of a mile is 40 rods? Is 80 rods?

5. How many rods are 33 feet? Are 22 yards?

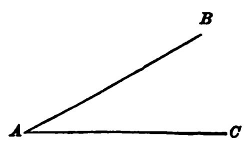
SURFACE MEASURES.

308. A **Straight Line** is a line that has only one direction.

Thus, the line *AB* is a straight line.

309. An **Angle** is the difference of direction of two lines drawn from the same point.

Thus, the lines *AB* and *AC*, meeting at *A*, form the angle *CAB*.

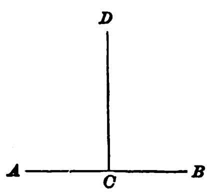


310. A **Perpendicular Line** is a straight line meeting another straight line so as to form two equal angles.

Thus, the line *CD* is a perpendicular line.

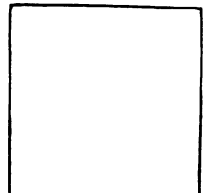
311. A **Right Angle** is an angle formed by two lines perpendicular to each other.

Thus, the angles *ACD* and *DCB* are each right angles.



312. A **Square** is a figure having four equal straight sides and four equal angles.

Thus, a *Square Inch* is a square having each of its sides 1 inch in length.



A Square Inch.

313. A **Rectangle** is any figure having four straight sides and four equal angles.

Thus, the figure in the margin, having four straight sides and four equal angles, is a rectangle.

314. **Surface** or **Square Measures** are those used in ascertaining the extent of surfaces.

TABLE.

144 square inches (sq. in.)	*are*	*1 square foot*	*sq. ft.*
9 square feet	"	*1 square yard*	*sq. yd.*
$30\frac{1}{4}$ square yards	"	*1 square rod*	*sq. rd.*
160 square rods	"	*1 Acre*	*A.*
640 acres	"	*1 square mile*	*sq. mi.*

1 A. = 160 sq. rd. = 4840 sq. yd. = 43560 sq. ft. = 6272640 sq. in.

315. In the **Measurement of Land** the square chain and its subdivisions are used.

625 square links (sq. li.)	*are*	*1 square rod*	*sq. rd.*
16 square rods	"	*1 square chain*	*sq. ch.*
10 square chains	"	*1 Acre*	*A.*

A *Perch* of surface is a square rod, and a *Rood* is 40 square rods. These denominations are now seldom used.

A *Section* of land is a square mile, and a *Quarter Section* is 160 acres.

A *Square* in architects' measure is 100 square feet.

A *Square Meter*, or a *Centiare*, is 1550 square inches, or 1.196 square yards; an *Are* is 119.6 square yards; and a *Hectare*, or 100 ares, is 2.471 acres.

316.—Ex. 1. How many square feet are 9 square yards?

2. In 81 square feet how many square yards?

3. How many square rods in 10 square chains?

4. In 160 square rods, how many square chains?

5. What part of an acre is 80 square rods? Is 120 square rods?

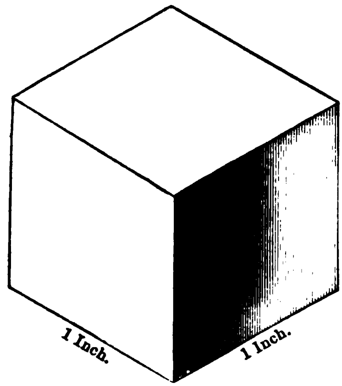
6. How many square rods in one fourth of an acre?

CUBIC MEASURES.

317. A **Cube** is a solid bounded by six *equal* squares, called its faces.

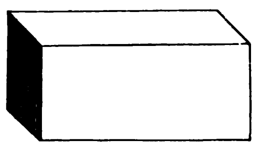
318. A **Cubic Inch** is a cube whose faces are each 1 square inch.

319. A **Rectangular Solid** is any volume bounded by six rectangular faces.



A CUBIC INCH.

Thus, the figure in the margin, having six rectangular faces, is a rectangular solid.


A RECTANGULAR SOLID.

320. **Cubic Measures** are those used in measuring things that have length, breadth and depth, or thickness.

TABLE.

1728 cubic inches (cu. in.) are 1 cubic foot...cu. ft.
27 cubic feet " 1 cubic yard..cu. yd.

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

321. In **Measuring Wood** the cord foot and cord are used. Thus,

16 cubic feet are 1 cord foot.....cd. ft.
8 cord feet, or 128 cubic feet, } are 1 cord.....cd.

A *Ton*, in computing the tonnage of ships and other vessels, is 100 cubic feet of their internal space.

A *Ton* of freight, for some articles, is estimated by the space occupied. The heavier articles are estimated by their weight.

A *Perch* of stone is $16\frac{1}{2}$ feet long, 1 foot high and $1\frac{1}{2}$ feet thick, and contains $24\frac{3}{4}$ cubic feet.

A *Cubic Meter*, or Stere, is 35.316 cubic feet, or .2759 of a cord.

Wood, as usually cut for market, is 4 feet long, and is piled in ranges 4 feet high. Of such ranges, a part that is 1 foot of the length of the range is 1 cord foot; a part that is 8 feet of the length is 1 cord.

WRITTEN EXERCISES.

322.—Ex. 1. How many feet are 46 rods?

$$\begin{array}{l} 46 \text{ rd.} \\ \underline{\quad 5\frac{1}{2}} \\ 230 \\ \underline{\;\; 23} \\ 253 \text{ yd.} \\ \underline{\quad 3} \\ 759 \text{ ft.} \end{array}$$

SOLUTION.—Since 1 rod is $5\frac{1}{2}$ yards, there must be $5\frac{1}{2}$ times as many yards as rods; hence, in 46 rods there must be $5\frac{1}{2}$ times 46 yards, or 253 yards.

Since 1 yard is 3 feet, there must be 3 times as many feet as yards; hence, in 253 yards there are 3 times 253 feet, or 759 feet, which is the answer required.

2. How many rods are 759 feet?

$$\begin{array}{rl} 3 \text{ ft.}\,) & \underline{759} \text{ ft.} \\ 5\frac{1}{2} \text{ yd.}\,) & 253 \text{ yd.} \\ \underline{2} & \underline{\;\;2\;\;} \\ 11 \text{ hf. yd.})\, & \underline{506} \text{ hf. yd.} \\ & 46 \text{ rd.} \end{array}$$

SOLUTION.—Since 3 feet are one yard, 759 feet must be as many yards as there are times 3 feet in 759 feet, which are 253 times. Hence, 759 feet = 253 yards.

Since $5\frac{1}{2}$ yards are 1 rod, 253 yards must be as many rods as there are times $5\frac{1}{2}$ yards in 253 yards, or times 11 half yards in 506 half yards, which are 46 times. Hence, 253 yards, or 759 feet, = 46 rods.

3. How many rods are 45 miles?

4. How many miles are 14400 rods?
5. How many square feet are 110 acres?
6. How many acres are 4791600 square feet?
7. How many square links are 5 acres?
8. How many acres are 500000 square links?
9. How many cubic feet in 312 cubic yards?
10. What will 25 cords of wood cost, at 75 cents per cord foot?

MEASURES OF CAPACITY.

323. Measures of Capacity are measures used in ascertaining the quantity of liquids, and some dry articles.

LIQUID MEASURES.

324. Liquid Measures are those used in measuring liquids.

TABLE.

4 gills (gi.)	*are*	*1 pint*	*pt.*
2 pints	"	*1 quart*	*qt.*
4 quarts	"	*1 gallon*	*gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

The *United States Standard Gallon* contains 231 cubic inches; the *Imperial Gallon* of Great Britain contains 277.274 cubic inches.

Malt liquors and milk were formerly sold by what was called *Beer Measure*, the gallon of which contained 282 cubic inches.

A *Barrel*, regarded as a measure of cisterns, vats, etc., is 31½ gallons, and a *Hogshead* is 63 gallons; but these terms in commerce are often applied to casks of various capacities.

325. Apothecaries' Fluid Measures, or those used in compounding medicines and in putting up medical prescriptions, are as follows—

60 minims (♏)	*are*	*1 fluid dram*	*f ʒ.*
8 fluid drams	"	*1 fluid ounce*	*f ℥.*
16 fluid ounces	"	*1 pint*	*O.*
8 pints	"	*1 gallon*	*Cong.*

326.—Ex. 1. How many gills in 3 quarts?

2. In 40 gills, how many quarts?

3. How many pints in 6 gallons?

4. In 40 pints, how many gallons?

5. What will 4 gallons of milk cost, at 5 cents a quart?

6. How many gallons of milk, at 5 cents a quart, can be bought for 60 cents?

DRY MEASURES.

327. Dry Measures are those used in measuring dry articles, such as grain, fruit, vegetables, coals, etc.

TABLE.

2 pints (pt.)	*are*	*1 quart*	*qt.*
8 quarts	"	*1 peck*	*pk.*
4 pecks	"	*1 bushel*	*bu.*

1 bu. = 4 pk. = 32 qt. = 64 pt.

The *United States Standard Bushel* contains 2150.42 cubic inches. The *Imperial Bushel* of Great Britain contains 2218.192 cubic inches.

Four *Heaped Pecks* are equal to 5 even pecks; and 6 quarts of dry measure are very nearly equal to 7 liquid quarts.

A *Liter* is 1.0567 liquid quarts, or .908 of a dry quart, or 61.022 cubic inches; and a *Hectoliter*, or 100 liters, is 2 bushels 3.35 pecks, or 3.531 cubic feet. A *Centiliter*, or $\frac{1}{10}$ of a liter, is .338 of a fluid ounce.

328.—Ex. 1. How many quarts are 6 pecks?

2. In 64 quarts, how many bushels?

3. At 15 cents a peck, what will 1 bushel of apples cost?

4. How many pecks of apples, at 60 cents a bushel, can be bought for 45 cents?

WRITTEN EXERCISES.

329.—Ex. 1. How many quarts in 6 hogsheads?

2. In 1512 quarts, how many hogsheads?

3. How many quarts in 112 bushels?

4. In 3584 quarts, how many bushels?

5. What will 5 bushels of chestnuts cost, at 6 cents a pint?

MEASURES OF WEIGHT.

330. Weight is the quantity of matter in bodies, as determined by the force with which they tend toward the earth.

AVOIRDUPOIS WEIGHTS.

331. Avoirdupois Weights are those used in weighing produce, groceries, iron, etc.

TABLE.

16 ounces (oz.)	*are*	*1 pound*	*lb.*
100 pounds	"	*1 hundred-weight*	*cwt.*
20 hundred-weight	"	*1 ton*	*T.*

1 T. = 20 cwt. = 2000 lb. = 32000 oz.

332. In collecting duties upon foreign goods, at the United States Custom Houses, and also in *freighting coal*, and *selling it by wholesale*,

28 pounds	*are*	*1 quarter.*
4 quarters, or 112 pounds,	"	*1 hundred-weight.*
20 hundred-weight, or 2240 lbs.,	"	*1 long ton.*

The ounce is considered as 16 drams, but the dram is not recognized in business.

The term *Cental* is beginning to be used for the hundred-weight.

A *Quarter of Grain* in Great Britain is 560 pounds, or 8 Imperial bushels.

A *Gram* is .03527 of an ounce; a *Kilogram*, or 1000 grams, is 2.2046 pounds; and a *Tonneau*, or 100 kilos, is 2204.6 pounds.

333. The following Units are sanctioned by custom or law—

32 lb. of oats	*are*	*1 bushel.*
45 lb. of timothy-seed	"	*1 bushel.*
48 lb. of barley	"	*1 bushel.*
56 lb. of rye	"	*1 bushel.*
56 lb. of Indian corn	"	*1 bushel.*
50 lb. of Indian meal	"	*1 bushel.*
60 lb. of wheat	"	*1 bushel.*
60 lb. of clover-seed	"	*1 bushel.*
60 lb. of potatoes	"	*1 bushel.*
56 lb. of butter	"	*1 firkin.*
100 lb. of meal or flour	"	*1 sack.*
100 lb. of grain or flour	"	*1 cental.*
100 lb. of dry fish	"	*1 quintal.*
100 lb. of nails	"	*1 cask.*
196 lb. of flour	"	*1 barrel.*
200 lb. of beef or pork	"	*1 barrel.*

334.—Ex. 1. How many ounces in 5 pounds?

2. In 64 ounces, how many pounds?

3. How many pounds in $\frac{9}{10}$ of a hundred-weight?

4. How many pounds in $\frac{3}{4}$ of a ton?

5. What part of a hundred-weight is 70 pounds?

6. How much must be paid for $\frac{7}{8}$ of a pound of spice, at 3 cents an ounce?

7. How much must be paid for $\frac{1}{2}$ of a quintal of fish, at 8 cents a pound?

8. How many pounds in 3 pecks of clover-seed?

9. How much will a barrel of beef cost, at 10 cents a pound?

10. How much will a cental of flour cost, at $5\frac{1}{2}$ cents a pound?

11. How much will a peck of timothy-seed cost, at 12 cents a pound?

12. How much will a ton of meal cost, at \$1.12$\frac{1}{2}$ per bushel?

13. What will be the cost of ten barrels of beef, at ten cents per pound?

TROY WEIGHT.

335. Troy Weights are those used in weighing gold, silver and gems.

TABLE.

24 grains (gr.)	*are*	*1 pennyweight*	*pwt.*
20 pennyweights	"	*1 ounce*	*oz.*
12 ounces	"	*1 pound*	*lb.*

1 lb. = 12 oz. = 240 pwt. = 5760 gr.

336. Apothecaries, in compounding medicines and in putting up medical prescriptions, either use only the units of grains, ounces and pounds, or subdivide the Troy pound. Thus—

20 grains (gr.)	*are*	*1 scruple*	℈.
3 scruples	"	*1 dram*	ℨ.
8 drams	"	*1 ounce*	*oz.*
12 ounces	"	*1 pound*	*lb.*

An ounce avoirdupois is 437½ grains, and an ounce Troy is 480 grains. A pound avoirdupois is 7000 grains, and a pound Troy is 5760 grains.

A *Gram* is 15.432 grains Troy.

337.—Ex. 1. How many ounces in 5 pounds Troy?

2. In 72 ounces Troy, how many pounds?

3. How many pennyweights are 8 ounces?

4. How many ounces are 200 pennyweights?

5. How much will $\frac{2}{3}$ of a pennyweight of metal cost, at 12 cents a grain?

WRITTEN EXERCISES.

338.—Ex. 1. How many ounces are 35 tons?

2. In 1120000 ounces, how many tons?

3. How many grains are 15½ pounds Troy?

4. In 89280 grains Troy, how many pounds?

5. What will 2565 pounds of coal cost, at $8 per ton?

6. What is the value of 11 pounds of fine silver, at $.06⅘ per pennyweight? *Ans.* $179.52.

7. How many pounds of fine silver can be bought for $179.52, at $.06⅘ per pennyweight?

MEASURES OF TIME.

339. Time is a definite portion of duration.

TABLE.

60 seconds (sec).	*are*	*1 minute*	*min.*
60 minutes	"	*1 hour*	*h.*
24 hours	"	*1 day*	*d.*
365 days	"	*1 common year*	*y.*
366 days	"	*1 leap year.*	

Also,

7 days	*are*	*1 week*	*wk.*
52 weeks 1 day	"	*1 common year*	*y.*
100 years	"	*1 century*	*cen.*

1 y. = 365 d. = 8760 h. = 525,600 min. = 31,536,000 sec.

340. The **Months,** their names, and the number of days in each, are—

	Days.		Days.
1st mo., January, has	31.	7th mo., July, has	31.
2d mo., February, "	28 or 29.	8th mo., August, "	31.
3d mo., March, "	31.	9th mo., September, "	30.
4th mo., April, "	30.	10th mo., October, "	31.
5th mo., May, "	31.	11th mo., November, "	30.
6th mo., June, "	30.	12th mo., December, "	31.

341. The exact length of the year, or the precise time in which the earth makes one revolution around the sun, is 365d. 5h. 48min. 49.7sec., or nearly $365\frac{1}{4}$ days. Hence, on account of this fractional part of a day, at certain points of time February has 29 days, and the year is leap year.

Every centennial year whose number is exactly divisible by 400, *and every year not a centennial year whose number is exactly divisible by* 4, *is leap year, or has* 366 *days.*

342.—Ex. 1. How many hours in $\frac{3}{4}$ of a day?

2. What part of a minute is 55 seconds?

3. How many months have 30 days each?

4. In 65 days, how many weeks, and what number of days remain?

5. From April 1 to September 1, how many months?

6. How much can be earned, at 12 cents per hour, in 6 working days of 10 hours each?

7. From January 12 to March 12, in a common year, how many days?

WRITTEN EXERCISES.

343.—Ex. 1. How many seconds in 48 hours?

2. How many hours in 172800 seconds?

3. How many hours in 16 years, allowing every fourth year to be a leap year?

4. How many years, of 365¼ days each, are in 140256 hours?

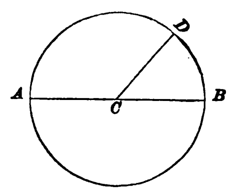
5. How many months in 5 centuries?

6. How many centuries in 6000 months?

7. How much will a man earn in 3 years, at $56 per month?

MEASURES OF CIRCLES.

344. A **Circle** is a plane surface, which is bounded by a line having all its parts equally distant from a point within, called the center.



345. A **Circumference** is the line that bounds a circle.

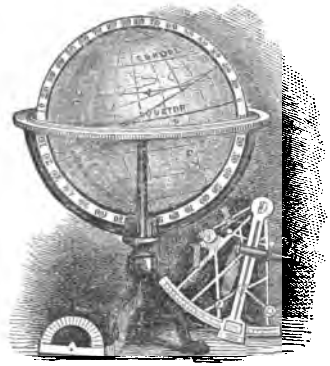
346. An **Arc** is any part of the circumference of a circle; as *AD*, *DB* or *BA*.

347. A **Diameter** of a circle is a straight line drawn through the center of that circle, and terminated both ways by the circumference; as the straight line *AB*.

348. A **Degree** is one of the 360 equal parts of a circumference.

349. The **Measure of an Angle**, whose sides meet at the center of a circle, is that part of the circumference included between the sides.

Thus, the arc *AD* is the measure of the angle *ACD*.



350. Circular and Angular Measures are those used for measuring angles and the difference of directions and in determining latitude and longitude, etc.

Seconds are usually subdivided into *tenths* or *hundredths*.

A *Sextant* is one sixth of a circumference, a *Quadrant* one fourth, and a *Semi-circumference* one half. A *Minute* of the circumference of the earth is a geographic mile.

TABLE.

60 seconds (″) are	*1 minute*	....	*′.*
60 minutes	*" 1 degree*	....	*°.*
360 degrees	*" 1 circumference*	....	*C.*

$$1\ C. = 360° = 21600' = 1296000''.$$

351. In **Astronomy**, the zodiac, or the apparent path of the sun in the heavens, is divided into 12 equal parts, called signs Hence,

30 degrees	*are*	*1 sign*	*S.*
12 signs	*"*	*1 circumference*.....	*C.*

Degrees of the earth's circumference on a meridian average 69.16 common miles.

Every circle, great or small, has the same number of degrees, minutes and seconds; hence, these parts of different circles have different lengths.

352.—Ex. 1. How many minutes are 3 degrees?

2. In 240 degrees, how many minutes?

3. What part of a minute is 45 seconds?

4. How many degrees are $\frac{3}{4}$ of a circumference?

5. What part of a degree is 50 minutes? What part of a circumference is 180 degrees?

MISCELLANEOUS MEASURES.

353. In the **Paper** business, the units used are given in the following

TABLE.

24 sheets are 1 quire.
20 quires " 1 ream.
2 reams " 1 bundle.
5 bundles " 1 bale.

354. In **Counting** certain articles, use is made of the following

TABLE.

12 ones are	*1 dozen*	.... *doz.*
12 dozen "	*1 gross*	.... *gro.*
12 gross "	*1 great gross*	.... *grt. gro.*

355.—Ex. 1. How many ones are 1 gross?

2. How many dozens is $\frac{3}{4}$ of a great gross?

3. How many quires are 5 reams?

4. In 5 bales of paper how many reams?

5. At \$1$\frac{1}{2}$ per dozen, what will 2 gross of writing-books cost?

6. How many reams of paper, at 10 cents a quire, can be bought for \$6?

WRITTEN EXERCISES.

356.—Ex. 1. In 3 common years, how many minutes?

2. In 1581120 minutes, how many common years?

3. By giving 40 minutes each day for 300 days to reading, how many working days of 10 hours each will be thus devoted?

4. How many dozen eggs can be put in a box which will contain 2880 eggs, and what is their value at 20 cents a dozen?

5. Name the leap years between 1870 and 1881.

6. What will 15 great gross of buttons cost, at 3 cents a button? *Ans.* \$777.60.

TEST QUESTIONS.

357.—1. What is MONEY? What is coin or specie? Currency? Recite the table of United States Money. How are computations in United States money performed?

2. What is PRICE? Cost? An aliquot part? Give the rule for finding the cost when the price is an aliquot part of a dollar. When the price is of 100 or 1000. When of 2000 pounds.

3. What is a BILL of goods? A bill of services? An invoice? The footing of a bill? An account? A statement of an account?

4. What is a MEASURE? What is quantity? How is quantity measured?

5. What is EXTENSION? A line? A surface? A volume?

6. What are LINEAR MEASURES? Recite the table. How is the yard divided in measuring cloth? What is used in measuring roads?

7. What are SURFACE or SQUARE MEASURES? What is a straight line? An angle? A perpendicular? A right angle? A square? A rectangle? Recite the table of square measures. What is used in the measurement of land?

8. What are CUBIC MEASURES? What is a cube? A rectangle? Give the table of cubic measures. What are used in measuring wood?

9. What are MEASURES OF CAPACITY? What are liquid measures? Give the table of liquid measures. What are apothecaries' fluid measures? What are dry measures? Give the table of dry measures.

10. What is WEIGHT? What are avoirdupois weights? Recite the table of avoirdupois weights. What are Troy weights? Recite the table of Troy weights. How do apothecaries subdivide the Troy pound?

11. What are the measures of TIME? Recite the table of time. What are the months? What years are leap years?

12. What are CIRCULAR MEASURES? What is a circle? A circumference? An arc? A diameter? A degree? The measure of an angle? Give the table of circular measures. How is the zodiac divided?

13. What units are used in COUNTING articles? Recite the table. What units are used in the paper business? Recite the table.

14. What is the DIVISOR in division? (Art. 80.) What is an exact divisor of a number? (Art. 101.) The greatest common divisor of two or more numbers? (Art. 112.) Of what is it the product? (Art. 113—1.)

15. What is a MULTIPLE of a number? (Art. 119.) How do a multiple and a divisor differ? Why may a multiple of a number be called the dividend of that number? What must the least common multiple of two or more numbers contain? (Art. 122.)

SECTION XXXV.

REDUCTION OF COMPOUND NUMBERS.

358.—Ex. 1. How many inches in 2 feet 6 inches? In 3 feet 7 inches? In 2 yards 1 foot?

2. How many feet in 30 inches? In 43 inches? How many yards in 84 inches?

3. How many gills in 2 quarts 1 pint? In 1 quart 1 pint 1 gill? In 3 quarts 1 pint?

4. How many quarts in 20 gills? In 13 gills? In 28 gills?

5. How many ounces in 5 pounds 6 ounces? In 3 pounds 8 ounces? In 10 pounds 10 ounces?

6. How many pounds in 86 ounces? In 56 ounces? In 170 ounces?

DEFINITIONS.

359. A **Simple Denominate Number** is a number expressed in units of only one denomination.

Thus, 3 yards, and 2 days, are each a simple denominate number.

360. A **Compound Denominate Number** is a number expressed in units of more than one denomination.

Thus, 2 feet 6 inches, 4 days 6 hours, are each a compound denominate number.

361. **Reduction of Denominate Numbers** is the process of changing them to equivalent numbers of a different denomination.

362. **Reduction Descending** is the process of changing a number to an equivalent number expressed in units of a lower denomination.

363. **Reduction Ascending** is the process of changing a number to an equivalent number expressed in units of higher denominations.

364. **Principle.**—*Reduction descending is performed by multiplication, and reduction ascending is performed by division.*

CASE I.

Reduction Descending.

365.—Ex. 1. How many quarts in 3 pecks 7 quarts? In 3 pecks 5 quarts? In 1 bushel 1 peck?

2. How many inches in 3 yards 2 feet? In 4 yards 1 foot?
3. How many pennyweights in 5 ounces 11 pennyweights? In 4 ounces 15 pennyweights?
4. How many days are 9 weeks 5 days?
5. How many inches in $\frac{5}{9}$ of a yard?

SOLUTION. — Since in 1 yard there are 3 feet, in $\frac{5}{9}$ of a yard there must be $\frac{5}{9}$ of 3 feet, or $\frac{15}{9}$ of 1 foot, which equals $\frac{5}{3}$ of a foot. And since in 1 foot there are 12 inches, in $\frac{5}{3}$ of a foot there must be $\frac{5}{3}$ of 12 inches, or 20 inches. Hence $\frac{5}{9}$ of a yard are 20 inches.

6. How many pounds in $\frac{3}{8}$ of a ton? In $\frac{7}{10}$ of a ton?
7. What part of a quart is $\frac{1}{5}$ of a gallon? $\frac{2}{9}$ of a gallon?

WRITTEN EXERCISES.

366.—Ex. 1. How many pints are 13 gal. 2 qt. 1 pt.

13 gal. 2 qt. 1 pt.
4
52 No. qt. in 13 gal.
2
54 No. qt. in 13 gal. 2 qt.
2
108 No. pt. in 13 gal. 2 qt.
1
109 No. pt. in 13 gal. 2 qt. 1 pt.

SOLUTION.—Since 1 gallon is 4 quarts, 13 gallons must be 13 times 4 quarts, or 52 quarts; and 52 quarts + 2 quarts are 54 quarts. Since 1 quart is 2 pints, 54 quarts must be 54 times 2 pints, or 108 pints; and 108 pints + 1 pint are 109 pints. Hence, 13 gal. 2 qt. 1 pt. are 109 pints.

2. How many yards are 53 mi. 132 rd. 4 yd.?
3. How many pounds are 5 T. 3 cwt. 15 lb.?

367. Rule for Reduction Descending.—*Multiply the number of the highest denomination given, by that number of the next lower which equals one of the higher,* *and to the product add the number, if any, of the lower denomination.*

Reduce this result in like manner, and so proceed until the given number is reduced to the required denomination.

PROBLEMS.

1. How many square yards are 37 A. 132 sq. rd.?
2. How many quarts are 308 bu. 1 pk. 6 qt.?
3. How many quarters are 68 yd. 1 qr.?
4. How many links are 2 mi. 40 ch. 25 li.?
5. How many gills in 16 hogsheads?
6. How many ounces in 1 long ton?
7. How many grains are 7 oz. 19 pwt. 13 gr.?
8. How many square rods in a quarter section of land?
9. How many square rods in 2 A. 5 sq. ch. 8 sq. rd.?
10. How many cubic feet are 20 cd. 6 cd. ft.?
11. How many seconds are 29 d. 12 h. 44 min. 3 sec.?
12. How many pints are $\frac{2}{5}$ of a bushel?

$$\frac{2}{5}\,bu. = \frac{2}{5}\times 4\,pk. = \frac{8}{5}\,pk.$$
$$\frac{8}{5}\,pk. = \frac{8}{5}\times 8\,qt. = \frac{64}{5}\,qt.$$
$$\frac{64}{5}\,qt. = \frac{64}{5}\times 2\,pt. = \frac{128}{5}\,pt.$$
$$\frac{128}{5}\,pt. = 25\frac{3}{5}\,pt.$$

SOLUTION.—Since 1 bushel is 4 pecks, $\frac{2}{5}$ of a bushel must be $\frac{2}{5}$ of 4 pecks, or $\frac{8}{5}$ of a peck.

Since 1 peck is 8 quarts, $\frac{8}{5}$ of a peck must be $\frac{8}{5}$ of 8 quarts, or $\frac{64}{5}$ of a quart.

Since 1 quart is 2 pints, $\frac{64}{5}$ of a quart must be $\frac{64}{5}$ of 2 pints, or $\frac{128}{5}$ of a pint $= 25\frac{3}{5}$ pints.

13. How many pounds are $\frac{3}{16}$ of a ton?
14. How many gills are $\frac{2}{8}$ of a hogshead?
15. What part of a second is $\frac{1}{1000000}$ of a day?
16. Reduce .0525 cwt. to ounces.

SOLUTION.—*.0525 cwt. = .0525 × 100 lb. = 5.25 lb. = 5.25 × 16 oz. = 84 oz.*

17. Express .09375 of an acre in square rods.
18. Express .7375 of a pound Troy in pennyweights.

19. Reduce $\frac{5}{8}$ of a rod to lower integers.

$$\frac{5}{8}\times 5\frac{1}{2}=\frac{55}{16}=3\frac{7}{16}=\text{No. of yd.}$$
$$\frac{7}{16}\times 3=\frac{21}{16}=1\frac{5}{16}=\text{No. of ft.}$$
$$\frac{5}{16}\times 12=\frac{60}{16}=3\frac{3}{4}=\text{No. of in.}$$
$$\frac{5}{8}\text{ rd.}=3\text{ yd. }1\text{ ft. }3\frac{3}{4}\text{ in.}$$

SOLUTION.—Since 1 rod is $5\frac{1}{2}$ yards, $\frac{5}{8}$ of a rod must be $\frac{5}{8}$ of $5\frac{1}{2}$ yards, or $3\frac{7}{16}$ yards.

Since 1 yard is 3 feet, $\frac{7}{16}$ of a yard must be $\frac{7}{16}$ of 3 feet, or $1\frac{5}{16}$ feet.

Since 1 foot is 12 inches, $\frac{5}{16}$ of a foot must be $\frac{5}{16}$ of 12 inches, or $3\frac{3}{4}$ inches. Hence, $\frac{5}{8}$ of a rod = 3 yd. 1 ft. $3\frac{3}{4}$ in.

20. Reduce $\frac{9}{18}$ of a day to a compound number.

21. Express $\frac{5}{8}$ of a hogshead as a compound number.

22. Express $\frac{4}{15}$ of a mile as a compound number.

23. Reduce .7375 of a pound Troy to a compound number.

SOLUTION.—*.7375 lb. = .7375 × 12 oz. = 8.85 oz.; .85 oz. = .85 × 20 pwt. = 17 pwt.* Hence, *.7375* of a pound = *8 oz. 17 pwt.*

24. Reduce .5625 of a day to a compound number.

25. How many pints are .015625 of a bushel?

26. Reduce .7625 of a degree to lower integers.

27. Express the value of 3.076 cubic yards as a compound number.

28. Express the value of 19.742 acres as a compound number.

CASE II.

Reduction Ascending.

368.—Ex. 1. How many pecks in 31 quarts? In 29 quarts? How many bushels in 40 quarts?

2. How many yards in 96 inches? In 60 inches?

3. How many ounces in 71 pennyweights? In 63 pennyweights?

4. How many yards in 20 inches?

SOLUTION.—Since 12 inches are 1 foot, there are in 20 inches as many feet as $\frac{1}{12}$ of 20, or $\frac{5}{3}$ of a foot. Since 3 feet are 1 yard, there are in $\frac{5}{3}$ of a foot as many yards as $\frac{1}{3}$ of $\frac{5}{3}$, or $\frac{5}{9}$ of a yard.

5. How many tons in 14 hundred-weight?

6. How many gallons in $\frac{5}{8}$ of a quart?

WRITTEN EXERCISES.

369.—Ex. 1. How many gallons are 109 pints?

$$\begin{array}{l} 2)\underline{109} \\ 4)\underline{54} \ldots\ldots 1 \text{ pt.} \\ \quad 13 \text{ gal.} \ldots 2 \text{ qt.} \end{array}$$

$$109 \text{ pt.} = 13 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.}$$

SOLUTION.—Since 2 pints are 1 quart, there must be one half as many quarts as pints, or 54 quarts, with a remainder of 1 pint.

Since 4 quarts are 1 gallon, there must be one fourth as many gallons as quarts, or 13 gallons, with a remainder of 2 quarts. Hence, 109 pt. = 13 gal. 2 qt. 1 pt.

2. How many miles are 14400 rods?
3. How many tons are 10315 pounds?

370. Rule for Reduction Ascending.—*Divide the given number by that number of its denomination which equals one of the next higher, and write the remainder, if any.*

Divide the quotient in like manner, and so continue until the given number is reduced to the required denomination.

The last quotient, with the remainders, if any, written in their order from the highest to the lowest, will be the required result.

Reduction Ascending and Reduction Descending, being performed by opposite processes, are proofs of each other.

PROBLEMS.

1. How many acres are 183073 square yards?
2. How many bushels are 9870 quarts?
3. How many yards are 275 quarters?
4. How many miles are 20025 links?
5. How many hogsheads are 32256 gills?
6. How many tons are 35840 ounces?
7. How many ounces are 3829 grains?
8. How many quarter sections of land are 25600 square rods?

9. How many acres, square chains and square rods are 408 square rods?

10. How many cords are 2556 cubic feet?

11. How many days are 2551443 seconds?

12. How many bushels are $25\frac{3}{5}$ pints?

$$25\frac{3}{5} = \frac{128}{5} = No.\ of\ pt.$$
$$\frac{128}{5} \div 2 = \frac{64}{5} = No.\ of\ qt.$$
$$\frac{64}{5} \div 8 = \frac{8}{5} = No.\ of\ pk.$$
$$\frac{8}{5} \div 4 = \frac{2}{5} = No.\ of\ bu.$$

SOLUTION.—Since 2 pints are 1 quart, there must be one half as many quarts as pints, or $\frac{64}{5}$ of a quart.

Since 8 quarts are 1 peck, there must be one eighth as many pecks as quarts, or $\frac{8}{5}$ of a peck.

Since 4 pecks are 1 bushel, there must be one fourth as many bushels as pecks, or $\frac{2}{5}$ of a bushel.

13. What part of a ton is 375 pounds?

14. What part of a hogshead is 1344 gills?

15. What part of a day is $\frac{54}{625}$ of a second?

16. What decimal of a hundred-weight is 84 ounces?

SOLUTION.—*84 oz. = 84 ÷ 16,* or *5.25 lb. = 5.25 ÷ 100,* or *.0525 cwt.*

17. What decimal of an acre is 15 square rods?

18. What decimal of a pound Troy is 177 pennyweights?

19. Reduce 3 yd. 1 ft. $3\frac{3}{4}$ in. to a fraction of a rod.

$$3\frac{3}{4}\ in. = \frac{15}{4}\ of\ an\ in.;\ \frac{15}{4} \div 12 = \frac{5}{16};\ hence,\ 3\frac{3}{4}\ in. = \frac{5}{16}\ ft.$$
$$1\frac{5}{16}\ ft. = \frac{21}{16}\ of\ a\ ft.;\ \frac{21}{16} \div 3 = \frac{7}{16};\ hence,\ 1\frac{5}{16}\ ft. = \frac{7}{16}\ yd.$$
$$3\frac{7}{16}\ yd. = \frac{55}{16}\ of\ a\ yd.;\ \frac{55}{16} \div \frac{11}{2} = \frac{5}{8};\ hence,\ 3\frac{7}{16}\ yd. = \frac{5}{8}\ rd.$$

SOLUTION.—Since 12 inches are 1 foot, there must be one twelfth as many feet as inches, or $\frac{5}{16}$ of a foot.

Since 3 feet are 1 yard, there must be one third as many yards as feet, or $\frac{7}{16}$ of a yard.

Since $5\frac{1}{2}$ yards, or $\frac{11}{2}$ of a yard, are 1 rod, there must be $\frac{2}{11}$ as many rods as yards, or $\frac{5}{8}$ of a rod.

20. What fraction of a day is 16 h. 36 min. $55\frac{5}{18}$ sec.?

21. What fraction of a hogshead is 39 gal. 8 pt. 3 gi.?

22. What fraction of a mile is 85 rd. 1 yd. 2 ft. 6 in.?

23. Reduce 8 oz. 17 pwt. to a decimal of a pound.

SOLUTION.—*17 pwt.* = *17* ÷ *20,* or *.85 oz.; 8.85 oz.* = *8.85* ÷ *12,* or *.7375 lb.*

24. Reduce 13 h. 30 min. to a decimal of a day.

25. Reduce 1 pint to a decimal of a bushel.

26. Reduce 45′ 45″ to a decimal of a degree.

27. Express 3 cu. yd. 2 cu. ft. 89.856 cu. in. as a mixed decimal of a cubic yard.

28. Express 19 A. 118 sq. rd. 21.78 sq. yd. as a mixed decimal of an acre.

CASE III.

One Compound Number Reduced to the Fraction of Another.

371.—Ex. 1. How many feet are 3 yards 2 feet? Are 5 yards 1 foot?

2. What fraction of 16 feet is 1 foot? Is 11 feet?

3. Reduce 3 yards 2 feet to a fraction of 5 yards 1 foot.

4. What fraction of 4 pounds 3 ounces is 2 pounds 5 ounces?

WRITTEN EXERCISES.

372.—Ex. 1. Reduce 3 wk. 5 d. to a fraction of 11 wk. 5 d. 1 h.

3 wk. 5 d. = *624 h.*

11 wk. 5 d. 1 h. = *1969 h.*

SOLUTION.—Since only similar numbers can be compared (Art. 214), we reduce each of the given numbers to hours, and have as their equivalent 624 hours and 1969 hours.

Since 1 hour is $\frac{1}{1969}$ of 1969 hours, 624 hours must be $\frac{624}{1969}$ of 1969 hours. Hence, 3 wk. 5 d. are $\frac{624}{1969}$ of 11 wk. 5 d. 1 h.

2. Reduce 5 mi. 40 rd. to a decimal of 8 mi. 20 rd.

5 mi. 40 rd. = *1640 rd.*

8 mi. 20 rd. = *2580 rd.*

$$\frac{1640}{2580} = \frac{82}{129} = .635+$$

SOLUTION.—5 mi. 40 rd. are $\frac{1640}{2580}$ = $\frac{82}{129}$, of 8 mi. 20 rd.; and $\frac{82}{129}$ = .635 +. Hence, 5 mi. 40 rd. are .635 + of 8 mi. 20 rd.

3. What fraction of 6 gal. 1 qt. 1 pt. is 2 gal. 0 qt. 1 pt.?

373. Rules for Reduction of one Compound Number to the Fraction of another.—*1. Reduce both of the given numbers to the same denomination; and then make the number denoting the part the numerator, and that denoting the whole the denominator of the fraction required.*

2. When the fraction required is a decimal, reduce the common fraction thus found to a decimal.

PROBLEMS.

1. What fraction of 25° 42′ 40″ is 7° 42′ 48″?

2. What decimal will express the relation of 5 cwt. 91 lb. to 2 T. 7 cwt. 28 lb.?

3. From a farm containing 170 A. 16 sq. rd., I sold 37 A. 128 sq. rd. What part of the farm did I sell? *Ans.* $\frac{2}{9}$.

MISCELLANEOUS PROBLEMS.

374.—1. What is the cost of .6725 of a hundred-weight of butter, at 40 cents per pound?

2. What decimal of a hundred-weight of butter, at 40 cents per pound, can be bought for $26.90?

3. What will 1 hogshead 4 gallons 1 quart of wine cost, at $5 per gallon?

4. How much wine, at $5 per gallon, can be bought for $336.25?

5. The distance between two places on the same parallel of latitude is 17° 30′; how far apart are they, a degree in that latitude being 54 miles? *Ans.* 945 miles.

6. A boy has 1 pk. 6 qt. $\frac{4}{9}$ pt. of chestnuts; what part of a bushel has he?

7. How many ounces of gold weigh as much as 4 pounds of lead? *Ans.* $58\frac{1}{3}$.

8. What decimal of a ton of nails, at 5 cents a pound, can be bought for $2.40?

9. A grocer has 8316 eggs to pack in 11 boxes; how many dozen must he pack in each box? *Ans.* 63.

10. What will 23 A. 120 sq. rd. of land cost, at $.50 per square rod? *Ans.* $1900.

11. How many sheets of paper are 12 reams 5 quires 18 sheets?

12. What number of silver spoons, each weighing 1 oz. 9 pwt., can be made from 2 pounds of silver?

TEST QUESTIONS.

375.—1. What is a NUMBER? A denominate number? A simple denominate number? A compound denominate number?

2. What is REDUCTION? How do reductions descending and ascending differ? Which is performed by multiplication? Which by division?

3. What is CANCELLATION? Upon what principle does cancellation depend? (Art. 129—3.) How is a fraction reduced to its lowest terms? Upon what principle does the process depend? (Art. 162.)

4. In what two ways may a fraction be MULTIPLIED by an integer? How may a number be multiplied by a fraction? (Arts. 189–195.)

5. In what two ways may a fraction be DIVIDED by an integer? In what two ways may a number be divided by a fraction? (Arts. 203–206.)

SECTION XXXVI.

ADDITION OF COMPOUND NUMBERS.

376.—Ex. 1. What is the sum of 2 T. 15 cwt. 25 lb.; 3 T. 0 cwt. 64 lb.; and 7 cwt. 16 lb.?

2 T.	*15 cwt.*	*25 lb.*
3	*0*	*64*
	7	*16*
6 T.	*3 cwt.*	*5 lb.*

SOLUTION.—Since only units of like kind can be added (Art. 46—1), write the numbers so that units of the same denomination shall stand in the same column.

Begin at the right, and add the numbers of each denomination in the order of the denominations.

The sum of the pounds is 105 lb., or 1 cwt. 5 lb. Write the 5 lb. as the pounds of the sum, and add the 1 cwt. with the sum of hundred-weights.

The sum of the hundred-weights is 23 cwt., or 1 T. 3 cwt. Write the 3 cwt. as the hundred-weights of the sum, and add the 1 T. with the column of tons.

The sum of the tons is 6 T., which we write as the tons of the sum.

Therefore, 6 T. 3 cwt. 5 lb. is the sum required.

2. What is the sum of 810 yd. 1 ft. 10 in.; 617 yd. 2 ft. 11 in.; 85 yd. 2 ft. 8 in.; 679 yd. 5 in.; and 6 yd. 3 in.?

3. What is the sum of 3 lb. 9 oz. 18 pwt. 11 gr.; 1 lb. 4 oz. 19 pwt. 20 gr.; and 1 oz. 0 pwt. 23 gr.?

377. Rule for Addition of Compound Numbers.—*Write the numbers so that units of the same denomination shall stand in the same column.*

Begin with the lowest denomination, and add the numbers of each denomination separately. If the sum be less than one of the next higher denomination, write it as a part of the required result.

If the sum is equal to one or more units of the next higher denomination, write the excess, if any, as a part of the required result, and add the number of the higher denomination with the number of that denomination.

PROBLEMS.

1. What is the sum of 13 lb. 6 oz. 11 pwt. 9 gr.; 1 lb. 4 oz. 13 pwt. 20 gr.; and 1 oz. 0 pwt. 13 gr.?

2. Find the sum of 4 gal. 1 qt. 1 pt. 1 gi.; 4 gal. 0 qt. 1 pt. 3 gi.; 5 gal. 3 qt. 0 pt. 2 gi.; and 10 gal. 2 qt. 1 pt. 4 gi.

3. Find the sum of 71° 9′ 59.5″; 20° 24′ 18.4″; and 19° 30′ 34″.

4. Find the sum of 46 bu. 2 pk. 6 qt. 1 pt.; 43 bu. 2 pk. 2 qt. 1 pt.; 86 bu. 1 pk. 3 qt.; 68 bu. 3 pk. 1 qt. 1 pt.; 76 bu. 2 pk. 3 qt.; and 69 bu. 2 pk. 1 qt. 1 pt. *Ans.* 391 bu. 2 pk. 2 qt.

5. Find the sum of 30 A. 120 sq. rd.; 42 A. 60 sq. rd.; 80 A. 20 sq. rd.; and 150 sq. rd. *Ans.* 154 A. 30 sq. rd.

6. What is the sum of 100 rd. 5 yd. 2 ft.; 150 rd. 0 yd. 2 ft.; and 105 rd. 3 yd. 1 ft.?

	100 rd.	*5 yd.*	*2 ft.*
	150	*0*	*2*
	105	*3*	*1*
1 mi.	*36 rd.*	*$3\frac{1}{2}$ yd.*	*2 ft.*

Or,

1 mi. 36 rd. 4 yd. 0 ft. 6 in.

SOLUTION.—Since $\frac{1}{2}$ yd. = 1 ft. 6 in., we may substitute this value for the $\frac{1}{2}$ yd., and thus obtain, as an expression of the result, 1 mi. 36 rd. 4 yd. 0 ft. 6 in.

7. What is the sum of 2 A. 120 sq. rd. 10 sq. yd.; 3 A. 0 sq. rd. 12 sq. yd.; and 140 sq. rd. 20 sq. yd.?

Ans. 6 A. 101 sq. rd. $11\frac{3}{4}$ sq. yd.

Or, 6 A. 101 sq. rd. 11 sq. yd. 6 sq. ft. 108 sq. in.

8. What is the sum of 66 y. 99 d. 8 h. 50 min.; 9 y. 1 d. 2 h. 57 min.; 6 y. 70 d. 1 h.; and 5 h. 50 min.?

9. What is the sum of 13 cu. yd. 8 cu. ft. 1030 cu. in.; 20 cu. yd. 11 cu. ft. 903 cu. in.; and 107 cu. yd. 11 cu. ft. 1240 cu. in.? *Ans.* 141 cu. yd. 4 cu. ft. 1445 cu. in.

10. Find the sum of $\frac{5}{9}$ of a mile and $\frac{7}{8}$ of a rod.

SOLUTION.—Since $\frac{5}{9}$ *mi.* = *177 rd. 4 yd. 0 ft. 10 in.*

and $\frac{7}{8}$ *rd.* = *4 2 $5\frac{1}{4}$*

$\frac{5}{9}$ *mi.* + $\frac{7}{8}$ *rd.* = *178 rd. $3\frac{1}{2}$ yd. 0 ft. $3\frac{1}{4}$ in.*

Or, *178 rd. 3 yd. 1 ft. $9\frac{1}{4}$ in.*

11. What is the sum of 5.141 tons and .3218 of a ton, expressed as a compound number?

12. What is the sum of .005 of a common year and $\frac{3}{5}$ of a week? *Ans.* 6 d. 0 h. 36 min.

13. What is the sum of 3 gal. 2 qt. 0 pt. 1.4 gi.; $\frac{4}{5}$ of a gallon; and .875 of a hogshead?

14. I have in one range of wood 13 cd. 3 cd. ft.; in a second, 21 cd. 48 cu. ft.; and in a third, 42 cd. 4 cd. ft. 8 cu. ft. How much have I in all? *Ans.* 77 cd. $2\frac{1}{2}$ cd. ft.

15. A ship sailing from Boston, in latitude 42° 20′ north, to Cape Horn, 55° 58′ 15″ south, passes through how many degrees of latitude?

16. Washington is 77° 2′ 48″ of longitude west of Greenwich, and the extreme west point of Alaska is 91° 14′ 12″ west of Washington. What is the longitude of that point, reckoned from Greenwich? *Ans.* 168° 17′ W.

17. What is the sum of the following measurements: 2 yd. 2 ft. 7 in.; $7\frac{1}{2}$ yd.; 3 yd. 1 ft. 11 in.; $1\frac{1}{2}$ rd. 5 yd. 1 ft. 6 in.; and 2 rd. 16 ft. 6 in.? *Ans.* 8 rd. 0 yd. 0 ft. 9 in.

SECTION XXXVII.

SUBTRACTION OF COMPOUND NUMBERS.

378.—Ex. 1. From 17 bu. 2 pk. 6 qt. take 8 bu. 3 pk. 4 qt.

17 bu.	*2 pk.*	*6 qt.*
8	*3*	*4*
8 bu.	*3 pk.*	*2 qt.*

SOLUTION.—Since only units of the same kind can be subtracted the one from the other (Art. 57—1), write the subtrahend under the minuend, so that units of the same kind shall stand in the same column.

Begin at the right, and subtract the units of each denomination of the subtrahend from those of the same kind in the minuend.

4 qt. from 6 qt. leave 2 qt., which is the difference of the quarts.

Since 3 pk. cannot be taken from 2 pk., take 1 bu. from the 17 bu., leaving 16 bu., and add it, reduced to pecks, to the 2 pk., thus obtaining 6 pk.; then, 3 pk. from 6 pk. leave 3 pk., which is the difference of the pecks.

8 bu. from 16 bu. leave 8 bu., which is the difference of the bushels.

Therefore, 8 bu. 3 pk. 2 qt. is the difference required.

2. From 35 lb. 14 oz. take 19 lb. 15 oz. *Ans.* 15 lb. 11 oz.

379. Rule for Subtraction of Compound Numbers.—*Write the subtrahend under the minuend, so that units of the same denomination shall stand in the same column.*

Begin with the lowest denomination, and subtract the number of units of each denomination of the subtrahend from the number of units of the same denomination of the minuend, if possible, and write the difference beneath as a part of the required difference.

If the number of any denomination of the subtrahend is greater than that above it, increase the upper number by adding to it as many units as are one of the next higher denomination, and subtract; then, regarding the number of units of the next higher denomination of the minuend as one less, proceed as before.

PROBLEMS.

1. From 12 cwt. 85 lb. 11 oz. take 7 cwt. 58 lb. 6 oz.

2. From 1 hhd. 34 gal. 2 qt. 1 pt. 3 gi. take 45 gal. 3 qt. 1pt. 2 gi. *Ans.* 51 gal. 3 qt. 1 gi.

3. From 14 yd. 2 qr. take 9 yd. 3 qr.

4. From 78° 55′ 0″ take 71° 4′ 20″. *Ans.* 7° 50′ 40″.

5. From 116 cd. 4 cd. ft. 6 cu. ft. 1620 cu. in. take 105 cd. 5 cd. ft. 7 cu. ft. 1511 cu. in.

6. Subtract 7 lb. 7 oz. 10 pwt. 23 gr. from 21 lb. 4 oz. 14 pwt. 13 gr. *Ans.* 13 lb. 9 oz. 3 pwt. 14 gr.

7. Subtract 5 mi. 215 rd. 5 yd. from 8 mi. 216 rd. 3 yd. *Ans.* 3 mi. $3\frac{1}{2}$ yd., or 3 mi. 3 yd. 1 ft. 6 in.

8. Take .0038 of a year from $\frac{2}{5}$ of a week.

SOLUTION.—Since $\frac{2}{5}$ *wk.* = *2 d. 19 h. 12 min. 0 sec.*
and *.0038 y.* = *1 9 17 16.8*
$\frac{2}{5}$ *wk.* — *.0038 y.* = *1 d. 9 h. 54 min. 43.2 sec.*

9. Take $\frac{13}{20}$ of a square yard from 1 rd. 21 sq. ft. 56 sq. in.

10. Take $\frac{5}{8}$ of a great gross from 9.125 gross.

11. From $1\frac{13}{24}$ of a bushel take 3 pk. 0 qt. 1 pt.

12. A man has travelled 4 mi. 64 rd. How much farther must he go to have travelled 6 miles?

13. From a cask containing 36 gal. 1 qt. of molasses, 19 gal. 2 qt. 1 pt. have been drawn. How much remains in the cask?

DIFFERENCE OF DATES.

380.—In computation of the **Difference of Dates,** centuries are numbered from the beginning of the Christian era, the months from the beginning of the year, and the days from the beginning of the month.

Thus, May 23, 1871, is the 23d day of the 5th month of the 71st year of the 19th century.

In estimating the difference between dates, the entire calendar months are found, and the remaining days counted.

Any number of days less than 30, in business transactions, are usually regarded as the same number of thirtieths of a month.

381.—Ex. 1. A man left home on a journey, July 17, 1867, and returned November 12, 1869. How long was he absent?

1869 y.	*11 mo.*	*12 d.*
1867	*7*	*17*
2 y.	*3 mo.*	*26 d.*

SOLUTION.

From July 17, 1867, to July 17, 1869 = 2 y.
" July 17, 1869, to Oct. 17, 1869 = 3 mo.
" Oct. 17, 1869, to Nov. 12, 1869 = 26 d.
Hence, the entire difference is 2 y. 3 mo. 26 d.

2. A man was born May 16, 1819; how old was he September 23, 1862? *Ans.* 43 y. 4 mo. 7 d.

3. If a note dated February 25, 1868, was paid July 11, 1869, how long did it remain unpaid? *Ans.* 1 y. 4 mo. 16 d.

4. The late civil war, which continued 4 y. 1 mo. 14 da., terminated May 26, 1865. When did it begin?

Ans. April 12, 1861.

5. The American Revolution began April 19, 1775, and terminated January 20, 1783. How many years did it continue?

SECTION XXXVIII.

MULTIPLICATION OF COMPOUND NUMBERS.

382.—Ex. 1. Multiply 6 gal. 3 qt. 1 pt. by 5.

6 gal.	*3 qt.*	*1 pt.*
		5
34 gal.	*1 qt.*	*1 pt.*

SOLUTION.—Write the multiplier under the lowest denomination of the multiplicand, and, beginning at the right, multiply the number of each denomination in the order of the denominations.

Five times 1 pt. are 5 pt., or 2 qt. 1 pt. Write the 1 pt. as the number of that denomination in the product, and reserve the 2 qt. to be added to the product of the quarts.

Five times 3 qt. are 15 qt.; 15 qt. and 2 qt. are 17 qt., or 4 gal. 1 qt. Write the 1 qt. as the number of that denomination in the product, and reserve the 4 gal. to be added to the product of the gallons.

Five times 6 gal. are 30 gal.; 30 gal. and 4 gal. are 34 gal., which write as the gallons of the product.

The entire product is 34 gal. 1 qt. 1 pt.

2. Multiply 12 bu. 3 pk. 1 qt. by 7.

3. Multiply 7 yd. $3\frac{3}{4}$ qr. by 8. *Ans.* 63 yd. 2 qr.

383. Rule for Multiplication of Compound Numbers.—*Write the multiplier under the lowest denomination of the multiplicand.*

Begin with the lowest denomination, and multiply the number of each denomination in its order. If the product is less than one of the next higher denomination, write it as a part of the required product.

If the product is equal to one or more units of the next higher denomination, write the excess, if any, as a part of the required product, and add the number of units of the next higher denomination to the number of that denomination.

PROBLEMS.

1. Multiply 16° 58′ 26$\frac{8}{9}$″ by 9. *Ans.* 152° 46′ 2″.
2. Multiply 15 lb. 5 oz. 13 pwt. by 11.
3. Multiply 2 gal. 1 qt. 1 pt. 2 gi. by 19.
   Ans. 46 gal. 1 qt. 0 pt. 2 gi.
4. Multiply 1 T. 17 cwt. 92 lb. by 28.
5. One ship is in 5° 15′ 45″ north latitude, and another is 5 times as far north. What is the latitude of the latter?
   Ans. 26° 18′ 45″ north.
6. If a team can draw in one load 1 cd. 1$\frac{1}{2}$ cd. ft. of wood, how much can it draw in 14 loads?
7. I bought 4 packages of medicine, each containing 3 lb. 4 ℥ 6 ʒ 1 ℈ 16 gr. What is the weight of the whole?
   Ans. 13 lb. 7 ℥ 2 ʒ 1 ℈ 4 gr.
8. A farm consists of 9 fields, each containing 12 A. 72 sq. rd. What is the extent of the farm?
9. If a steamer sail 211 mi. 192 rd. a day, how far will it sail in 15 days?
10. How much time in 100 years, each 365 d. 5 h. 48 min. 49.7 sec. long? *Ans.* 36524 d. 5 h. 22 min 50 sec.
11. A lot of land is divided into 6 house-lots, each of which contains 1 A. 4 sq. rd. 120 sq. ft. How much land is there in all the lots? *Ans.* 6 A. 26 sq. rd. 175$\frac{1}{2}$ sq. ft.

SECTION XXXIX.

DIVISION OF COMPOUND NUMBERS.

384.—Ex. 1. Divide 34 gal. 1 qt. 1 pt. by 5.

5)34 gal. 1 qt. 1 pt.
6 gal. 3 qt. 1 pt.

SOLUTION.—Write the divisor at the left of the dividend, and beginning at the left, divide the number of each denomination in its order.

One fifth of 34 gal. is 6 gal., with a remainder of 4 gal. Write the 6 gal. as the gallons of the quotient; the 4 gal. = 16 qt., which added to the 1 qt. = 17 qt.

One fifth of 17 qt. is 3 qt., with a remainder of 2 qt. Write the 3 qt. as the quarts of the quotient; the 2 qt. are 4 pt., which, added to the 1 pt., = 5 pt.

One fifth of 5 pt. is 1 pt., which write as a part of the quotient.

The entire quotient is 6 gal. 3 qt. 1 pt.

2. Divide 89 bu. 1 pk. 7 qt. by 7. *Ans.* 12 bu. 3 pk. 1 qt.
3. Divide 39 lb. 7 oz. 8 pwt. 9 gr. by 6.
   Ans. 6 lb. 7 oz. 4 pwt. 17½ gr.

385. Rule for Division of Compound Numbers. — *Beginning with the highest denomination, divide the number of each denomination in its order, and write the several quotients as the parts of the same denominations of the required quotient.*

If there are partial remainders, reduce each to the next lower denomination, and add the same to the number of that denomination before dividing it.

When divisor and dividend are both compound numbers, they must be reduced to simple denominate numbers of the same denomination before dividing.

PROBLEMS.

1. Divide 67 yd. 2 qr. by 8. *Ans.* 8 yd. 1¾ qr.
2. Divide 23 cu. yd. 0 cu. ft. 12 cu. in. by 4.
   Ans. 5 cu. yd. 20 cu. ft. 435 cu. in.
3. Divide 53 T. 1 cwt. 76 lb. by 28.

4. In 9 equal lots of land, taken together, there are 112 A. 8 sq. rd. What is the extent of each lot?

Ans. 12 A. 72 sq. rd.

5. A quantity of tea, consisting of 19 equal parcels, contains 3 cwt. 32 lb. 8 oz. What is the weight of a single parcel?

Ans. 17 lb. 8 oz.

6. Divide 30° 2′ by 2° 30′ 10″.

$$30^\circ\ 2' = 108120''$$
$$2^\circ\ 30'\ 10'' = 9010''$$
$$108120'' \div 9010'' = 12$$

SOLUTION.—When, as in this problem, the divisor and dividend are similar compound numbers, reduce both to the lowest denomination mentioned in either, and divide as in simple numbers.

7. How many kegs, each containing 6 gal. 3 qt. 1 pt., can be filled from a cask containing 34 gal. 1 qt. 1 pt.?

8. If 8 yd. $1\frac{3}{4}$ qr. are required for a suit of clothes, how many suits can be made from 67 yd. 2 qr.?

9. How many loads, of 1 T. 17 cwt. 92 lb. each, are there in 53 T. 1 cwt. 76 lb. of hay? *Ans.* 28.

LONGITUDE AND TIME.

386.—The earth turns on its axis from west to east once in 24 hours. This causes the sun to appear to pass around the earth from east to west in the same time.

The sun appears sooner to places east of any given point on the earth than to those west of it. Hence, of any two given places, the one farthest east has the later time, and the one farthest west the earlier time.

Since the circumference of any circle is 360°, the sun appears to pass over $\frac{1}{24}$ of 360° of the earth's circumference, or 15° of longitude, in 1 hour; $\frac{1}{60}$ of 15°, or 15′, in 1 minute; and $\frac{1}{60}$ of 15′, or 15″, in 1 second. Hence the following

COMPARISON OF LONGITUDE AND TIME.

15° of Longitude	*correspond to*	*1 hour in time.*
15′ of Longitude	"	*1 minute in time.*
15″ of Longitude	"	*1 second in time.*

387.—Ex. 1. The difference in longitude between Washington and San Francisco is 45° 23′ 27″; what is the difference in time?

$$15\,)\,\underline{45^\circ \quad 23' \quad 27''}$$
$$3\text{ h.} \quad 1\text{ min.} \quad 33\tfrac{4}{5}\text{ sec.}$$

SOLUTION.—Since 15° of difference in longitude correspond to 1 hour difference in time, 15′ difference in longitude correspond to 1 minute difference in time, and 15″ difference in longitude correspond to 1 second difference in time, 45° 23′ 27″ of difference in longitude must correspond to $\frac{1}{15}$ as many hours, minutes and seconds respectively, or to 3 h. 1 min. $33\frac{4}{5}$ sec.

2. The difference in time between Washington and San Francisco is 3 h. 1 min. $33\frac{4}{5}$ sec.; what is the difference in longitude?

$$3\text{ h.} \quad 1\text{ min.} \quad 33\tfrac{4}{5}\text{ sec.}$$
$$\underline{\qquad\qquad\qquad 15}$$
$$45^\circ \quad 23' \quad 27''$$

SOLUTION.—Since 1 second difference in time corresponds to 15″ difference in longitude, 1 minute difference in time to 15′ of difference in longitude, and 1 hour difference in time to 15° difference in longitude, 3 h. 1 min. $33\frac{4}{5}$ sec. difference in time must correspond to 15 times as many seconds, minutes and degrees of longitude respectively, or to 45° 23′ 27″.

388. Rules for Longitude and Time.—*1. Divide the difference of longitude by 15, and the number of degrees, minutes and seconds of the quotient, respectively, will be the hours, minutes and seconds of the difference of time.*

2. Multiply the difference of time by 15, and the number of the seconds, minutes and hours of the product, respectively, will be the seconds, minutes and degrees of longitude.

PROBLEMS.

1. When it is 12 o'clock M. at that part of Alaska which is 87° 14′ 30″ west of Boston, what is the time in Boston?

Ans. 48 min. 58 sec. past 5 o'clock P.M.

2. The time at Philadelphia is 5 h. 0 min. 40 sec. earlier than that of Greenwich; what is the longitude of Philadelphia?

Ans. 75° 10′ W.

3. The difference in time between Portland and Chicago is 1 h. 9 min. $25\frac{11}{15}$ sec.; what is the difference in longitude?

Ans. 17° 21′ 26″.

4. When it is midnight at Canton, 113° 15′ east, what time is it at New Orleans, 90° 7′ west?

Ans. 26 min. 32 sec. after 10 o'clock A. M.

TEST QUESTIONS.

389.—1. In what respects is the ADDITION of compound numbers like addition of simple numbers? Why cannot dissimilar denominate numbers be added?

2. What is the difference between SUBTRACTION of simple and compound numbers? How may the subtraction be proved?

3. From what are the centuries numbered? Months? Days? What is the process of finding the difference between dates?

4. In what respects is MULTIPLICATION of compound numbers like multiplication of simple numbers? Why cannot a compound number be multiplied by a denominate number? (Art. 71—1.)

5. When in DIVISION of compound numbers the divisor is an abstract number, what kind of a number will the quotient be? When the divisor and dividend are similar compound numbers, what must be done before dividing? What kind of a number will the quotient be?

6. Why does the sun appear to pass around the earth from east to west? Of two places on the earth having different longitude, which has the earlier time?

7. What part of the earth's circumference does the sun appear to pass over in 1 hour? To what in time do 15° of longitude correspond? 15′ of longitude? 15″ of longitude? What are the rules for longitude and time?

8. What is the RULE for addition of simple numbers? (Art. 49.) The rule for addition of compound numbers? For subtraction of simple numbers? (Art. 60.) For subtraction of compound numbers?

9. What is the rule for multiplication of simple numbers? (Art. 75.) For the multiplication of compound numbers? For the division of simple numbers? (Art. 91.) For the division of compound numbers?

SECTION XL.

ANALYSIS BY ALIQUOT PARTS.

390.—Ex. 1. How many hundred-weight is one half of a ton? One fourth of a ton? One fifth of a ton? One eighth of a ton? One tenth of a ton?

2. What part of an acre is 80 square rods? Is 40 square rods? 32 square rods? 20 square rods?

3. What part of a year is 6 months? Is 4 months? 3 months? 2 months? What part of a month is 15 days? Is 10 days? 3 days?

4. What will one half a ton of hay cost, at $22 per ton? One fourth of a ton, at $24 per ton?

5. How much will 2 A. 80 sq. rd. of land cost, at $30 per acre? 5 A. 40 sq. rd., at $20 per acre?

6. How much will it cost a man for 2 mo. 3 d. board, at $20 per month?

DEFINITION.

Analysis by Aliquot Parts, or Practice, is a concise method of computation by employing aliquot parts. (Art. 279.)

WRITTEN EXERCISES.

391.—1. What is the rent of a store for 1 year 5 months 10 days, at $300 a year?

SOLUTION.

The rent for	*1 y.*	*is*		*$300.00*
" "	*4 mo.*	"	$\frac{1}{3}$ *of $300 =*	*100.00*
" "	*1 mo.*	"	$\frac{1}{4}$ *of 100 =*	*25.00*
" "	*10 d.*	"	$\frac{1}{3}$ *of 25 =*	*8.33*$\frac{1}{3}$
" "	*1 y. 5 mo. 10 d.*	"		*$433.33*$\frac{1}{3}$

2. At $80 a ton, what will 5 T. 15 cwt. 50 lb. of iron cost? *Ans.* $462.

3. At $.96 per gallon, what will 3 gal. 2 qt. 1 pt. of molasses cost?

4. How much will 9 months 24 days of labor amount to, at $600 a year? *Ans.* $490.

5. What is the cost of constructing 20 mi. 120 rd. of road, at $4000 per mile? *Ans.* $81500.

6. At $6 a hundred-weight, what must be paid for 1375 pounds of fish? *Ans.* $82.50.

7. What is the cost of 360 yards of camlet, at $.62½ per yard?

SOLUTION.

At $.50	*a yard, the cost of*	*360 yd. is*	$\frac{1}{2}$ *of $360 =*	*$180.00*
" $.12\frac{1}{2}$	" "	*360 yd.* "	$\frac{1}{4}$ *of 180 =*	*45.00*
" *$.62*$\frac{1}{2}$	" "	*360 yd.* "		*$225.00*

$.62½ = ½ of $1 + ¼ of ½ of $1. At ½ of a dollar a yard the cost of 360 yards is ½ of $360, or $180. At ¼ of ½ of $1 a yard the cost is ¼ of $180, or $45. Hence, at $.62½ a yard, the cost is $180 + $45, or $225.

8. What is the cost of 180 bushels of corn, at $1.12½ per bushel?

9. What is the cost of 460 yards of cloth, at $5.75 per yard?

10. What is the cost of 172 bushels of rye, at $.87½ per bushel?

SOLUTION.

At $1.00	*per bu., the cost*	*of 172 bu. is*		*$172.00*
" $.12\frac{1}{2}$	" "	*172 bu.* "	$\frac{1}{8}$ *of $172 =*	*21.50*
" *$ .87*$\frac{1}{2}$	" "	*172 bu.* "		*$150.50*

Since $.87½ is ⅛ of a dollar less than a dollar, at $.87½ per bu. the cost of 172 bu. will be ⅛ of $172 less than $172, or $150.50.

11. What is the cost of 1671 pounds of tea, at $.83⅓ per pound? *Ans.* $1392.50.

12. What is the cost of 4 T. 16 cwt. 20 lb. of hay, at $25 per ton? *Ans.* $120.25.

SECTION XLI.

RECTANGULAR MEASUREMENTS.

CASE I.

Surfaces.

392.—Ex. 1. How many square inches are there in a rectangular surface which is 15 inches long and 1 inch wide?

2. At 3 cents per square foot, how much must be paid for a rectangular board which is 25 feet long and 1 foot broad?

3. How many square rods are there in a walk which is 50 rods long and 1 rod wide?

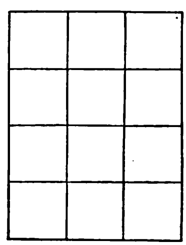
DEFINITIONS.

393. The **Dimensions** of a rectangular surface are the length and breadth, or width, of that surface.

394. The **Unit of Measure** for surfaces is always a square whose dimensions are known; as 1 square inch, 1 square foot, etc.

395. The **Area** of a surface is the number of times the surface contains a given unit of measure.

Thus, the rectangle in the margin will be seen to contain 12 square inches, if it be supposed to be 4 inches long and 3 inches wide.



For, upon each inch of length there may be conceived to be 1 square inch, making a row of 4 square inches, and as there will be as many such rows as there are inches in the width, or 3 rows, the area of the rectangle must be 3 times 4 square inches, or 12 square inches.

396. Principle.—*The area of a rectangle is equal to the number of square units denoted by the product of the number of linear units in the length, multiplied by the number of the same linear units in the width, the square units having the same name as the linear units.*

WRITTEN EXERCISES.

397.—Ex. 1. How many square feet of surface has a rectangular table whose length is 7 feet 5 inches and width 5 feet 4 inches?

89 × 64 = 5696 No. of sq. in.
5696 sq. in. = 39 sq. ft. 80 sq. in.

SOLUTION.—7 ft. 5 in. = 89 inches, and 5 ft. 4 in. = 64 inches.

The product of 89 by 64, or 5696, must denote the number of square inches of surface; and 5696 square inches are equal to 39 sq. ft. 80 sq. in., which is the surface required.

2. In a floor 16 feet long and 11 feet wide, are how many square feet? *Ans.* 176.

3. The area of a rectangular floor is 176 square feet, and its length is 16 feet. What is its width?

176 ÷ 16 = 11 ft.

SOLUTION.—Since the product of the number of linear units in the length by the number in the width is equal to the number of square units in the area, the number of linear units in the required dimension must equal the quotient of the number of the square units of the area divided by the number of linear units in the given dimension, or 11 feet.

4. The area of a board is 45 square feet, and its width $1\frac{1}{2}$ feet. What is its length?

398. Rules for Measurements of Rectangular Surfaces.—*1. Multiply the length by the width, and the product will denote the area.*

2. Divide the area by either of the dimensions, and the quotient will denote the other dimension.

PROBLEMS.

1. How many acres are there in a rectangular field whose length is 80 rd. and width 20 rd.? *Ans.* 10.

2. The area of a field is 4608 sq. rd., and its width is 16 rd.; what is its length?

3. How many square yards of carpeting will cover a room $13\frac{1}{2}$ ft. square? *Ans.* $20\frac{1}{4}$.

4. A path is 18 ft. 8 in. long, and 5 ft. 3 in. wide. What is its area in square feet?

SOLUTION.—18 ft. 8 in. $= 18\frac{2}{3}$ ft. $= \frac{56}{3}$ ft., and 5 ft. 3 in. $= 5\frac{1}{4}$ ft. $= \frac{21}{4}$ ft. $\frac{56}{3} \times \frac{21}{4} = 98$. Hence, the area required is 98 sq. ft.

5. If it take $32\frac{2}{3}$ sq. yd. of carpeting to cover a floor whose width is 14 ft., what is the length of the floor?

CASE II.

Solids.

399.—Ex. 1. How many cubic feet are there in a rectangular beam whose length is 20 ft., width 1 ft. and thickness 1 ft.?

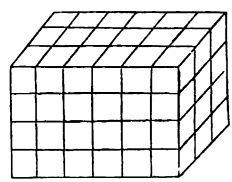
2. What is the value of a stick of timber 12 ft. long, 1 ft. wide and 1 ft. thick, at 10 cents per cubic foot?

DEFINITIONS.

400. The **Dimensions** of a rectangular solid, or volume (Art. 319), are the length, breadth or width, and thickness, depth or height.

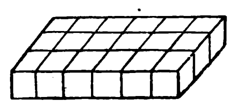
401. The **Unit of Measure** for solids, or volumes, is always some cube (Art. 317) whose dimensions are known; as, 1 cubic inch, 1 cubic foot, etc.

402. The **Cubic Contents,** or capacity, of a solid, or volume, is the number of times the solid or volume contains a given unit of measure.



Thus, the rectangular solid, or volume, in the margin will be seen to contain 72 cubic inches, if it be supposed to be 6 inches long, 3 inches wide and 4 inches thick.

For, upon each of the 18 square inches of the lower face there may be conceived to be 1 cubic inch, making a layer of 18 cubic inches; and as there will be as many such layers as there are inches of thickness, or 4, the contents of the volume must be 4 times 18 cubic inches, or 72 cubic inches.



403. Principle.—*The cubic contents of a solid, or volume, are equal to the number of cubic units denoted by the product of the number of the same linear units in the length, width and thickness, the cubic units having the same name as the linear units.*

WRITTEN EXERCISES.

404.—Ex. 1. How many cubic feet are there in a rectangular block of marble which is 8 ft. long, 3 ft. 6 in. wide and 2 ft. 3 in. thick?

$$\begin{array}{r} 8 \\ \underline{3\frac{1}{2}} \\ 28 \\ \underline{2\frac{1}{4}} \\ 63 \text{ cu. ft.} \end{array}$$

SOLUTION.—3 ft. 6 in. = $3\frac{1}{2}$ ft., and 2 ft. 3 in. = $2\frac{1}{4}$ ft. The product of the number of units in the length, width and thickness is 63, which must be the number of cubic feet required.

2. What are the cubic contents of a body 20 ft. long, 6 ft. wide and 4 ft. thick?

3. A block, containing 15625 cubic inches, is 2 feet 1 inch wide and 2 feet 1 inch thick. What is its length?

Ans. 2 ft. 1 in.

4. A rectangular body whose cubic contents are 480 cu. ft. is 20 ft. long and 6 ft. wide. What is its thickness?

$$20 \times 6 = 120$$

$$120\overline{)480}$$
$$4$$

SOLUTION.—Since the number of cubic feet of contents must be the product of the number of units in the length, width and thickness, the quotient of 480 divided by 120, the product of the units in the two given dimensions, must be the number of feet of thickness required. 480 ÷ 120 = 4. Hence, 4 feet must be the thickness required.

405. Rules for Measurement of Rectangular Solids.—*1. Multiply the length, width and thickness together, and the product will denote the cubic contents.*

2. Divide the cubic contents by the product of any two of the dimensions, and the quotient will denote the other dimension.

PROBLEMS.

1. How many cubic feet is the capacity of a bin whose inside measures 12 feet long, $6\frac{1}{4}$ feet wide and $5\frac{1}{3}$ feet deep?
Ans. 400.

2. What are the contents of a cube whose edge measures 5.5 feet? *Ans.* 166.375 cu. ft.

3. How many cord feet are there in a load of wood 8 feet long, $3\frac{1}{2}$ feet wide and 5 feet high?

4. How much wood of the usual length is there in a range 163 feet long and 4 feet high?

5. If a load of wood is 8 feet long and 3 feet wide, how high must it be to contain a cord? *Ans.* 5 ft. 4 in.

SECTION XLII.

REVIEW PROBLEMS.

MENTAL EXERCISES.

406.—Ex. 1. How much will 2 pecks of berries cost, at $12\frac{1}{2}$ cents per quart?

2. How much is gained by selling a gross of buttons, which cost 75 cents, for $8\frac{1}{3}$ cents per dozen?

3. If the forenoon and afternoon sessions of a school are each 3 hours, what part of the two sessions are two recesses of 20 minutes each?

4. When it is 9 o'clock in the morning at Philadelphia, what time is it at a point 15° 45′ west of Philadelphia?

5. How much must be paid for a board 22 feet long and 1 foot 6 inches wide, at the rate of $30 per thousand square feet?

6. If you should leave home and travel till your watch is 35 minutes fast, how far in longitude would you have travelled, and in what direction?

7. Two boys were employed to measure the length of a ditch; one reported it to be 1 rd. 16 ft. 11 in., and the other 1 rd. 5 yd. 1ft. 11 in. The true length was 2 rd. 5 in. How much did each of the measurements differ from the true length?

WRITTEN EXERCISES.

407.—Ex. 1. How many hogsheads are 2217 quarts?

2. How many quarts are 8 hhd. 50 gal. 1 qt.?

3. In 1000000 seconds are how many weeks?

4. In 1 wk. 4 d. 46 min. 40 sec. are how many seconds?

5. How much wood in a range 40 feet long, $7\frac{1}{2}$ feet high and 4 feet wide? *Ans.* $9\frac{3}{8}$ cords.

6. What must be the height of a range of wood which is 40 feet long and 4 feet wide, to contain 9 cd. 3 cd. ft.?

7. What decimal part of 3 gallons is 3 pints?

8. What is the sum of $\frac{3}{4}$ of a foot, $\frac{5}{8}$ of a yard and $\frac{7}{8}$ of a mile? *Ans.* 280 rd. 0 yd. 2 ft. $7\frac{1}{2}$ in.

9. What is the value of $.13\frac{1}{3}$ of 5 hours?

10. What decimal part of 5 hours is 40 seconds?

11. How many years after the battle of Lexington, April 18, 1775, was that of New Orleans, January 8, 1815?

12. How many days will there be from October 10, 1871, to March 17, 1872?

13. If the 16th of May is Sunday, what day of the week is the 20th of the next October?

Solution.—The difference in the given dates is 157 days, or 22 weeks 3 days. 3 days after Sunday, the ending of the 22 weeks, must be Wednesday, the day of the week required.

14. What part of a day is 6 h. 3 min. 4 sec.?

15. If a clock tick 172800 times a day, how many times will it tick in 6 h. 3 min. 4 sec.?

16. When 2 bu. 1 pk. of clover-seed is sold for \$5.94, what is the price of a peck?

17. How much molasses in 43 casks, each holding 97 gal. 1 pt. 2 gi.? *Ans.* 4179 gal. 0 qt. 0 pt. 2 gi.

18. How long must a rectangular lot of land be, whose width is 16 rods, to contain 2 acres? *Ans.* 20 rods.

19. How much is $\frac{3}{7}$ of 45 T. 15 cwt. 25 lb.?

20. How many yards of cambric, which is $\frac{3}{4}$ of a yard wide, will line $6\frac{3}{4}$ yards of cloth, which is $1\frac{1}{4}$ yards wide?

21. What will it cost to carpet a room 20 feet wide and 18 feet long with carpeting 4 feet wide, costing \$2.33$\frac{1}{3}$ per yard?

SECTION XLIII.

PERCENTAGE.

408.—Ex. 1. How much is $\frac{1}{100}$ of 100 yards? $\frac{2}{100}$?

2. How much is $\frac{3}{100}$ of 100? $\frac{5}{100}$? $\frac{7}{100}$? $\frac{25}{100}$?

3. What part of 100 yards is 1 yard? Is 11 yards?

4. How many hundredths of a hundred-weight are 3 pounds?

5. How many hundredths of anything is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it?

6. What part of 100 hundredths is 25 hundredths?

DEFINITIONS.

409. Any **Per cent.** of a number is so many hundredths of that number.

The term *per cent.* is a contraction of the Latin *per centum*, and means *by the hundred.*

Thus, 6 per cent. of 25 is .06 of 25; and 5 per cent. of 4 is .05 of 4.

410. The **Sign** % is generally used by business-men, instead of the words *per cent.*

Thus, 8% means 8 per cent.

411. Any per cent. less than 100 per cent. may be expressed by a decimal or common fraction; any per cent. equal to or greater than 100 per cent. may be expressed by an integer, a mixed number or an improper fraction; and a fractional part of 1 per cent. may be expressed as a common fraction at the right of the figure in the hundredths order. Thus,

1 per cent.	may be written,	*1%, .01,*	or	$\frac{1}{100}$
6 per cent.	"	*6%, .06,*	"	$\frac{6}{100}$
7 per cent.	"	*7%, .07,*	"	$\frac{7}{100}$
$7\frac{3}{10}$ per cent.	"	$7\frac{3}{10}\%$, $.07\frac{3}{10}$,	"	$\frac{7\frac{3}{10}}{100}$
$12\frac{1}{2}$ per cent.	"	$12\frac{1}{2}\%$, $.12\frac{1}{2}$,	"	$\frac{12\frac{1}{2}}{100}$
100 per cent.	"	*100%, 1.00,*	"	$\frac{100}{100}$
125 per cent.	"	*125%, 1.25,*	"	$\frac{125}{100}$

WRITTEN EXERCISES.

412. Write and read—

1. 4%.	4. 80%.	7. $2\frac{1}{3}$ per cent.
2. 5%.	5. 17%.	8. $16\frac{2}{3}$ per cent.
3. 16%.	6. 106%.	9. 217 per cent.

10. Write decimally 3%; 11%; 14%; 93%.
11. Write as a common fraction 5%; 17%; 31%.
12. Express as a number of hundredths $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{5}$.
13. Express in per cent. $\frac{3}{4}$; $\frac{2}{5}$; $2\frac{1}{4}$; $6\frac{1}{4}$; $4\frac{1}{2}$.

GENERAL CASES OF PERCENTAGE.

413.—Ex. 1. To take 5% of a number is to take how many hundredths of that number?

2. What is 5% of 100 bushels? Of 200 yards?
3. What is 1% of \$500? 7% of \$500?
4. What per cent. of 100 bushels is 5 bushels? Is 6 bushels? Is 25 bushels?
5. What per cent. of \$500 is \$5? \$35?
6. Five bushels are 5% of what number of bushels?
7. Thirty-five dollars are 7% of what number of dollars?
8. Twelve dollars are 2% of what number of dollars?
9. Of what number are 15 yards 6%?
10. Of what number are 27 gallons 9%?
11. To what will \$400 amount if increased by 5% of itself?
12. How much is \$200 diminished by 25% of itself?
13. A certain number increased by 5% of itself is 420; what is that number?
14. A certain number diminished by 25% of itself is 150; what is that number?

DEFINITIONS.

414. Percentage is the process of computing by hundredths.

415. The **Base** is the number or quantity of which the hundredths are computed.

416. The **Rate Per Cent.**, or **Rate**, is the number denoting the number of hundredths of the base which are taken.

417. The **Percentage** is the result of finding a number of hundredths of the base.

The percentage is also sometimes called the per cent.

418. The **Amount** is the base with the percentage added to it.

419. The **Difference** is the base with the percentage subtracted from it.

CASE I.

Base and Rate given, to find the Percentage, Amount or Difference.

420.—Ex. 1. What is 20% of 455 yards, and what is the amount and difference?

$$\begin{array}{r} 455\ yd. \\ .20 \\ \hline 91.00\ yd. \end{array}$$

Or,

$$455\ yd. \times \frac{1}{5} = \frac{455}{5}\ yd. = 91\ yd.$$

$$Amount = 455\ yd. + 91\ yd. = 546.$$

$$Difference = 455\ yd. - 91\ yd. = 364\ yd.$$

SOLUTION.—Since 20% of any number is .20 of that number, 20% of 455 yards must be .20 of 455 yards, or 91 yards.

Or, since 20% of any number is $\frac{1}{5}$ of that number, 20% of 455 yards must be $\frac{1}{5}$ of 455 yards, or 91 yards.

The amount is 455 yards plus the percentage, or 546 yards; and the difference is 455 yards minus the percentage, or 364 yards.

2. How much is 15% of $460?

3. How much is 2% of 6550 pounds, and what is the amount?

421. Principles.—1. *The percentage is equal to the product of the base by the rate.*

2. *The amount is equal to the sum of the base and percentage.*

3. *The difference is equal to the base less the percentage.*

422. Rules for Finding the Percentage, Amount or Difference.—
1. Multiply the base by the rate, and the product will be the percentage.

2. Add the percentage to the base, and the sum will be the amount; or subtract the percentage from the base, and the result will be the difference.

PROBLEMS.

Find the percentage of—

1. 516 bushels at 5%.
2. \$360 at 15%. *Ans.* \$54.
3. 455 gallons at 3%.
4. 93 acres at 7%. *Ans.* 6.51 A.
5. 812 men at 25%.

How much is—

6. 7% of 210 pounds?
7. $\frac{3}{4}$% of 703 yards?
8. 33$\frac{1}{3}$% of 942 sheep?
9. 125% of 1215 tons?
10. 350% of \$1600?

11. If a man, whose income is \$1250 a year, spends 80% of it for his living, how many dollars does he spend?

Ans. \$1000.

CASE II.

Base and Percentage given, to find the Rate.

423.—Ex. 1. What per cent. of 455 yd. is 91 yd.?

$\frac{91}{455} = .20 = 20\%$

Or,

$\frac{91}{455}$ of $100\% = 20\%$

Solution.—91 yd. are $\frac{91}{455}$ of 455 yd., or .20, or 20% of 455 yd. Or, 91 yd. are $\frac{91}{455}$ of 455 yd. $\frac{91}{455} = .20$; hence, 91 yd. equal .20 of 455 yd., or 20%.

2. What per cent. of \$390 is \$11.70? *Ans.* 3.
3. What per cent. of 3240 T. is 21.6 T.? *Ans.* $\frac{2}{3}$ of 1.

424. Principle.—*The rate is equal to the quotient of the percentage divided by the base.*

425. Rules for Finding the Rate Per Cent.—*1. Divide the percentage by the base. Or,*

2. Take such a part of 100 per cent. as the percentage is of the base.

PROBLEMS.

What per cent. of—

1. 516 bu. is 25.80 bu.?
2. 455 gal. is 13.65 gal.?
3. 93 A. is 6.51 A.?
4. 14.70 lb. is 14.70 lb.?
5. \$1600 is \$4000? *Ans.* 250.
6. 460 ft. is 368 ft.? *Ans.* 80.
7. 514$\frac{1}{2}$ rd. is 84 rd.?
8. 942 sheep is 314 sheep?
9. 703 yd. is 5.26$\frac{3}{4}$ yd.?
10. \$6300 is \$173.25?

11. What per cent. of 2 A. 72 sq. rd. is 144 sq. rd.?

SOLUTION.

2 A. 72 sq. rd. = 392 sq. rd.

$$144 \text{ sq. rd.} = \frac{144}{392} = \frac{18}{49} = 36\frac{36}{49}\% \text{ of } 392 \text{ sq. rd.}$$

12. A pound Troy is what per cent. of a pound Avoirdupois?

13. A merchant had 1 T. 15 lb. 10 oz. of sugar, and sold 10 cwt. 46 lb. 4 oz. What per cent. did he sell? *Ans.* $51\frac{39}{48}$.

CASE III.

Rate and Percentage Given, to find the Base.

426.—Ex. 1. 91 yd. are 20% of how many yards?

$$\frac{91}{.20} = 455$$

Or,

$$91 \times 5 = 455$$

SOLUTION.—20% of any number is .20 of that number. Since 91 yd. are 20% of some number, 1% of that number must be $\frac{1}{20}$ of 91 yd.; and 100%, or the number required, must be 100 times $\frac{1}{20}$ of 91 yd., or $\frac{100}{20}$ of 91 yd., which is the same as the quotient obtained by dividing 91 yd. by .20. Or,

Since 100% is 5 times 20%, and 91 yd. are 20% of some number, that number must be 5 times 91 yd.; or, since 20% is $\frac{1}{5}$ of a number, and 91 yd. are 20% of some number, that number must be the quotient obtained by dividing 91 yd. by $\frac{1}{5}$.

2. Of what number are 71 mi. $16\frac{2}{3}$%?
3. Of what number are 203 men 50%? *Ans.* 406 men.

427. Principle.—*The base is equal to the quotient of the percentage divided by the rate.*

428. Rules for Finding the Base of the Percentage.—*1. Divide the percentage by the rate. Or,*

2. Take as many times the percentage as 100 per cent. is times the rate.

PROBLEMS.

Of what are—

1. 13.65 gal., 3%?	4. $5.27\frac{1}{4}$ yd., $\frac{3}{4}$%?
2. 6.51 A., 7%?	5. $5600, 350%?
3. $22.61, $\frac{7}{8}$%? *Ans.* $2584.	6. 1724 ft., 400%? *Ans.* 431 ft.

7. If the percentage be 23.8 lb. and the rate 4%, what must be the base?

8. If the rent of a store at $1020 is 12% of its valuation, what is its valuation? *Ans.* $8500.

9. A farmer sold 85 A. 100 sq. rd. of land, which was just 25% of his farm. What was the extent of his farm?

CASE IV.

Amount or Difference and Rate Given, to Find the Base.

429.—Ex. 1. A certain number increased by 8% of itself is 189; what is that number?

1.08)189.00(175
108
810
756
540
540

Solution.—A number increased by 8% of itself is 108% of itself, or 1.08 times itself.

If 189 is 1.08 times a number, that number must be $\frac{1}{1.08}$ of 189, or 175.

2. A certain number diminished by 25% of itself is 327. What is the number?

.75)327.00(436
300
270
225
450
450

Solution.—A number diminished by 25% of itself is 75% of itself, or .75 of itself.

If 327 is .75 of a number, that number must be $\frac{1}{.75}$ of 327, or 436.

3. If the amount is 124.20, and the rate 15%, what is the base? *Ans.* 108.

4. If the difference is 278.30, and the rate 45%, what is the base?

430. Principle.—*The base is equal to the quotient of the amount divided by* 1 *plus the rate, or to the quotient of the difference divided by* 1 *minus the rate.*

431. Rule.—*Divide the amount by 1 plus the rate, or divide the difference by 1 minus the rate.*

PROBLEMS.

1. An infantry regiment, after losing $7\frac{1}{2}\%$ of its men, had 740 left. How many had it at first? *Ans.* 800.

2. By running 15% faster than usual, a locomotive ran 552 miles in a day. What was the usual daily speed? *Ans.* 480 miles.

3. A farmer purchased a farm for a certain sum, expended for tools and stock 11% of the price of the farm, and found that the whole cost was $7215. What was the cost of the farm alone?

4. Osgood raised 800 bushels of corn, which was 25% more than $\frac{1}{2}$ of what Benton raised. How many bushels did Benton raise? *Ans.* 1280.

SECTION XLIV.

PROFIT AND LOSS.

432.—Ex. 1. I sold a barrel of flour, which cost $8, at an advance of 25%, or for $\frac{1}{4}$ more than the cost. How much did I gain?

2. John bought a hat for $5, and sold it at a loss of 20%, or for $\frac{1}{5}$ less than the cost. How much did he lose?

3. If I sell a horse, which cost me $120, at a profit of 10%, how much do I get for him?

4. I had 40 sheep, but have sold 4 of them. What per cent. of the 40 have I sold?

5. What per cent. shall I gain by selling for $10, flour which cost me $8?

6. What per cent. does a merchant lose by selling goods at $\frac{4}{5}$ of their cost?

7. I sold flour at $10 and gained 25%. What did it cost me?

DEFINITIONS.

433. Profit and **Loss** are terms used to denote the gain or loss in business transactions.

The gain or loss may be regarded as a certain per cent. of the cost. Hence,

434. The **Base** of computation of profit or loss is the cost; and, by the principles of percentage, we have the—

435. Rules for Profit or Loss.—*1. Multiply the cost by the rate, and the product will be the profit or loss.*

2. Divide the profit or loss by the cost, and the quotient will be the rate.

3. Multiply the cost by 1 plus the rate of profit, or by 1 minus the rate of loss, and the product will be the selling price.

4. Divide the profit or loss by the rate; or, divide the selling price by 1 plus the rate of profit, or by 1 minus the rate of loss, and the quotient will be the cost.

PROBLEMS.

Ex. 1. I bought a house for \$3500, and sold it at a gain of $12\frac{1}{2}\%$. How much was the profit?

2. Goods which cost \$5400, were sold at 9% below cost. How much was the loss? *Ans.* \$486.

3. A farmer had 460 sheep, which cost him \$3 each, but he lost 5% of them. How much was the loss? *Ans.* \$69.

4. A merchant bought 112 barrels of flour, at \$7 a barrel, and sold it so as to gain 15%. How much was the profit?

5. How much will be my loss on 3 casks of molasses, of 63 gallons each, which cost me 80 cents a gallon, if I am obliged to sell it at 10% below cost?

6. Williams bought coal at \$7.50 per ton, and sold it at \$7.05 per ton. What was the loss per cent.? *Ans.* 6.

7. A field produced the value of \$67 one year, and \$16.75 more the next year. What was the gain per cent.?

8. If I sell $\frac{1}{2}$ of a house at $\frac{3}{5}$ of the cost of the whole house, do I gain or lose, and at what rate? *Ans.* Gain 20%.

9. By selling coal at \$8.05 per ton, I gained 15%. What was the cost per ton? *Ans.* \$7.

10. A watch not proving as good as I expected, I was content to sell it at a loss of \$6, which was $7\frac{1}{2}\%$ of the cost. What was the cost? *Ans.* \$80.

11. By selling an article at $87\frac{1}{2}\%$ of its cost, I lost \$125. What was the cost?

12. I sold molasses at 115% of its cost, and thereby gained 9 cents on a gallon. What was the cost per gallon?

13. Henry bought cloth at \$6.50 per yard; at what price must it be marked to allow of 12% profit?

14. At what price must I sell a house which cost me \$5400, to gain 20%? *Ans.* \$6480.

15. John paid \$225 for a horse, and \$15 for his keeping. For how much must he be sold above the first cost to allow of $5\frac{1}{2}\%$ profit? *Ans.* \$28.20.

16. I bought 60 yd. of cloth for \$240; what must be my selling price per yard to make $12\frac{1}{2}\%$?

17. I sold a house for \$6976, and thereby gained 9%. What was the cost? *Ans.* \$6400.

18. By selling tea at 76 cts. per pound, I suffer a loss of 20%. What was the cost?

TEST QUESTIONS.

436.—1. What is any PER CENT. of a number? Of what is the term a contraction? What sign is generally used for the words Per Cent.?

2. How may any per cent. less than 100 per cent. be expressed? A per cent. equal to or greater than 100 per cent.?

3. What is PERCENTAGE? The base of percentage? The rate? The amount? The difference?

4. To what is the percentage equal? How is the percentage found when the base and rate are given? How is the amount found? How is the difference found?

5. To what is the RATE equal? How is the rate found when the base and percentage are given?

6. To what is the BASE equal? How is the base found when the rate and percentage are given? How is the base found when the amount or difference and rate are given?

7. What are PROFIT and LOSS? What is the base of computation of profit and loss? To what is the profit or loss equal? The rate? The cost?

8. To what is the SELLING PRICE equal? How is the cost found when the selling price and the rate of profit or loss are known?

SECTION XLV.

COMMISSION.

437.—Ex. 1. How much should I receive for selling goods to the amount of $500, if allowed $2\frac{1}{2}\%$?

2. If I am allowed $2\frac{1}{2}\%$ for making purchases, how much per ton should I receive for purchasing coal at $8 per ton?

3. To how much will a collector be entitled for collecting $300, at 2%?

4. How much will remain of a collection of $300, after deducting the collector's fees at the rate of $1\frac{1}{2}\%$?

DEFINITIONS.

438. An **Agent, Commission Merchant** or **Broker** is a person who, by authority, buys or sells goods or property, or collects money for another.

439. A **Consignee** is a person to whom goods are sent for sale, and a **Consignor** is the person sending the goods.

440. **Commission** is the percentage allowed an agent or commission merchant as pay for transacting business.

441. The **Base** of commission is the sum expended or received.

442. The **Amount** is the sum expended or received plus the commission.

443. The **Net Proceeds** of a sale or collection are the sum left after the commission and other charges, if any, are deducted.

444. **Rules for Commission.**—*1. Multiply the base by the rate, and the product is the commission.*

2. Subtract from the base the commission, and the other charges, if any, and the result will be the proceeds.

3. Divide the commission by the base, and the quotient is the rate.

4. Divide the commission by the rate, or divide the amount by 1 plus the rate, and the quotient is the base.

PROBLEMS.

1. An agent has sold goods for me to the amount of $5000, at $2\frac{1}{2}\%$ commission. What is his commission? *Ans.* $125.

2. A commission merchant sells 4520 bu. of wheat, at $2.50 per bushel. How much is his commission at 3%?

3. If I collect as an agent $390, and am entitled to 5% commission, how much must I pay over? *Ans.* $370.50.

4. An agent bought 80 barrels of beef, at $22 per barrel, and paid $16 for insurance and $9 for cartage. His commission was $2\frac{1}{4}\%$. What was the amount of his bill to his employer? *Ans.* $1824.60.

5. A commission merchant having sold some goods, paid $4168.80 to his employer, and retained as his commission $151.20. What was the rate of his commission?

6. My agent collected $390, and, retaining his commission, paid over $370.50. At what rate was his commission?

7. A commission merchant bought some goods, paid for cartage $21.50, and charged for storage $31, and for commission $112.50. His entire bill was $5165. What was the rate of commission? *Ans.* $2\frac{1}{4}\%$.

8. I paid a commission merchant $12.56 for selling goods, at the rate of 4%. What amount did he sell? *Ans.* $314.

9. A treasurer's commission for collecting taxes in one year, at $1\frac{1}{2}\%$, is $413.10. What was the amount collected?

10. A commission merchant in Chicago received $$1665.62\frac{1}{2}$ with which to purchase flour at $6.50 per barrel, after deducting his commission of $2\frac{1}{2}\%$. How much was his commission, and how many barrels did he purchase?

Ans. Commission, $$40.62\frac{1}{2}$; barrels, 250.

11. An agent received $5922 with which to purchase goods, after deducting his commission of 5%. How much was his commission, and what was the sum to be expended?

Ans. Commission, $282; sum, $5640.

12. I remit to an agent $360.70, with which to purchase goods; deduct his commission of 5%, and pay $3.70 for insurance. What sum can he expend for the goods?

SECTION XLVI.

INSURANCE.

445. **Insurance** is indemnity secured for loss.

446. **Fire Insurance** is indemnity secured for loss of property by fire or lightning.

447. **Marine Insurance** is indemnity secured for loss of property by casualties of navigation.

448. **Health, or Accident, Insurance** is indemnity secured for loss by sickness or accident.

449. **Life Insurance** is indemnity secured for loss of life.

450. The **Policy** is the contract between the insurer and the insured.

451. The **Premium** is the sum paid for insurance.

452. In **Property Insurance** the premium is computed at a certain rate per cent. on the value insured.

453. In **Life Insurance** the premium is computed at a certain sum or rate per \$100 or \$1000 insured.

454. **Rules for Insurance.**—*1. Multiply the value insured by the rate, and the product is the premium.*

2. Divide the premium by the value insured, and the quotient is the rate.

3. Divide the value insured by 1 minus the rate, and the quotient will be the amount to be insured to cover the value insured and premium of insurance.

PROBLEMS.

1. What premium must be paid for insurance of \$3000 on a house, at $2\frac{1}{2}\%$? *Ans.* \$75.

2. What is the expense of insuring $\frac{3}{4}$ of a mill valued at \$8400, at 5%, the policy being \$1? *Ans.* \$316.

3. Hall paid \$91, including the policy at \$1, for insuring

$2500 on his house and $2000 on his saw-mill. What was the rate of insurance? *Ans.* 2%.

4. If $75 is paid for insuring $3000 on a house, what is the rate of premium?

5. What sum must be insured on $5600, at 3%, to cover property and premium in case of loss? *Ans.* $5773.19 +.

6. For what sum must property, valued at $4000, be insured, at 5%, to cover $\frac{3}{4}$ of the property, the premium and the policy at $2? *Ans.* $3160.

7. A man 40 years old has obtained a life policy for $6000, at the rate of $29.60 per $1000. What is the annual premium?

8. A man 31 years old took out a life policy for $7500 for the benefit of his family, on the plan of semi-annual payments, at the rate of $23.78 per $1000. He died at the age of 35. How much did the amount due his family exceed the payments he had made? *Ans.* $6073.20.

9. If $3160 must be insured on a house to cover $\frac{3}{4}$ of its value, the premium at 5% and the policy at $2, what is the value of the house? *Ans.* $4000.

TEST QUESTIONS.

455.—1. What is a COMMISSION MERCHANT or BROKER? A consignee? A consignor?

2. What is COMMISSION? The base of commission? The amount? What are the net proceeds?

3. To what is the COMMISSION equal? How is it found when the base and rate are given?

4. To what are the NET PROCEEDS equal? How are they found when the base and commission are given?

5. To what is the RATE equal? How is the rate found when the commission and base are given?

6. To what is the BASE equal? How is it found when the commission and rate are given? When the amount and rate are given?

7. What is INSURANCE? Fire insurance? Marine insurance? Health and accident insurance? Life insurance?

8. What is the POLICY? The premium? How is the premium computed in property insurance? In life insurance?

SECTION XLVII.

REVIEW PROBLEMS.

MENTAL EXERCISES.

456.—Ex. 1. At what price must tea which cost 75 cts. be sold, to make $16\frac{2}{3}\%$ profit?

2. I have 60 cts.; how much of it must I spend to have 88% of it left?

3. If I had 35 sheep and sold 14, what per cent. of them did I sell?

4. I bought a cart for $75, and sold it for $90. What per cent. did I gain?

5. In a certain school, 5 of every 8 pupils are girls. What per cent. are boys?

6. 6% of a number is what per cent. of 30% of the number?

7. A merchant sold tea at a loss of 16 cts. a pound, which was 25% of the cost. What was the cost?

8. Henry is 11 yr. old and John is 125% as old. How many years is the difference in their ages?

9. If $37\frac{1}{2}\%$ is lost by selling goods for $90, what was their cost?

10. 75 is 25% more than what number? 42 is 20% more than what number?

11. What amount of money must be forwarded to a commission merchant, to cover a purchase of $180 and his commission of 10%?

12. A horse was sold for $90, at which price $12\frac{1}{2}\%$ was gained. What per cent. would have been gained by selling him for $100?

13. I bought a wagon for $72, which was 20% more than its value. I sold it at 5% less than its value. How much did I lose?

14. Higgins sold a cow for $30, and by the transaction lost $16\frac{2}{3}\%$. He sold another cow at an advance of 16% for just enough to cover, by the profits, the loss upon the first cow. What did he get for the last cow?

WRITTEN EXERCISES.

457.—Ex. 1. If a cubic foot of pine timber weighs, when green, 44 lb. 12 oz., and when seasoned, 30 lb. 11 oz., what per cent. does it lose in seasoning? *Ans.* $31\frac{76}{179}$.

2. How large a sale must a merchant make, at a profit of 15%, to clear \$3750? *Ans.* \$25000.

3. What must be the selling price and profit of coal whose first cost is \$6, freight 10% and rate of gain 20%?

Ans. Selling price, \$7.92; profit, \$1.32.

4. If a merchant closed out his goods at a loss of 10%, how much did he lose on calico that cost $12\frac{1}{2}$ cts. per yard, and on sugar that cost 15 cts. per pound?

Ans. On calico, $1\frac{1}{4}$ cts. per yard.
On sugar, $1\frac{1}{2}$ cts. per pound.

5. My horse cost $\frac{3}{5}$ as much as my carriage; what per cent. of the cost of the one was the cost of the other?

Ans. The horse, 60% of the carriage.
The carriage, $166\frac{2}{3}$% of the horse.

6. I sold one half of a lot of goods which cost me \$456, at a loss of 25%, and the other half at a profit of \$69.54. What was the gain per cent. on the whole transaction? *Ans.* $2\frac{3}{4}$.

7. I bought a farm for a certain sum, and after expending $5\frac{3}{4}$% of the cost for repairs and improvements, and paying a tax of $1\frac{1}{4}$%, I sold it for \$6420, which just made up what I had paid out. What was the original cost? *Ans.* \$6000.

8. By selling cloth at \$6 per yard, I gain 25%. What per cent. shall I gain by selling it at \$5.28?

$$
\begin{array}{r}
1.25)\$6.0000(\$4.80 \\
\underline{5\,00} \\
\underline{1000} \\
\underline{1000}
\end{array}
$$

$$\$5.28-\$4.80=\$.48;\ \tfrac{48}{480}=.10=10\%$$

SOLUTION.—If the gain at \$6 per yard is 25%, \$6 must be 125%, or 1.25 times the cost, and the cost must be $\frac{1}{1.25}$ of \$6, or \$4.80.

By selling that which cost \$4.80 at \$5.28, the gain is \$.48, which is 10% of \$4.80.

9. A man drew from a bank \$264, which was 8¼% of his deposit. What was his deposit?

10. I sold a watch for \$69, and lost 20 per cent. What per cent. should I have gained if I had sold it for \$93.50?

11. Do I make or lose in selling an article marked 25% above cost, if I deduct 20%?

SOLUTION.—The marked price is 25% above cost, or 125% of cost. 20%, or ⅕ of 125% of cost, is 25% of cost. 125% — 25% of cost is 100% of cost, or the cost. Hence, I neither make nor lose.

12. I sold goods marked 40% above cost, at a deduction of 35%. What per cent. did I lose? *Ans.* 9.

13. A merchant sold two bills of goods of \$75 each; on the one he made 20%, and on the other he lost 20%. What was his gain or loss? *Ans.* Loss, \$6.25.

14. Find the net proceeds of the following account of sales, rendered by Barnard & Smith, commission merchants, to Henry Law, consignor:

Sales of Wheat for Acct. of HENRY LAW, *Elgin.*

1871.		TO WHOM SOLD.	DESCRIPTION.		PRICE.		
Jan.	*4*	*Albert Ward.*	*101 bu.*	*No. 1 Spring.*	*\$1.50*	*\$151*	*50*
"	*8*	*Snyder & Co.*	*552* "	*Mixed Spring.*	*1.40*	*772*	*80*
Feb.	*3*	*O. Smith & Co.*	*310* "	*Amber State.*	*1.45*	*449*	*50*
"	*10*	*H. A. Stein.*	*500* "	*White.*	*1.60*	*800*	*00*
"	*17*	*Thomas Prince.*	*75* "	*Mixed Spring.*	*1.40*	*105*	*00*
Mar.	*2*	*H. Dunster.*	*124* "	*Amber State.*	*1.45*	*179*	*80*
						\$2458	*60*
		CHARGES.					
		Freight and Drayage			*\$51.00*		
		Insurance on \$2000 @ 1½%			*30.00*		
		Storage			*11.00*		
		Commission, 5%, on \$2458.60			*122.93*	*214*	*93*
		Net proceeds					

BARNARD & SMITH.

CHICAGO, *March* 4, 1871.

SECTION XLVIII.

SIMPLE INTEREST.

458.—Ex. 1. When the allowance for the use of money is 6 per cent., how many hundredths of the money is the allowance?

2. When the allowance for the use of money is 7 per cent., how many hundredths of the money is the allowance?

3. When the allowance for the use of money is 5 per cent. per year, what is the allowance for the use of $1 for 1 year? For 2 years? For 3½ years?

4. When the allowance for the use of money is 6 per cent. per year, how many hundredths of the money is it for 12 months? For 2 months?

5. At the rate of 6 per cent. for the use of money per year, to how much will $300 amount in 2 years?

6. When the allowance for the use of $200 per year is $30, what is the yearly rate?

7. If I should lend $500 at a yearly rate of 4 per cent. for its use, to how much would it amount in 2½ years?

DEFINITIONS.

459. **Interest** is an allowance for the use of money.

460. The **Principal** is the sum for the use of which interest is paid.

461. The **Amount** is the sum of the principal and interest.

462. The **Rate of Interest** is the rate per cent. of the principal allowed for its use one year.

463. A **Legal Rate of Interest** is any rate allowed by law, and **Usury** is interest reckoned at a higher rate than the law allows.

When in a contract between parties, no rate of interest is named, in most of the States and on debts due the United States, the legal rate is 6%; in some of the States it is 7%, in several 8%, and in others 10%.

In this book, when no particular rate is named or implied, 6% is understood.

464. In the **Computation** of interest it is customary to reckon 30 days a month, and 12 months, or 360 days, a year; and in finding the difference between dates, to

Take the number of entire calendar months, and the actual number of days left.

Thus, from January 20 to August 4 is 6 mo. 15 da., or $6\frac{1}{2}$ mo.

Months are often conveniently expressed as twelfths of a year, and 3 days as a tenth of a month.

Thus, 6 mo. 15 da. may be expressed as 6.5 mo., or as $\frac{6.5}{12}$ of a year.

465. In the process of reckoning interest, partial results, if necessary, may be carried to four orders of decimals. But in answers it is sufficiently exact to reject mills if less than 5, and if 5 or more than 5, to call them 1 cent.

466. Simple Interest is interest on the principal alone.

CASE I.

Principal, Rate and Time Given, to Find the Interest or Amount.

GENERAL METHOD.

467.—Ex. 1. What is the interest and what is the amount of \$576 for 2 y. 6 mo., at 6%?

$$\begin{array}{r} \$576 \\ .06 \\ \hline \$34.56 \\ 2\frac{1}{2} \\ \hline \$69.12 \\ 17.28 \\ \hline \$86.40 \\ 576.00 \\ \hline \$662.40 \end{array}$$

SOLUTION.—2 y. 6 mo. are $2\frac{1}{2}$ years.

The interest of \$576 for 1 year at 6% is .06 of \$576, or \$34.56.

Since the interest for 1 year is \$34.56, for $2\frac{1}{2}$ years it must be $2\frac{1}{2}$ times \$34.56, or \$86.40.

The principal added to the interest, or the amount, is \$662.40.

2. What is the interest of \$760 for 4 y. 8 mo.?

3. What is the interest of \$662.50 for 3 y. 4 mo.?

4. What is the interest of $480.50 for 2 y. 7 mo. 12 da., at 8%?

$$
\begin{array}{r}
\$480.50 \\
.08 \\
\hline
\$38.4400 \\
31.4 \\
\hline
15376 \\
3844 \\
11532 \\
\hline
12)\$1207.016 \\
\hline
\$100.58
\end{array}
$$

SOLUTION.—2 y. 7 mo. 12 da. are 31.4 mo., or $\frac{31.4}{12}$ y.

The interest of $480.50 for 1 year, at 8%, is .08 of $480.50, or $38.44.

Since the interest for 1 year is $38.44, for 31.4 mo., or $\frac{31.4}{12}$ of a year, it must be $\frac{31.4}{12}$ of $38.44, or $100.58.

5. What is the interest of $78.50 for 3 y. 3 mo., at 7%?

6. What is the amount of $110.25 for 1 y. 8 mo., at 6%? *Ans.* $121.28.

468. Principle.—*The interest is equal to the product of the principal, rate and time.*

469. Rules for Interest by the General Method.—*1. Multiply the principal by the rate, and that product by the time expressed as years. Or,*

2. Multiply the principal by the rate, and that product by the time expressed as months, and divide the result by 12.

3. Add the interest and principal, and the result will be the amount.

PROBLEMS.

What is the interest of—

1. $5631 for 1 year, at 6%? *Ans.* $337.86.
2. $860 for 2 years, at 7%? *Ans.* $120.40.
3. $325 for 5 years, at 8%?
4. $1450 for 2 years 6 months, at 6%? *Ans.* $217.50.
5. $111.42 for 4 years 2 months, at 5%?
6. $19000 for 2 y. 4 mo., at $7\frac{3}{10}$%? *Ans.* 3236.33\frac{1}{3}$.
7. $6600 for 3 y. 6 mo. 20 da., at 6%? *Ans.* $1408.

8. What is the interest of \$750 from Jan. 9, 1869, to Nov. 9, 1870? *Ans.* \$82.50.

9. What is the amount of \$3350 for 5 y. 9 mo., at 8%?

10. What is the amount of \$1242 from July 3, 1868, to Jan. 18, 1870? *Ans.* \$1356.89.

SIX PER CENT. METHOD.

470. The **Interest** on any principal, at 6%,

For	*12 months,*	or	*1 year,*	is	*.06*	of the principal.
"	*2 months,*	"	$\frac{1}{6}$ *year,*	"	*.01*	"
"	*1 month,*	"	$\frac{1}{12}$ *year,*	"	*.005*	"
"	*6 days,*	"	$\frac{1}{5}$ *month,*	"	*.001*	"
"	*3 days,*	"	$\frac{1}{10}$ *month,*	"	$.000\frac{1}{2}$	"
"	*1 day,*	"	$\frac{1}{30}$ *month,*	"	$.000\frac{1}{6}$	"

Hence, the following

471. Principles.—1. *The interest at* 6 *per cent. for any number of months is equal to one half as many hundredths of the principal as there are months; and*

2. *The interest at* 6 *per cent. for any number of days is equal to one sixth as many thousandths of the principal as there are days.*

WRITTEN EXERCISES.

472.—**Ex.** 1. What is the interest of \$576 for 1 y. 7 mo., at 6%?

$$\begin{array}{r} \$576 \\ .09\frac{1}{2} \\ \hline 5184 \\ 288 \\ \hline \$54.72 \end{array}$$

SOLUTION.—1 y. 7 mo. are 19 months.

Since the interest at 6% is one half as many hundredths of the principal as there are months in the time, it must be one half of 19 hundredths, or .09½, of \$576, which is \$54.72.

2. What is the interest of \$950 for 2 y. 8 mo., at 6%?

3. What is the interest of \$420 for 3 y. 4 mo., at 7%?

4. What is the interest of $455 for 2 y. 6 mo. 12 da., at 7%?

$455
.152
910
2275
455
6)$69.160
11.5266 +
$80.687

SOLUTION.—2 y. 6 mo. 12 da. are 30.4 mo. Since the interest at 6% is one half as many hundredths of the principal as there are months in the time, it must be one half of 30.4 hundredths, or .152, of $455, which is $69.160.

7% interest is $\frac{1}{6}$ more than 6% interest; hence, $69.160 plus $\frac{1}{6}$ of $69.160, or $80.687, is the interest required.

5. What is the interest of $940 for 33 da. at 6%?

$940
$.005\frac{1}{2}$
4700
470
$5.17

SOLUTION.—Since the interest at 6% is one sixth as many thousandths of the principal as there are days in the time, it must be one sixth of 33 thousandths, or $.005\frac{1}{2}$, of $940, which is $5.17.

6. What is the interest of $756 for 8 mo., at 6%?

Ans. $30.24.

7. What is the interest of $631.20 for 11 mo., at 8%?

473. Rules for Interest by the Six Per Cent. Method.—*1. Multiply the principal by one half as many hundredths as there are months, or by one sixth as many thousandths as there are days in the time, and the result will be the interest at 6 per cent.*

2. For interest at any other rate than 6 per cent., increase or diminish the interest at 6 per cent. by such a part of itself as will make the required interest.

PROBLEMS.

What is the interest of—

1. $38.60 for 6 mo. 24 da., at 5%?
2. $1090 for 14 da., at 6%? *Ans.* $2.54.
3. $400.50 for 7 mo. 6 da., at 7%?
4. $5000 for 63 da., at 9%? *Ans.* $78.75.

5. \$342 for 93 da., at 7% ?
6. \$1200 for 1 mo. 21 da., at 6% ? *Ans.* \$10.20.
7. \$1560 for 1 y. 8 mo., at 7% ?
8. \$1920 for 2 y. 3 mo., at 5% ? *Ans.* \$216.
9. \$500 from January 15 to December 2, at 7½% ?
10. What is the amount of \$1345 from April 9, 1870, to September 5, 1871, at 7% ? \$1477.59.
11. What is the amount of \$3000 from June 11, 1870, to August 17, 1871, at 8% ? *Ans.* \$3284.

SPECIAL METHODS FOR DAYS.

474. The **Interest** of any principal at 6% for 2 months, or 60 days, is one hundredth of the principal. Hence the following—

475. Principle.—*The interest of any principal at 6% for any number of days is as many hundredths of the principal as 60 is contained times in the number of days.*

WRITTEN EXERCISES.

476.—Ex. 1. What is the interest of \$240 for 93 da., at 6% ?

Int. of \$240 for 60 da. = \$2.40

$$\begin{array}{r} \$2.40 \\ 93 \\ \hline 720 \\ 2160 \\ \hline 60)\$223.20 \\ \hline \$3.72 \end{array}$$

SOLUTION.—The interest at 6% for 60 days is one hundredth of the principal, or \$2.40.

Since for any number of days it is as many hundredths of the principal as 60 is contained times in the number of days, for 93 da. it must be $\frac{1}{60}$ of 93 times \$2.40, or \$3.72. Or,

\$240	*= Principal.*
\$2.40	*= Int. for 60 da.*
1.20	*= " 30 da.*
.12	*= " 3 da.*
\$3.72	*= " 93 da.*

SOLUTION.—Since the interest for 60 days is \$2.40, for 30 days, or ½ of 60 days, it must be ½ of \$2.40, or \$1.20, and for 3 days, or $\frac{1}{10}$ of 30 days, it must be $\frac{1}{10}$ of \$1.20, or \$.12. The sum of these results is \$3.72, which is the interest required.

2. What is the interest of $120.60 for 11 da., at 6% ?
3. What is the interest of $500 for 123 da., at 5% ?

Ans. $8.54.

477. Rule for Interest by Special Method for Days.—*1. Remove the decimal point in the principal two orders to the left, for the interest at 6 per cent. for 60 days.*

For the interest for any other number of days, multiply the interest for 60 days by the number of days, and divide by 60; or, take any convenient multiples or aliquot parts of the interest for 60 days.

PROBLEMS.

What is the interest of—

1. $318.20 for 36 da., at 6% ? *Ans.* $1.91.
2. $415 for 19 da., at 6% ?
3. $31.25 for 16 days, at 6% ? *Ans.* $.08.
4. $1120 for 153 days, at 7% ?
5. $6000 for 8 days, at 12% ? *Ans.* $16.00.
6. $311.50 for 35 days, at 6% ? *Ans.* $1.82.
7. $65.20 for 130 days, at 7% ?
8. Find the amount of $17000 for 75 days, at 8%.

CASE II.

Principal, Interest and Time Given, to Find the Rate.

478.—Ex. 1. At what rate will $576 earn $86.40 in 2 years 6 months?

$$\$576 \times .01 \times 2\tfrac{1}{2} = \$14.40$$

$$\$86.40 \div \$14.40 = 6$$

SOLUTION.—The interest of $576 at 1% for $2\frac{1}{2}$ years is $14.40.

Since the principal at 1% in $2\frac{1}{2}$ years earns $14.40, to earn $86.40 it must be at as many per cent. as $86.40 is times $14.40, or at 6%.

2. At what rate will $1450 earn $217.50 in 2 years 6 months?

479. Rule for Finding the Rate of Interest.—*Divide the given interest by the interest of the principal for the given time at 1 per cent.*

PROBLEMS.

At what rate will—

1. \$1760 earn \$246.40 in 2 years? *Ans.* 7%.
2. \$110.25 gain \$11.02½ in 1 year 8 months? *Ans.* 6%.
3. \$6600 gain \$1412.40 in 3 years 6 months 20 days?
4. \$19000 gain \$3236.33⅓ in 2 years 4 months? *Ans.* $7\frac{3}{10}$%.

5. At what rate will \$100 double itself in 16 years 8 months? *Ans.* 6%.

6. At what rate will any principal double itself in 12½ years?

7. If you should pay, January 22, 1871, \$735, as principal and interest for \$700 borrowed July 22, 1870, what would be the rate? *Ans.* 10%.

CASE III.

Principal, Rate, and Interest, or Amount Given, to Find the Time.

480.—Ex. 1. In what time will \$576 gain \$86.40, interest being at 6%?

$$\$576 \times .06 = \$34.56$$

$$\$86.40 \div \$34.56 = 2\tfrac{1}{2}$$

SOLUTION.—The interest of \$576 for 1 year, at 6%, is \$34.56.

Hence, it must require as many years to gain \$86.40 interest as \$86.40 is times \$34.56, or 2½ years, which are 2 years 6 months.

2. In what time will \$400 amount to \$435, at 7% interest?

$$\$435 - \$400 = \$35$$

$$\$400 \times .07 = \$28$$

$$\$35 \div \$28 = 1\tfrac{1}{4}$$

SOLUTION.—The amount \$435 less the principal \$400 is \$35, which is the interest.

The interest of \$400 for 1 year, at 7%, is \$28.

Hence, it must require as many years to gain \$35 interest, or for \$400 to amount to \$435, as \$35 is times \$28, or 1¼ years, which are 1 year 3 months.

3. In what time will \$650 gain \$55.25, interest being at 6%?

481. Rule for Finding the Time that a Sum has been at Interest.—*Divide the given interest by the interest of the principal for one year at the given rate.*

PROBLEMS.

In what time will—

1. \$7000 gain \$588, at 7% interest? *Ans.* 1 y. 2 mo. 12 d.
2. \$6000 gain \$355, at 5% interest?
3. \$100 double itself, at 7% interest? *Ans.* 14 y. $3\frac{3}{7}$ mo.
4. \$1250 amount to \$1253.12$\frac{1}{2}$, at 6% interest? *Ans.* 15 days.
5. Any principal double itself, at 8% interest?
6. \$1828.50 amount to \$2331.33$\frac{3}{4}$, at 10% interest? *Ans.* 2 y. 9 mo.

CASE IV.

Time, Rate, and Interest, or Amount Given, to Find the Principal.

482.—Ex. 1. What principal will earn \$86.40 in 2 y. 6 mo., at 6%?

$$\$1 \times .06 \times 2\tfrac{1}{2} = \$.15$$
$$\$86.40 \div \$.15 = 576$$

SOLUTION.—The interest of \$1 for 2 years 6 months, at 6%, is \$.15.

Hence, it must require as many dollars to earn \$86.40 as \$86.40 is times \$.15, or \$576.

2. What principal will amount to \$435 in 1 year 3 months, at 7%?

$$\$1 + \$.08\tfrac{3}{4} = \$1.08\tfrac{3}{4}$$
$$\$435 \div \$1.08\tfrac{3}{4} = 400$$

SOLUTION.—The amount of \$1 for 1 year 3 months, at 7%, is \$1.08$\frac{3}{4}$.

Hence, it must require as many dollars to amount to \$435 as \$435 is times \$1.08$\frac{3}{4}$, or \$400.

3. What principal will gain \$45.24$\frac{1}{2}$ in 3 years, at 6%? *Ans.* \$251.36.

483. Rule for Finding the Principal that has been at Interest.—*Divide the given interest or amount by the interest or amount of \$1 for the given time and rate.*

PROBLEMS.

What principal will—

1. Gain \$18.75$\frac{3}{4}$ in 90 days, at 6%? *Ans.* \$1250.50.
2. Gain \$246.40 in 2 years, at 7%?

3. Gain \$352.50 in 1 year 2 months 3 days, at 5%?

Ans. \$6000.

4. Gain \$3236.33$\frac{1}{3}$ in 2 years 4 months, at 7$\frac{3}{10}$%?

5. Amount to \$355.60 in 1 year 7 months 10 days, at 8%?

Ans. \$315.

6. Amount to \$200 in 14 years 3$\frac{3}{7}$ months, at 7%?

Ans. \$100.

TEST QUESTIONS.

484.—1. What is INTEREST? What is the principal? The amount? The rate?

2. What is a LEGAL RATE of interest? What is usury? What is the rate in most of the States, when no rate is named? What are the rates in other States? On debts due the United States?

3. In the COMPUTATION of interest, how is it customary to reckon time? In finding the difference between dates? How far in the processes of reckoning interest may partial results be carried? What is sufficient when there are mills in the answer?

4. What is SIMPLE INTEREST? To what is simple interest equal? What is the rule for the general method of computing interest?

5. To what is interest at 6% for any number of months equal? For any number of days? What is the rule for the 6% method of computing interest?

6. What part of the principal is the interest at 6% for 2 months? How do you most readily find the interest at 6% for 60 days? For any other number of days?

7. When the principal, interest and time are given, how do you find the rate? When the principal, rate and interest, or amount, are given, how do you find the time? When the time rate and interest are given, how do you find the principal?

8. What is PERCENTAGE? To what is the percentage of a number equal? To what is the rate equal? To what is the base equal?

9. To what is PROFIT and LOSS equal? What are the rules for profit or loss?

10. What is the base of COMMISSION? The amount? The net proceeds? What are the rules for commission?

11. How do property and life insurance differ? What are the rules for insurance?

SECTION XLIX.

PARTIAL PAYMENTS.

485. A **Note,** or a **Promissory Note,** is a written promise to pay a certain sum of money for value received.

486. The **Maker** of a note is the party who signs it.

487. The **Payee** of a note is the party to whom, or to whose order, it is to be paid.

488. An **Indorser** of a note is a party who writes his name upon the back of a note or other obligation, to transfer it or to guarantee its payment.

489. The **Face** of a note is the sum named in it.

The number of dollars should be written in words.

490. A **Time Note** is one made payable at a specified time.

When no time for its payment is specified, the note is due on demand.

491. Three days, called **Days of Grace,** are usually allowed for the payment of such a note.

Thus, a note payable 30 days after date is really due on the last day of grace, or 33 days after date.

492. A **Negotiable Note** is one so made that it can be sold or transferred.

FORM OF A NEGOTIABLE NOTE.

860\frac{50}{100}$. ST. LOUIS, *January* 4, 1871.

For value received, I promise to pay to the order of Andrew Hale Eight Hundred Sixty $\frac{50}{100}$ Dollars, on demand, with interest.

Daniel Wright.

Andrew Hale can transfer this note by simply writing his name on its back, or he may transfer it to John Jones by writing upon its back,

Pay to the order of John Jones.

Andrew Hale.

493. Partial Payments are part payments of notes or other obligations bearing interest.

494. Indorsements of partial payments are statements of the payments on the back of the instrument.

495. The **Supreme Court** of the United States, and most of the States, adopt for partial payments a rule based upon the following:

496. Principles.—1. *Payments must be applied first to the discharge of interest due, and the balance, if any, toward the discharge of the principal.*

2. *Interest must not be added to the principal to draw interest.*

3. *Interest must accrue only on unpaid principal.*

WRITTEN EXERCISES.

497.—Ex. 1. A note for $2000 was given July 1, 1870. Upon it was paid, as by indorsement, January 1, 1871, $260; July 1, 1871, $50; and May 1, 1872, $96. Required the balance due November 25, 1872.

SOLUTION.

Principal,		*$2000.00*
Int. to Jan. 1, 1871, 6 mo.,		*60.00*
Amount,		*$2060.00*
First payment, Jan. 1, 1871,		*260.00*
New principal,		*$1800.00*
Int. to May 1, 1872, 16 mo.,		*144.00*
Amount,		*$1944.00*
Second payment, July 1, 1871,	*$50*	
Third payment, May 1, 1872,	*96*	*146.00*
New principal,		*$1798.00*
Int. to Nov. 25, 1872, 6.8 mo.		*61.13*
Amount or balance due,		*$1859.13*

498. Rule of the Supreme Court of the United States for Partial Payments.—*Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due, and subtract the payment or the sum of the payments.*

Regard the remainder as a new principal, and proceed as before.

PROBLEMS.

1. Find the amount due December 29, 1871, at 7%, upon a note for $960, dated Albany, N. Y., March 11, 1869; and on which there has been paid, November 1, 1870, $63.52; and April 17, 1871, $70.60.

2. Find the balance due April 1, 1872, on the following note:

$500. HARRISBURG, *May* 16, 1869.

For value received, on demand, I promise to pay Charles Berger, or bearer, Five Hundred Dollars, with interest, without defalcation.

John Hofland

INDORSEMENTS: *Nov. 22, 1869, received Forty-five Dollars; May 28, 1870, received Seventy $\frac{50}{100}$ Dollars.*

Ans. $460.47.

3. Find the balance due July 25, 1872, at 6%, on the following note:

$6000. PROVIDENCE, *January* 1, 1870.

On demand, for value received, I promise to pay the bearer Six Thousand Dollars, with interest.

James D. Mowry.

INDORSEMENTS: *July 1, 1870, received One Thousand Dollars; May 1, 1872, received Three Thousand Dollars.*

4. Find the balance due August 1, 1870, at 8%, on the following note:

$4000. RICHMOND, *June* 5, 1869.

I promise to pay, to the order of Andrew L. Brown, Four Thousand Dollars, with interest. Value received.

H. M. Reeves.

INDORSEMENTS: *August 17, 1869, received One Thousand Dollars; January 29, 1870, received One Hundred Dollars.* *Ans.* $3198.91.

499. Business men often settle notes and interest accounts, running not more than a year, by the following, called—

500. The Merchants' Rule for Partial Payments.—*Find the amount of the principal at the time of settlement.*

Find the amount of each payment from the time it was made until settlement; and from the amount of the principal subtract the amounts of the payments.

In mercantile accounts, settlements are made, according as the custom may be, either at the end of the civil year or at the end of some understood number of months.

PROBLEMS.

1. A note for $1500, on demand with interest, dated January 1, 1870, had paid on it, May 1, 1870, $800, and May 16, 1870, $300. How much was due at the end of the first six months, interest being at 7%? *Ans.* $440.54.

2. Find the balance due at the end of the year on a note for $650, given April 9, 1870, on which has been paid, June 9, $150, and November 30, $200. *Ans.* $322.33.

3. Find the balance due August 5, 1871, on a note for $1275, given September 30, 1870, on which has been paid, December 5, 1870, $55; January 9, 1871, $760; June 3, 1871, $400.

SECTION L.

PRESENT WORTH AND DISCOUNT.

501. Discount is a sum deducted from a price or debt. Its computation may have reference to time, or not, according to the kind of deduction understood.

COMMERCIAL DISCOUNT.

502. Commercial Discount is a per cent. deducted from a price or from the face of a bill, without reference to time.

503. The **Net Price** of an article is the selling price less the discount.

504. The **Cash Value,** or Net Proceeds, of a bill is its face less the discount.

505. The **Base** of commercial discount is the selling price, or the face of a bill.

506. The **Rate** is the rate per cent. of deduction.

507. The **Discount** is the percentage of the deduction.

508. Rules for Commercial Discount.—*Multiply the selling price, or the face of the bill, by the rate per cent. of deduction, and the result will be the commercial discount.*

Subtract from the selling price, or from the face of the bill, the commercial discount, and the result will be the net price, cash value, or net proceeds.

PROBLEMS.

1. What is the net cash price of flour invoiced at $12.40 per barrel, on 30 days' time, or 5% off for cash?

Ans. $11.78 per barrel.

2. What is the cash value of goods amounting as by bill to $1565, discount off being 20%, and 10% off for cash?

Ans. $1126.80.

3. I sold a bill of books amounting to \$980, taking 10% off for 60 days, and 5% off for cash. What was the cash value of the bill? *Ans.* \$837.90.

TRUE DISCOUNT.

509. The **Present Worth** of a debt payable at a future time, without interest, is such a sum as, being placed at interest, will amount to the given debt when it becomes due.

Thus, \$100 is the present worth of \$106 due 1 year hence, at 6%.

510. The **True Discount** is the difference between the face of the debt and the present worth; or,

It is the interest on the present worth for the time intervening between the payment of the debt and the time of its becoming due.

511. The *present worth* corresponds to the *principal*, the *debt* to the *amount*, and the *discount* to the *interest*. (Art. 483.) Hence, the following—

512. Rules for True Discount.—*1. Divide the face of the debt, or the given sum, by the amount of \$1 for the given time and rate, and the result will be the present worth.*

2. Subtract the present worth from the given sum, and the result will be the true discount.

PROBLEMS.

1. What is the present worth and the true discount of \$408, payable in 4 months, at 6%?

Ans. Present worth, \$400; true discount, \$8.

2. What sum of ready money is equivalent to \$3350, due 1 year 8 months hence, when money is worth 7%?

3. What is the true discount on \$625, due 3 months hence, at 8%? *Ans.* \$12.25½.

4. If I pay a debt of \$1200, 2 years 6 months before it is due, what discount should be made, money being worth 6%?

5. I am offered goods for \$4050 cash, or for \$4253.75 on 3 months. Which is the better offer, and how much better is it, if money is worth 10%? *Ans.* Cash, by \$100.

SECTION LI.

BANKING.

513. A **Bank** is an institution established by law for receiving deposits, loaning money, or issuing notes or bills to circulate as money.

514. A **Deposit** is money, or its equivalent, entrusted to the care of a bank.

515. A **Check** is a written order, or request, for money, addressed to a bank by a person having a deposit.

FORM OF A BANK CHECK.

No. 13. HARTFORD, *January* 3, 1872.

Traders' National Bank.

*Pay to*__________*Henry Brown,*__________*or bearer,*

__________*Six Thousand*__________ $\frac{75}{100}$ *Dollars.*

\$6000$\frac{75}{100}$. *J. F. Pratt.*

516. Bank discount is the interest deducted from the face of a note, for the payment of its proceeds before the note becomes due.

517. The **Proceeds** or **Avails** of a note discounted, are the face, or amount, of the note, less the discount.

FORM OF A NOTE PAYABLE AT A BANK.

$755\frac{50}{100}$. NEW ORLEANS, *May* 16, 1871.

Ninety days after date, I promise to pay to the order of ——— Jean Remond ——— at the First National Bank ——— Seven Hundred Fifty-five ——— $\frac{50}{100}$ Dollars. Value received.

Paul Ganot.

518. The **Maturity** of a note is the time on which it is legally due, and that is the last day of grace. When the last day of grace happens on Sunday or a legal holiday, the note is payable a day earlier.

A note is *payable* or *nominally due* at the time specified in the note. The times of being payable and of maturity are usually indicated by writing the numbers of the days of the two dates with a line between them. Thus, July $^{13}/_{16}$, 1871.

A bank, on discounting a note, retains the discount, holds the note and pays the proceeds, or avails, to the former holder.

519. The **Term of Discount** is the number of days from the time of discounting to the maturity of the note.

The term of discount is called *the time to run.*

When a note is not paid at maturity, a written notice, called a *protest,* should be made out and sent to the indorser or indorsers by an officer known as a Notary Public.

In order to hold the indorsers for the payment of the note, this should be done on the last day of grace.

CASE I.

The Face of a Note, Time and Rate given, to Find the Bank Discount, or Proceeds.

520.—Ex. 1. Required the bank discount and proceeds of a note for $800, for 3 months, dated Wilmington, Del., August 5, and discounted September 6, at 6%.

Int. of \$800 for 60 da. $= \$8.00$

$$\begin{array}{r} \$8.00 \\ 63 \\ \hline 60)\$504.00 \\ \hline \$8.40, \textit{Discount.} \end{array}$$

$\$800 - \$8.40 = \$791.60$, *Proceeds.*

SOLUTION.—Maturity, or 3 months plus 3 days of grace from August 5, is November 8. The term of discount, or the number of days from September 6 to November 8, is 63 days.

Interest of \$800 for 60 days at 6% is .01 of \$800, or \$8.00, and for 63 days at 6% is $\frac{63}{60}$ of \$8, or \$8.40, which is the required discount.

The face of the note less the discount is \$800 — \$8.40, or \$791.60; which is the proceeds.

2. What is the bank discount of a note for \$500, at 30 days, at 6%? *Ans.* \$2.75.

521. Rules for Bank Discount.—*1. Find the interest on the face of the note, at the given rate, for the term of bank discount, and the result will be the bank discount.*

2. Subtract the bank discount from the face of the note, and the difference will be the proceeds.

PROBLEMS.

1. I bought a horse for \$250, and gave my note at 90 days. How much ready money did he cost me, discount being at 8%? *Ans.* \$244.83.

2. What are the proceeds of a note for \$1220, due in 4 months, discount at 7%?

3. What is the bank discount of a note for \$193, dated July 1, payable in 60 days, with interest at 5%, discounted 15 days after date, at 6%? *Ans.* \$1.56.

4. Find the time of becoming due, term of discount, bank discount and proceeds of a note, dated May 5, 1871, for \$1450.25, payable in 6 months, and discounted, July 11, 1871, at 7%.

Ans. Due, Nov. $^5/_8$; term of discount, 120 days; discount, \$33.84; proceeds, \$1416.41.

5. Find the time of becoming due, term of discount, bank discount and proceeds of the following note, which was discounted August 8.

$5670. READING, *July* 5, 1871.

Ninety days after date, I promise to pay to the order of Luther Getz & Co., Five Thousand Six Hundred Seventy Dollars, at the First National Bank, without defalcation. Value received.

George Hartman.

Ans. Due, Oct. $^3/_6$; term of discount, 59 days; discount, $55.76; proceeds, $5614.24.

CASE II.

Proceeds or Bank Discount, Time and Rate given, to find the Face of the Note.

522.—Ex. 1. The proceeds of a note discounted for 90 days, at 6%, are $787.60. Required the face of the note.

Bank discount of $1 for 93 days = $.0155

Proceeds of $1 for 93 days = $.9845

$787.60 ÷ .9845 = $800

SOLUTION. — The bank discount of $1 for 93 days is $.0155. The proceeds of $1 for 93 days are $1 less $.0155, or $.9845.

Since $.9845 is the proceeds of $1 for the given time and rate, $787.60 must be the proceeds of as many dollars as $787.60 is times $.9845, or $800.

2. For what sum must a 6 months' promissory note be written, that the proceeds, at 7% discount, may be $484.75?

Ans. $502.64.

523. Rules for finding the Face of a Note.—*1. Divide the given proceeds by the proceeds of $1 for the given time and rate. Or,*

2. Divide the given discount by the discount of $1 for the given time and rate.

PROBLEMS.

1. The bank discount on a certain note discounted for 3 months, at 6%, was \$12.40. What was the face of the note?
Ans. \$800.

2. What must be the face of a note in order that when discounted for 60 days, at 6%, its proceeds shall be \$394?

3. The proceeds of a 30 days' note, discounted at 8%, were \$1335.63. What sum will pay the note at maturity?
Ans. \$1345.50.

4. For what sum must a note be made to obtain \$650 from a bank, for 57 days, at 6% discount?

5. For what sum must a note, dated March 9, on 4 months, be drawn, so that if discounted at 5%, May 9, it shall yield \$1090.90? *Ans.* \$1100.68.

TEST QUESTIONS.

524.—1. What is a PROMISSORY NOTE? Who is the maker? The payee? An indorser? What is the face of a note?

2. What is a TIME NOTE? What are days of grace? When is a time note really due?

3. What is a NEGOTIABLE NOTE? How may a negotiable note be transferred?

4. What are PARTIAL PAYMENTS? Indorsements? What is the United States rule? The merchants' rule?

5. What is DISCOUNT? Commercial discount? The net price of an article? The cash value of a bill?

6. What is the BASE of commercial discount? Of what is the discount the percentage? What is the rule for computing commercial discount?

7. What is the PRESENT WORTH of a debt? What is true discount? What is the rule for the computation of true discount and present worth?

8. What is a BANK? A deposit? A check? Bank discount? What are the proceeds or avails of a note discounted?

9. What is the MATURITY of a note? When is a note payable? What is done in discounting a note? What is the term of discount?

10. What is the RULE for finding the bank discount or proceeds of a note? When the proceeds or bank discount, time and rate are given, how is the face of the note found?

SECTION LII.

ANNUAL INTEREST.

525. Annual Interest is simple interest on the principal and on each year's interest from the time of its accruing, till settlement.

526.—Ex. 1. How much interest is due on a debt of \$500, at 6% annual interest, at the end of 3 years 6 months?

SOLUTIONS.

\$500 × .06 = \$30 = each year's int. on the principal.

\$500 × .21 =	*\$105.00 =*	*simple int. " "*	*for 3 y. 6 mo.*
\$30 × .15 =	*4.50 =*	*" on 1st year's int.*	*for 2 y. 6 mo.*
\$30 × .09 =	*2.70 =*	*" on 2d " "*	*for 1 y. 6 mo.*
\$30 × .03 =	*.90 =*	*" on 3d " "*	*for 6 mo.*
	\$113.10 =	*the annual interest.*	

Or,

\$500 × .21 =	*\$105.00 =*	*simple int. on the prin. for 3 y. 6 mo.*
\$30 × .27 =	*8.10 =*	*" on \$30 for 4 y. 6 mo.*
	\$113.10 =	*the annual interest.*

In the second solution the computation is abridged by finding the interest on one year's interest for 4 y. 6 mo., which is the sum of the periods for which the yearly interests remained unpaid.

2. What is the amount of \$720 for 2 years 8 months, at 7% annual interest? *Ans.* \$862.63.

527. Rule for Annual Interest.—*Compute interest on the principal for the entire time, and on one year's interest for the sum of the periods during which the yearly interests remain unpaid.*

The sum of these interests will be the interest required.

PROBLEMS.

1. What is the annual interest for 3 years on a debt of $1000, at 6%, annual interest?

2. How much is due May 1, 1870, on a note for $350, dated January 1, 1868, with interest payable annually at 8%?

Ans. $419.06.

3. How much is due May 16, 1871, on a note for $600, dated March 3, 1868, with interest payable annually at 6%, provided the yearly interest has been promptly paid?

Ans. $607.30.

528. When **Partial Payments** have been made on notes or other obligations, with "interest annually," by the law of VERMONT,

Find the interest on each payment to the time the interest on the principal next becomes due, and finding the amount, apply it—first, to liquidate any interest that may have accrued upon unpaid yearly interests; secondly, to liquidate yearly interests that may have become due; and thirdly, to the extinguishment of the principal.

By the NEW HAMPSHIRE RULE, payments only that exceed the interest due at the time made are allowed interest.

PROBLEMS.

1. Find the balance due May 1, 1871, on the following note, by the Vermont and by the New Hampshire Rules:

$600. CONCORD, N. H., *May* 1, 1868.

On demand, I promise to pay to the order of James Upham, for value received, Six Hundred Dollars, with interest annually.

Benjamin Hill.

INDORSEMENTS: *July 1, 1868, received $100; August 9, 1869, received $25; Sept. 1, 1870, received $300; Jan. 1, 1871, $110.*

Ans. By Vt. Rule, $144.78; by N. H. Rule, $147.43.

2. Required the balance due December 3, 1871, on a note for $500, dated September 3, 1867, bearing interest annually, by the New Hampshire Rule; and indorsed, April 19, 1869, received $63; November 9, 1870, received $321.50.

Ans. $224.86.

SECTION LIII.

COMPOUND INTEREST.

529. Compound Interest is interest on interest and principal combined, at specified intervals.

Interest may be compounded, or made part of the principal, annually, semi-annually, quarterly, etc.

530.—Ex. 1. What is the compound interest of $744 for 2 years 4 months, at 5% ?

$744	
.05	
$37.20	*Int. for 1st y.*
744	
$781.20	*Amt. for 1st y.*
.05	
$39.0600	*Int. for 2d y.*
781.20	
$820.26	*Amt. for 2d y.*
$.01\frac{2}{3}$	
$13.6710	*Int. for 4 mo.*
820.26	
$833.9310	*Amt. for 2 y. 4 mo.*
744	
$89.93	*Comp. Int. for 2 y. 4 mo.*

SOLUTION. — The amount of $744 for the first year is $781.20, which we make a principal for the second year.

The amount at compound interest of $744 for 2 years is $820.26, which we make a principal for the remaining 4 months of the time.

The amount at compound interest of $744 for 2 years 4 months is $833.93 +.

The last amount, minus the given principal, is $89.93, which is the compound interest required.

2. What is the amount at compound interest of $520 for 4 years, at 6% ?

Ans. $656.49.

531. Rule for Compound Interest.—*Find the amount of the given principal for the first interval, at the given rate.*

Make that amount a principal for the second interval, and so on.

From the last amount subtract the given principal, and the remainder will be the compound interest.

PROBLEMS.

1. What is the compound interest of $3010 for 3 years 3 months, at 7% ? *Ans.* $741.91.

2. What is the interest of $9000 for 5 years 6 months, interest compounding at 3%, semi-annually? *Ans.* $3458.09.

3. What is the amount of $506.50 for 2 years 3 months, interest compounding at $1\frac{1}{2}$%, quarterly?

4. What sum will pay, January 1, 1872, $5000, borrowed July 1, 1869, at 7%, compound interest? *Ans.* $5924.86.

5. Find the amount of $150.25, at compound interest, from May 16, 1870, to July 1, 1873, at 6%.

6. Find the compound interest of $3000 from October 3, 1869, to April 3, 1872, at 8%. *Ans.* $639.17.

TEST QUESTIONS.

532.—1. What is SIMPLE INTEREST? The general method of computing simple interest?

2. What is DISCOUNT? Commercial discount? How do true discount and bank discount differ? What are the rules for bank discount?

3. What are PARTIAL PAYMENTS? What is the United States Rule for computing partial payments?

4. What is ANNUAL INTEREST? How does it differ from simple interest? What is the method of computing annual interest?

5. What is the RULE for computing partial payments by the Vermont Annual Interest Method? How does the New Hampshire Method differ from that of Vermont?

6. What is COMPOUND INTEREST? How does it differ from annual interest? What is the rule for computing compound interest?

SECTION LIV.

REVIEW PROBLEMS.

MENTAL EXERCISES.

533.—Ex. 1. What is the interest of \$15 for 4 years, at 6%? For 2 years, at 5%?

2. What is the interest of \$50 for 8 months, at 8%? For 18 months, at 10%?

3. What is the interest of \$300 for 1 month 20 days, at 6%?

SOLUTION.—The interest of \$300 for 1 year is \$18; hence, for 1 month, or $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of \$18, or \$1½, and for 20 days, or ⅔ of a month, it is ⅔ of \$1½, or \$1; \$1½ and \$1 is \$2.50.

4. What is the interest of \$600 for 3 months 15 days, at 8%?

5. What is the interest of \$30 for 3 months 3 days, at 7%?

6. In what time will \$100 gain \$18 interest, at 6%? \$20 interest, at 5%?

7. In what time will \$200 gain \$21 interest, at 7%? \$26, at 8%?

8. In what time will \$60 gain \$.30 interest, at 6%? \$2.10, at 7%?

9. What is the interest of \$60 for 60 days, at 6%? For 93 days, at 6%?

10. In what time will a given principal double itself at 1%? At 5%? At 6%?

11. In what time will \$5 gain \$5, at 8%? At 10%?

12. At what per cent. will \$400 in 2 years 6 months gain \$60 interest?

SOLUTION.—\$400 in 2 years 6 months, at 1%, will gain \$10; hence, it will gain \$60 in the same time at as many per cent. as \$10 is contained times in \$60, or 6 per cent.

13. At what per cent. will \$120 in 1 year 9 months gain \$21?

14. At what per cent. will \$100 in 2 years 3 months gain \$18?

15. At what per cent. will a given principal double itself in 20 years? In 16⅔ years?

16. What is the amount of $150 for 3 years 9 months, at 8%?

17. What principal, at 6%, in 5 years will amount to $130?

18. What sum put at interest, at 7%, will earn for me $210 yearly?

19. How much greater is the annual than the simple interest on $500 for 4 years, at 10%?

20. What are the proceeds of a 30-day note for $400, discounted at 12%?

WRITTEN EXERCISES.

534.—Ex. 1. If you borrow, June 12, 1870, $500, at 6% interest, what amount will you owe by delaying payment to July 15, 1871? *Ans.* $532.75.

2. I borrowed $500, June 12, 1870, and paid July 15, 1871, as principal and interest, $532.75. What was the rate of interest?

3. What is the amount of $5000 for 2½ months, at 1½% per month? *Ans.* $5187.50.

4. What sum, put at interest at 6%, will give an income of $40 per month? *Ans.* $8000.

5. The amount of money I have at interest is $7300. It gives me exactly $1 a day, each common year. What is the rate of interest? *Ans.* 5%.

6. Chandler's money, at 6% interest, earns for him $1140, and Benton's, at 8%, earns for him $60 more. Which has the more money, and how much more?

Ans. Chandler, $4000.

7. What sum must a father invest for his son when he is 16 years 8 months old, at 6%, that on coming of age, he may have $2100? *Ans.* $1666.66⅔.

8. What was due November 12, 1870, on a note for $350, dated May 4, 1869, at 6% interest, and on which are indorsed, January 14, 1870, received $50, and June 13, 1870, received $25?

9. I bought a bill of goods amounting to $4500 on 2 months' time, or 5% off for cash. Should I have gained or lost by

borrowing the money of the bank on a 60-day note, discount being at 6%? *Ans.* Gained $179.64.

10. If a legacy of $2400 is left me on the 3d of May, to be paid on the Christmas-day following, what is its present value, money being worth 5%? *Ans.* $2325.08.

11. How much greater is the bank than the true discount on $7400 due 4 years 8 months hence, without grace, at 7%?

12. How much more is the compound than either the annual or simple interest on $5000 for 6 years, at 6%?

Ans. $22.60 more than the annual interest; $292.60 more than the simple interest.

SECTION LV.

RATIO.

535.—Ex. 1. If one house is 12 feet high and another 24 feet high, how many times as high as the first is the second?

2. What part of $25 is $5? Of $60 is $3?

3. John is 11 years old, and his uncle is 33 years old. How does his age compare with that of his uncle?

4. What part of 33 years is 11 years? How many times 11 years are 33 years?

5. What relation has 1 to 6? 8 to 24?

6. What is the relative value of $15 compared with $3? $62 compared with $9?

DEFINITIONS.

536. **Ratio** is the relation which one of two similar numbers bears to the other with respect to value. It is ascertained by the division of the first of the numbers by the second.

Thus, the ratio of $6 to $3 is 2.

537. The **Terms** of a ratio are the two numbers whose values are compared.

The first term is called the *Antecedent,* and the second term is called the *Consequent.*

538. The **Sign** of ratio is the colon (:), which is the sign of division (÷) with the line between the dots left out.

Thus, 7 : 8 denotes the ratio of 7 to 8, or $\frac{7}{8}$.

539. A **Simple Ratio** is a ratio each term of which is a single number.

Thus, 9 : 3 is a simple ratio.

540. A **Compound Ratio** is a ratio formed by multiplying together the corresponding terms of two or more simple ratios.

Thus, (6 : 2) × (5 : 3), or $\frac{6}{2} \times \frac{5}{3}$, or $\frac{6:2}{5:3}$, expresses a compound ratio.

541. Principles.—1. *The terms of a ratio must be similar numbers.*

2. *The ratio of two numbers is the quotient of the antecedent divided by the consequent.*

3. *Both terms of a ratio may be multiplied or divided by the same number without affecting its value.*

4. *A compound ratio is the ratio of the product of its antecedents to the product of its consequents.*

EXERCISES.

542.—Ex. 1. Indicate the ratio of 8 to 5. Of 6 to 4. *Ans.* 8 : 5; 6 : 4.

2. Express by a fraction the ratio of 11 to 12. *Ans.* $\frac{11}{12}$.

3. Find the ratio of 75 to 25. Of 19 to 6. *Ans.* 3; $3\frac{1}{6}$.

4. Find the ratio of 13 to 11; of 15 to 31.

5. What is the ratio of 8 yards to 2 feet? *Ans.* 12.

6. If the antecedent be 63, and the consequent 9, what is the ratio? *Ans.* 7.

7. If the ratio be 7, and the consequent 9, what is the antecedent?

8. If the consequent be 7, and the ratio 9, what is the antecedent?

9. If the ratio be 7, and the antecedent 63, what is the consequent?

10. Express the ratio of $\frac{7}{8}$ to $\frac{3}{4}$.

11. The antecedents of a compound ratio are 3, 8 and 10, and the consequents are 8, 4 and 5. What is the compound ratio?

12. Express the ratio of a field 6 rods long and 4 rods wide, to a field 3 rods long and 2 rods wide.

13. In 12 days, 8 men can do how many times as much work as 3 men can do in 16 days?

SECTION LVI.

PROPORTION.

543.—Ex. 1. What is the ratio of 2 to 1? Of 10 to 5? Of 12 to 6?

2. What two numbers have the same quotient as $10 \div 5$? As $18 \div 6$?

3. What two numbers have the same ratio to each other as 18 has to 6? As 12 has to 8?

4. When 2 oranges cost 8 cents, what will 10 oranges cost?

5. What is the ratio of 10 oranges to 2 oranges? Of 40 cents to 8 cents?

6. What is the ratio of 40 to 8 expressed as a fraction? Of 14 : 3 expressed as a fraction?

7. 8 cents are $\frac{10}{2}$ of how many cents?

8. What number has to 8 cents a ratio equal to the ratio of 10 to 2?

9. If 9 men can do a piece of work in 25 days, how many men can do the same in 5 days?

10. 9 men are $\frac{5}{25}$ of how many men?

11. What number of men has to 9 men a ratio equal to the ratio of 25 to 5?

12. 12 is $\frac{4}{7}$ of what number? 12 is to what number as 4 : 7?

13. 27 dollars are $\frac{54}{6}$ of how many dollars?

14. To what number has 27 dollars a ratio equal to the ratio of 54 to 6?

DEFINITIONS.

544. Proportion is an equality of ratios.

Thus, $4 : 2 = 6 : 3$ expresses a proportion.

545. The **Sign** of proportion is a double colon (: :), which, instead of the sign of equality, may be placed with one of the equal ratios before and the other after it.

Thus, $4 : 2 :: 6 : 3$ expresses a proportion.

Each ratio is a *Couplet*, and each term is a *Proportional*.

546. The **Antecedents** of a proportion are the antecedents of its ratios, and the **Consequents** are the consequents of its ratios.

547. The **Extremes** of a proportion are its first and fourth terms, and the **Means** are its second and third terms.

When the means of a proportion are equal, either is called the *Mean Proportional* between the extremes.

548. A **Missing Term** in a proportion may be denoted by x.

Thus, in the proportion $84 : 8 :: 21 : x$, the missing term is denoted by the x.

549. The ratio of one number to another is always expressed by an abstract number; hence, any couplet of a proportion, in a process of computation, may be regarded as formed of abstract numbers.

550. Principles.—1. *The product of the extremes is equal to the product of the means.*

For, since every ratio may be expressed in the form of a fraction, the proportion $4 : 6 :: 2 : 3$ may be expressed thus: $\frac{4}{6} = \frac{2}{3}$. Reducing these fractions to similar fractions, we have $\frac{4 \times 3}{6 \times 3} = \frac{2 \times 6}{3 \times 6}$. The resulting fractions being equal, and having the same denominators, the numerators must be equal. Hence, $4 \times 3 = 2 \times 6$. But the factors 4 and 3 are the extremes, and the factors 6 and 2 are the means. Hence, also,

2. *Either extreme is equal to the product of the means divided by the other extreme; and*

3. *Either mean is equal to the product of the extremes divided by the other mean.*

EXERCISES.

551. Find the missing terms in the following proportions—

1. $8 : 4 :: 28 : x$ *Ans.* 14.	6. $3\frac{1}{2} : x :: 2 : \frac{3}{4}$. *Ans.* $1\frac{5}{16}$.
2. $2 : 7 :: x : 10$. *Ans.* $2\frac{6}{7}$.	7. $\frac{5}{8} : \frac{7}{8} :: \frac{3}{7} : x$. *Ans.* $\frac{3}{5}$.
3. $9 : 6 :: 6 : x$.	8. $3 : \frac{1}{2} :: \frac{3}{8} : x$. *Ans.* $\frac{1}{16}$.
4. $x : 13 :: 35 : 7$. *Ans.* 65.	9. x : 11 T. :: \$4.32 : \$5.94.
5. \$7 : x :: 9 bu. : 63 bu.	10. 21 yd. : 45 yd. :: x : \$50.

SIMPLE PROPORTION.

552. Simple Proportion is the equality of two simple ratios.

Simple proportion is sometimes termed the *Rule of Three*, because, when three of its proportionals are given, they may be used to find the fourth.

553.—Ex. 1. If a man earn \$24 in 2 months, how much will he earn in 9 months?

$$\$24 : \$x :: 2\ mo. : 9\ mo.$$

Or,

$$2\ mo. : 9\ mo. :: \$24 : \$x$$

$$\$x = \frac{\overset{12}{\cancel{\$24}} \times 9}{\cancel{2}} = \$108$$

Solution.—There is a ratio between \$24 and \$$x$, or required term, for they are similar, both dollars; likewise between 2 months and 9 months, for they are similar.

The ratios are equal, for the money earned in 9 months will be as many times the \$24 earned in 2 months as the 9 months are times 2 months.

Then, expressing the proportion, we find the unknown term, by Prin. 3, Art. 550, to be \$108.

2. If 6 men use a barrel of flour in 60 days, how long will it last 9 men?

$$6\ men : 9\ men :: x\ days : 60\ days$$

$$x\ days = \frac{\overset{20}{\cancel{60}}\ days \times \overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{9}}} = 40\ days$$

Solution.—If 6 men use a barrel of flour in 60 days, 9 men will use it in $\frac{6}{9}$ as many as 60 days. Hence, the proportion, 6 men : 9 men :: x days : 60 days; which gives for the required term, 40 days.

3. How many casks, of 32 gallons each, will hold as much as 48 casks, of 42 gallons each?

4. If 18 men can mow a field in 12 days, how many men can mow a similar field in 9 days? *Ans.* 24.

554. Rule for Simple Proportion.—*Select the ratios upon which the question depends, and form from them a proportion.*

Then, if the required term be an extreme, divide the product of the means by the given extreme; or, if it be a mean, divide the product of the extremes by the given mean.

PROBLEMS.

1. If a man earn \$24 in 2 weeks, how much will he earn in 52 weeks? *Ans.* \$624.

2. When 385 yards of muslin can be bought for \$63, how much can be bought for \$18?

3. When 42 casks of molasses cost \$492.66, what is the cost of 5 casks? *Ans.* \$58.65.

4. If 3 yards of cloth that is $2\frac{1}{2}$ yards wide will line a garment, how much cloth that is only $\frac{3}{4}$ of a yard wide will line the same garment? *Ans.* 10 yards.

5. If 17 carpenters can do a piece of work in $11\frac{9}{17}$ days, in what time can 7 carpenters do it?

6. If a cistern can be filled in 3 hours 25 minutes by 2 pipes, in what time can it be filled by 5 pipes of like size? *Ans.* 1 hour 22 minutes.

7. A person travelled a certain distance in 12 days, walking 10 hours a day. How many days would the same journey have required had he walked only 8 hours a day? *Ans.* 15.

8. When $19\frac{1}{2}$ acres of land sell for \$1800, for how much will 9 acres sell?

9. How many men would perform in 168 days a piece of work which 108 men can perform in 266 days? *Ans.* 171.

10. When 27 tons of hay cost \$675, what is the cost of 21 tons?

11. If a ship have provisions sufficient to last 24 men 80 days, how long will the same last if the ship take on board 8 more men? *Ans.* 60 days.

12. If 8 horses eat 24 bushels 3 pecks of oats in 10 days, how many bushels will 6 horses eat in the same time?

13. When 40 yards of cloth cost \$82$\frac{2}{7}$, what will $\frac{1}{2}$ of $\frac{7}{8}$ of a yard cost? *Ans.* \$.90.

14. If $\frac{21}{26}$ of a ship cost \$42000, what will $\frac{8}{15}$ of a ship cost?

15. When \$7.49 pays for $\frac{7}{9}$ of a ton of coal, what will 16$\frac{2}{3}$ tons cost? *Ans.* \$160.50.

16. If the earth in its orbit moves 19320 miles in 16 minutes 48 seconds, how far does it move in one hour? *Ans.* 69000 miles.

COMPOUND PROPORTION.

555. A Compound Proportion is a proportion in which one or both ratios are compound.

Thus, $\begin{matrix} 9:18 \\ 6:5 \end{matrix} :: 99:165$, and $\begin{matrix} 12:3 \\ 3:4 \end{matrix} :: \begin{matrix} 6:3 \\ 9:6 \end{matrix}$ are compound proportions.

556. The formation of a compound proportion may often be facilitated, in the solution of problems, by applying the following

557. Principle.—*The ratio between two causes at work for the same end will equal the ratio between the effects produced.*

558.—Ex. 1. If 2 men can mow 15 acres in 6 days, how many acres can 3 men mow in 8 days?

$$\begin{matrix} 2 \text{ men} : 3 \text{ men} \\ 6 \text{ days} : 8 \text{ days} \end{matrix} :: 15 \text{ acres} : x \text{ acres}.$$

$$x \text{ acres} = \frac{15 \text{ acres} \times \cancel{3} \times \overset{2}{\cancel{8}}}{\cancel{2} \times \underset{4}{\cancel{6}}} = 30 \text{ acres}.$$

SOLUTION.—If 2 men can mow 15 acres, 3 men can mow $\frac{3}{2}$ as many acres; hence, the proportion 2 men : 3 men :: 15 acres : x acres.

If in 6 days 15 acres can be mown, in 8 days there can be mown $\frac{8}{6}$ as many as 15 acres; hence, the proportion 6 days : 8 days :: 15 acres : x acres.

By the first proportion, the required term would be $\frac{3}{2}$ of 15 acres, and by the second proportion, $\frac{8}{6}$ of 15 acres; hence, by the two combined it will be $\frac{8}{6}$ of $\frac{3}{2}$ of 15 acres, or 15 aeres $\times \frac{3}{2} \times \frac{8}{6}$, or 30 acres.

Or, selecting the ratios upon which the question depends, and forming a proportion, we have $\begin{matrix} 2:3 \\ 6:8 \end{matrix}$ :: 15 acres : x acres, or $2 \times 6 : 3 \times 8$: : 15 acres : x acres, which gives, as before, 30 acres for the required term.

2. If 6 men in 8 weeks can build a wall 400 feet long and 8 feet high, how long a wall that is 2 feet high can 12 men build in 4 weeks?

SOLUTION.

$$\begin{matrix} 6 \text{ men} & : 12 \text{ men} \\ 8 \text{ weeks} : & 4 \text{ weeks} \end{matrix} :: \begin{matrix} 400 \text{ feet long} : x \text{ feet long.} \\ 8 \text{ feet high} : 2 \text{ feet high.} \end{matrix}$$

Or,

$$\left.\begin{matrix} \text{1st cause.} \\ 6 \\ 8 \end{matrix}\right\} : \left\{\begin{matrix} \text{2d cause.} \\ 12 \\ 4 \end{matrix}\right\} :: \left\{\begin{matrix} \text{1st effect.} \\ 400 \\ 8 \end{matrix}\right\} : \left\{\begin{matrix} \text{2d effect.} \\ x \\ 2 \end{matrix}\right.$$

$$x \text{ feet} = \frac{400 \text{ feet} \times \cancel{8} \times \cancel{12} \times 4}{\cancel{2} \times \cancel{6} \times \cancel{8}} = 1600 \text{ feet.}$$

Applying the principle of causes and effects (Art. 557),

We have, as the first cause, 6 men at work 8 weeks, and, as the second cause, 12 men at work 4 weeks. These causes are similar, and constitute a compound ratio.

The effect of the first cause is a wall 400 feet long and 8 feet high; the effect of the second cause is a wall x feet long and 2 feet high. These effects are similar, and constitute a compound ratio.

These compound ratios are equal, and form a compound proportion.

The required number is a factor of one of the extremes; hence, dividing the product of the means by the product of the given factors of the extremes, we have that number, which is 1600 feet.

3. If 5 compositors, in 16 days, 11 hours long, can compose 25 sheets of 24 pages in each sheet, 44 lines in each page, and 40 letters in a line, in how many days, each 10 hours long, may 9 compositors compose a volume to be printed in the same letter, consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters to a line?

SOLUTION.

$$\left.\begin{matrix} \text{1st cause.} \\ 5 \\ 16 \\ 11 \end{matrix}\right\} : \left\{\begin{matrix} \text{2d cause.} \\ 9 \\ x \\ 10 \end{matrix}\right\} :: \left\{\begin{matrix} \text{1st effect.} \\ 25 \\ 24 \\ 44 \\ 40 \end{matrix}\right\} : \left\{\begin{matrix} \text{2d effect.} \\ 36 \\ 16 \\ 50 \\ 45 \end{matrix}\right.$$

$$x \text{ days} = \frac{\cancel{5} \times \overset{2}{\cancel{16}} \times \cancel{11} \times \overset{\cancel{4}}{\cancel{36}} \times \overset{2}{\cancel{16}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{\overset{8}{\cancel{9}}}{\cancel{45}}}{\cancel{9} \times \cancel{10} \times \underset{\cancel{5}}{\cancel{25}} \times \underset{\cancel{3}}{\cancel{24}} \times \underset{\cancel{4}}{\cancel{44}} \times \underset{\cancel{8}}{\cancel{40}}} = 12 \text{ days.}$$

Here, the first cause, 5 compositors working 16 days 11 hours long : the second cause, 9 compositors working x days 10 hours long : : the first effect, 25 sheets of 24 pages in each sheet, 44 lines in each page, and 40 letters in a line : the second effect, 36 sheets of 16 pages in each sheet, 50 lines in each page and 45 letters in a line.

Regarding the terms as abstract, and dividing the product of the extremes by the product of the means, since the required number belongs to one of the means, we have as that number 12.

559. Rule for Compound Proportion.—*Select the ratios upon which the question depends, and form from them a proportion.*

Then, if the required number belongs to either extreme, divide the product of the means by the product of the given extremes; or, if it belongs to either mean, divide the product of the extremes by the product of the given means.

PROBLEMS.

1. If you travel 360 miles in 12 days of 8 hours each, how many miles can you travel, at the same rate, in 60 days of 6 hours each? *Ans.* 1350.

2. When 12 men can harvest 32 acres of corn in 18 days, how many men, at the same rate, can harvest 128 acres in 36 days? *Ans.* 24.

3. If 2 horses consume 30 centals of corn in 365 days, how many centals will 11 horses consume in 73 days, at the same rate?

4. If 6 men can build a wall 20 feet long, 6 feet high and 4 feet thick in 16 days, in what time can 24 men build one 200 feet long, 8 feet high and 6 feet thick? *Ans.* 80 days.

5. If a barrel of flour will feed a family of 8 persons for 13 days, how many barrels will feed a family of 3 times as many persons for a year of 365 days? *Ans.* $84\frac{3}{13}$.

6. When 84 men can mow 72 acres of grass in 15 days, how many acres can 96 men mow in 12 days?

7. A farmer employed 16 laborers to harvest his wheat, calculating that they would do it in 16 days, but after 8 days' work he resolved to have the remainder harvested in 4 days. How many additional laborers must be employed for that purpose? *Ans.* 16.

8. If I travel 300 miles in 6 days of 8 hours each, in how many days of 10 hours each can I travel 450 miles, if I travel one half faster than at first? *Ans.* $4\frac{4}{5}$ days.

9. If 320,000 bricks, 9 inches long, 5 inches broad and $2\frac{1}{4}$ inches thick, are required for the construction of a building, how many bricks, 12 inches long, 6 inches broad and 3 inches thick, would have been required for the same purpose?

Ans. 150,000.

TEST QUESTIONS.

560.—1. What is RATIO? How is the ratio of one number to another ascertained?

2. What are the TERMS of a ratio? What is the first term called? The second term? When is a ratio simple? When compound?

3. What kind of numbers must the terms of a ratio be? Why may both terms of a ratio be either multiplied or divided by the same number without affecting its value?

4. What is PROPORTION? What is the sign of proportion? How does it differ from the sign of ratio?

5. What are the TERMS of a proportion? What is each ratio of a proportion called? What is each term of a proportion called?

6. What are the antecedents? The consequents? The extremes? The means?

7. How may a MISSING TERM of a proportion be denoted? How may any couplet of a proportion be regarded in computation?

8. To what is the product of the means of a proportion equal? To what is either extreme equal? Either mean?

9. What is SIMPLE PROPORTION? What is it sometimes called? What is the rule for simple proportion?

10. What is COMPOUND PROPORTION? What is the rule for compound proportion?

SECTION LVII.

DISTRIBUTIVE PROPORTION.

561.—Ex. 1. What number is $\frac{2}{5}$ of 25? Is $\frac{3}{5}$ of 25?

2. If 25 apples be divided between two boys, so that one shall have 2 as often as the other shall have 3, what part of 25 apples will each have? How many apples will each have?

3. Divide 15 into two parts having the ratio of 2 to 5.

4. If 27 cents be divided among 3 boys, so that their shares shall be to one another as 2, 3 and 4, what part of 27 cents will the share of each be?

5. Two boys buy 42 oranges, and divide them into two parts in the ratio of 3 : 4. How many will there be in each part?

DEFINITIONS.

562. Proportional Parts of a number are such parts of the number as are proportional to given numbers.

Thus, 5, 10 and 15 are parts of 30 proportional to 1, 2 and 3.

563. Distributive Proportion is the process of separating a number into parts proportional to given numbers.

WRITTEN EXERCISES.

564.—Ex. 1. Divide 1035 into two parts which shall be to each other as 2 to 7.

$$7+2=9$$

$$\tfrac{2}{9}\times 1035=230$$

$$\tfrac{7}{9}\times 1035=805$$

Or,

$$9:2::1035:230$$

$$9:7::1035:805$$

PROOF.

$$230+805=1035$$

Solution.—The parts are to be to each other as 2 to 7. The sum of 2 and 7 is 9. The smaller of the parts, then, must be $\frac{2}{9}$ of 1035, which is 230; and the larger of the parts, $\frac{7}{9}$ of 1035, which is 805. Or,

Since the parts are to be in the ratio of 2 to 7, and the sum of the parts is 9, as 9 : 2 : : 1035 : 230, the smaller part; and, as 9 : 7 : : 1035 : 805, the larger part.

The correctness of the solution is tested by adding the parts, for their sum must equal the given number.

2. Divide 500 into 3 parts proportional to the numbers 2, 3 and 5.

565. Rule for Distributive Proportion.—*Take, for the required parts, such fractions of the quantity to be divided as each of the numbers to which the parts are to be proportional is of the sum of these numbers.*

PROBLEMS.

1. Two kinds of tea are mixed in the ratio of 15 pounds of Oolong to 9 pounds of Japan. How much of each is there in a mixture weighing 120 pounds?

Ans. 75 lb. Oolong and 45 lb. Japan.

2. Gunpowder contains $\frac{3}{4}$ of its weight of nitre; nitre is composed of 39 parts of potassium, 14 of nitrogen and 48 of oxygen. How many pounds of potassium are there in 909 pounds of gunpowder?

3. Four regiments have for duty, respectively, 60, 90, 150 and 225 men; the commander requires a special detail of 70 men. How many men must each regiment furnish?

Ans. 8, 12, 20, 30.

SECTION LVIII.

PARTNERSHIP.

566. A **Partnership,** or a company, is an association of persons for the transaction of business.

567. Partners are the persons associated in business.

568. Capital, or **Stock,** is that which is invested in the business.

569. A **Dividend** is that which is divided among the partners as their profits from the business; and an **Assessment** is that required to be paid by each partner for increase of capital, or for meeting expenses or losses.

570. The **Liabilities** of a company are its debts, and the **Assets** are its property.

CASE I.

Profits or Losses Apportioned according to the Capital.

571.—Ex. 1. A, B and C entered into partnership. A put in $1500, B $1600, and C $900. The company gained $500. What was each partner's share?

SOLUTIONS.

$$\$1500+\$1600+\$900=\$4000=\textit{entire capital.}$$

$$\textit{A's part of the capital}=\frac{1500}{4000}=\frac{3}{8}$$

$$\textit{B's " "}=\frac{1600}{4000}=\frac{2}{5}$$

$$\textit{C's " "}=\frac{900}{4000}=\frac{9}{40}$$

$$\textit{A's share of gain}=\frac{3}{8}\textit{ of }\$500=\$187.50$$

$$\textit{B's " "}=\frac{2}{5}\textit{ of }\$500=\$200.00$$

$$\textit{C's " "}=\frac{9}{40}\textit{ of }\$500=\$112.50$$

Or,

$$\textit{The gain }\$500=\frac{500}{4000}=\frac{1}{8}=12\tfrac{1}{2}\%\textit{ of the entire capital.}$$

$$\textit{A's share of gain}=\frac{1}{8}\textit{ of }\$1500=\$187.50$$

$$\textit{B's " "}=\frac{1}{8}\textit{ of }\$1600=\$200.00$$

$$\textit{C's " "}=\frac{1}{8}\textit{ of }\$900=\$112.50$$

Or,

$$\textit{A's share of gain}=12\tfrac{1}{2}\%\textit{ of }\$1500=\$187.50$$

$$\textit{B's " "}=12\tfrac{1}{2}\%\textit{ of }\$1600=\$200.00$$

$$\textit{C's " "}=12\tfrac{1}{2}\%\textit{ of }\$900=\$112.50$$

$$\textit{Proof,}\qquad \$500.00$$

2. A, B and C engaged in business together. A furnished $875, B $1680, and C $945. They gained $1000. What was each partner's share of the profits?

Ans. A's, $250; B's, $480; and C's, $270.

572. Rule. *Apportion the profit or loss among the partners in proportion to their capital in the business. Or,*

Find such a part of each man's capital as the profit or loss is of the entire capital, and the result is his share of the profit or loss.

PROBLEMS.

Ex. 1. Thayer, Lane & Co.'s profits are \$6400. Thayer's capital is \$50000, and that of each of his two partners is \$15000. How much should each receive?

Ans. Thayer, \$4000, and each of his partners, \$1200.

2. A and B bought a ship together, A contributing $\frac{1}{4}$, and B the remainder. C was employed to manage the ship, and was to have for his compensation half of the profits. The expenses were \$7500, and the income \$25500. What are their respective shares of the profits?

Ans. A's, \$2250; B's, \$6750; and C's, \$9000.

3. A bankrupt, whose assets are \$6800, owes M \$4500, N \$3600, and O \$2100. How much can he pay each of these creditors? *Ans.* M, \$3000; N, \$2400; O, \$1400.

4. A, B, C and D traded in company. They furnished of the capital, \$9500, \$6500, \$4000 and \$5000 respectively. They gained \$6125. What was each partner's share of the profits?

5. A, B and C joined in a speculation with a capital of \$1440. They gained \$1080, of which A is entitled to \$3 as often as B is to \$5, and as C is to \$7. What was the capital and the gain of each?

Ans. A's capital, \$288; B's, \$480; C's, \$672.
A's gain, \$216; B's, \$360; C's, 504.

CASE II.

Profits or Losses Apportioned according to Capital and Time.

573.—Ex. 1. A goes into business at the beginning of the year with a capital of \$2000. After 6 months he admits as a partner B, with a capital of \$6000. If the net profits at the end of the year are \$2000, what is each partner's share?

SOLUTION.

$$\begin{aligned}
&A\text{'s } \$2000 \text{ for } 12 \text{ mo.} = \$24000 \text{ for } 1 \text{ mo.}\\
&B\text{'s } \$6000 \text{ for } 6 \text{ mo.} = \$36000 \quad \text{"} \quad \text{"}\\
&A\text{'s and } B\text{'s together} \quad = \$60000 \quad \text{"} \quad \text{"}
\end{aligned}$$

$$A\text{'s share of profits} = \tfrac{24000}{60000} = \tfrac{2}{5} \text{ of } \$2000 = \quad \$800$$

$$B\text{'s share of profits} = \tfrac{36000}{60000} = \tfrac{3}{5} \text{ of } \$2000 = \$1200$$

$$\textit{Proof,} \quad \$2000$$

Or,

$$\begin{aligned}
&\text{Interest of } A\text{'s } \$2000 \text{ for } 12 \text{ mo.} = \$120\\
&\text{Interest of } B\text{'s } \$6000 \text{ for } 6 \text{ mo.} \; = \$180\\
&\text{Interest of } A\text{'s and } B\text{'s capital} \quad = \$300
\end{aligned}$$

$$A\text{'s share of profits} = \tfrac{120}{300} = \tfrac{2}{5} \text{ of } \$2000 = \quad \$800$$

$$B\text{'s share of profits} = \tfrac{180}{300} = \tfrac{3}{5} \text{ of } \$2000 = \$1200$$

$$\textit{Proof,} \quad \$2000$$

2. Wilson, Hayes and Jones in partnership gained $936. Wilson had in business $4680 for 4 months, Hayes $5616 for 5 months, and Jones $2880 for 13 months. Required their respective shares of the profits.

574. Rule.—*Apportion the profit or loss among the partners in proportion to the products of their capital by the time in business. Or,*

Apportion the profit or loss among the partners in proportion to the interests of their capital for the time in business.

PROBLEMS.

1. A, B and C rent a pasture together, for which they agree to pay $80. A put in 8 cattle for 180 days, B, 6 cattle for 150 days, and C, 20 cattle for 123 days. How much of the rent should each pay? *Ans.* A, $24; B, $15; C, $41.

2. Anson and Potter entered into partnership the first of January, and each put in $3000. The first of April, Anson put in $1000; and the first of September, Potter put in $500. At the end of the year the profits proved to be $2000. What should be each partner's dividend?

Ans. Anson's, $1084.34; Potter's, $915.66.

3. Benson commenced business at the beginning of the year with $4500. April 1, he took into partnership Colfax with $5000. At the end of the year it was found necessary to contribute $660 to meet liabilities. What should be each partner's share of the assessment?

4. Albert Whiting, John Walker and Peter Woodman formed a company, under the firm-name of Albert Whiting & Co. Whiting put in $7000 for 10 months; Walker, $18000 for 5 months; and Woodman $20000 for 3 months. They gained $13200. What was each partner's share?

Ans. Whiting's, $4200; Walker's, $5400; Woodman's, $3600.

5. A and B entered into partnership. At the commencement, A put in $5000, but at the end of 4 months took out $3000, and continued the remainder in the business 6 months longer. At the commencement, B put in $3000, but at the end of 5 months put in $4000, and continued the whole in business 3 months longer. On settlement it was found that they had lost $1768. What was each partner's share of the loss?

Ans. A's, $832, and B's, $936.

TEST QUESTIONS.

575.—1. What is DISTRIBUTIVE PROPORTION? What are the proportional parts of a number?

2. What is PARTNERSHIP? Who are partners? What is the capital or stock?

3. What is a DIVIDEND by a company? An assessment? What are the liabilities of a company? The assets?

4. What is the RULE for apportioning partnership profits or losses when the capitals of the partners are in the business for equal times? When the capitals of the partners are in the business for unequal times?

SECTION LIX.

AVERAGE OF PAYMENTS.

576. **Average of Payments** is the process of finding the average time for the payment of several sums due at different times.

577. The **Average Time** is the date on which debts due at different times may be equitably discharged by one payment.

578. The **Term of Credit** is the time which is to elapse before a debt becomes due.

579. The **Average Term of Credit** is the time which is to elapse before the average time.

CASE I.

Terms of Credit Beginning at the Same Time.

580.—Ex. 1. On January 1, Belden Wilder owes me $100, due that day; $600, due March 1; and $800, due July 1. At what date may he equitably cancel his indebtedness?

$$
\begin{array}{rcr}
\$100 \times 0 = & & \$0000 \\
600 \times 2 = & & 1200 \\
800 \times 6 = & & \underline{4800} \\
\overline{\$1500} &) & \underline{\$6000} \\
 & & 4
\end{array}
$$

January 1 + 4 mo. = May 1

SOLUTION.—The $100 being due Jan. 1, its term of credit is 0 months.

The credit of $600 for 2 months is the same as the credit of 2 times $600, or $1200, for 1 month.

The credit of $800 for 6 months is the same as the credit of 6 times $800, or $4800, for 1 month.

Hence, the credit of the entire indebtedness, or $1500, is the same as that of $1200 + $4800, or $6000, for 1 month, which is equal to the credit of $1500 for as many months as $1500 is contained times in $6000, or for 4 months. January 1 + 4 months equals May 1.

We may also consider the debtor as entitled to the use, or interest, of each of the debts for its term of credit. Hence, the following

Interest of	*$100*	*for 0 mo. =*	*$00.00*
"	*600*	" *2 mo. =*	*6.00*
"	*800*	" *6 mo. =*	*24.00*
Debts,	*$1500*	*Total Int.,*	*$30.00*

SOLUTION.—Reckoning the interest at 6%, the aggregate of interest for the terms of credit is $6 + $24, or $30.

The interest of $1500, or the sum of the debts, at the same rate for 1 month is $7.50. Wilder should therefore have the use of the $1500 as many months from January 1 as $7.50 is contained times in $30, which is 4; and 4 months from January 1 is May 1.

Any rate of interest might have been used in the computation, and the result would have been the same.

2. Three debts are due me—one of $120 in 5 months, another of $125 in 4 months, and a third of $500 in 8 months. What is the average time of their payments?

581. **Rules.**—*Multiply each of the debts by its term of credit, and divide the sum of the products by the sum of the debts; the quotient will be the average term of credit. Or,*

Find the interest of each debt for its term of credit, and divide the sum of their interests by the interest of the sum of the debts for one month or one day; the quotient will be the average term of credit.

The date of the debts, plus the average term of credit, will be the average time.

In finding the average term of credit when any of the debts have cents, it is customary to neglect them if less than 50; and if 50 or more to regard them as $1.

In a result, if there be a fraction of a day, reject it when less than ½; and when otherwise, call it 1 day.

PROBLEMS.

1. A merchant owes $60 due in 72 days, $85 due in 128 days, $70 due in 176 days, and $105 due in 320 days. Required the average time at which the whole will be due.

2. January 1, Alfred Day bought bills of goods payable as follows: $70 at date, $110 on March 2, $80 on May 5, $120 on July 20, $48 on September 27, and $50 on October 7. Required the average time of payment. *Ans.* May 22.

3. July 5, Johnson Paterson bought bills of goods payable as follows: $500 on August 5, $600 on September 5, and $1000 on September 20. Required the average time of payment.

CASE II.

Terms of Credit Beginning at Different Times.

582.—Ex. 1. I bought goods of James Hunt & Co. as follows: March 1, a bill of \$500, on 4 months; March 22, a bill of \$200, on 2 months, and April 29, a bill of \$680, on 5 months. What is the average time of payment of the whole?

SOLUTION.

March 22 + 2 mo. = May 22,	*\$200 × 0*	=	*\$00000*
March 1 + 4 " = July 1,	*500 × 40*	=	*20000*
April 29 + 5 " = Sept. 29,	*680 × 130*	=	*88400*
	\$1380		*)\$108400(78 760/1380*
			9660
			11800
May 22 + 79 days = Aug. 9.			*11040*
			760

Or,

Interest of \$200	*for 0 days*	=	*\$00.00*
" *500*	" *40 days*	=	*3.33*
" *680*	" *130 days*	=	*14.73*
Sum of bills, \$1380	*Total interest,*		*\$18.06*

Interest of \$1380 for 1 day = \$.23

$\$18.06 \div \$.23 = 78\frac{12}{23}$. *May 22 + 79 da. = Aug. 9.*

The several bills are due July 1, May 22 and September 29, respectively. Selecting the earliest day of maturity as the day from which to reckon, \$200 has no term of credit, the \$500 has a credit of 40 days, and the \$680 has a credit of 130 days, from May 22. The average term of credits, by either form of solution, is 79 days, nearly.

Hence, May 22 + 79 days, which is August 9, is the average time required.

The first form of solution is called the *Product Method,* and the second the *Interest Method.* Accountants generally prefer the latter.

The date of the first debt's maturity was selected to reckon from for convenience. Had the latest date of maturity been selected, the average time would have been counted back from that date.

2. Robert Hendricks gave me, June 4, a note for $315.63 on 4 months; June 15, a note for $535.47 on 2 months; and July 3, a note for $300 on 3 months. Regarding these notes without grace, should he wish to take them up by giving one note for their amount, when should it be payable?

Ans. September 11.

583. Rule.—*Find the date at which each debt becomes due. Select the earliest date at which any of the debts matures, and reckoning from it, as in the previous case, find the average term of credit; and the selected date, plus the average term of credit, will be the average time.*

In working by the interest method, it may be most convenient to take for the selected date the *first day of the month in which the first credit begins.*

When the terms of credit are all equal, we may simply find the average date of the debts, and add the common term credit, for the average time.

PROBLEMS

1. When should a note to settle the following account be made payable?

LEWIS MANLY,

To STONE, DEXTER & CO. *Dr.*

1871.				
May	*13*	*To Merchandise @ 4 mo., as per bill*	*$500*	*15*
"	*24*	" " *@ 2 mo.,* "	*300*	*00*
June	*15*	" " *Cash*	*99*	*83*
			$899	*98*

Ans. August 17.

2. Purchased of Jonas Munger, on a credit of 90 days, January 6, a bill of $600, and February 15, a bill of $200. Required the average date of purchase and the average time of payment. *Ans.* January 16; April 16.

3. What is the average time of the following bills, allowing to each term of credit 3 days' grace?—Sold, April 3, a bill of \$500 on 3 months; April 4, a bill of \$200 on 2 months; April 4, a bill of \$200 for cash; and April 10, a bill of \$500 on 3 months. *Ans.* June 21.

CASE III.

Debit and Credit Account.

584.—Ex. 1. From what time should a note draw interest for the balance of the following account, allowing 3 days' time to the items on time?

Dr. JAMES BLAKE *in account with* GEORGE HILL. *Cr.*

1870.					1870.				
May	*8*	*To Mdse., Cash*	*\$ 40*	*00*	*May*	*10*	*By Cash*	*\$ 30*	*00*
"	*17*	" " *30 days*	*240*	*00*	"	*30*	" "	*205*	*00*
"	*20*	" " *Cash*	*240*	*00*	*June*	*13*	" "	*165*	*00*

SOLUTIONS.

Int. of	*\$40 for 0 days =*	*\$0.00*
"	*240 " 12 days =*	*.48*
"	*240 " 42 days =*	*1.68*
	\$520	*\$2.16*
	400	*1.7516*
Balance,	*\$120*	*\$ .4084*

\$.4084 ÷ .02 = 20.42.

Int. of	*\$30 for 2 days =*	*\$.01*
"	*205 " 22 days =*	*.7516*
"	*165 " 36 days =*	*.99*
	\$400	*\$1.7516*

Int. of \$120 for 1 da. = \$.02

May 8 + 20 da. = May 28.

Or,

May 8,	*\$40 × 0 =*	*\$00000*
" *20,*	*240 × 12 =*	*2880*
June 19,	*240 × 42 =*	*10080*
	\$520	*\$12960*
	400	*10510*
Balance,	*\$120*	*\$2450*

May 10,	*\$30 × 2 =*	*\$60*
" *30,*	*205 × 22 =*	*4510*
June 13,	*165 × 36 =*	*5940*
	\$400	*\$10510*

$\$2450 \div \$120 = 20\frac{5}{12}$

May 8 + 20 da. = May 28, the average time.

The item on the debit side, on 30 days, by adding 3 days' grace, is due June 19. The other items, being cash, are due at their respective dates.

Selecting the earliest date of maturity of any item as that from which to reckon, we find that the balance of account, \$120, and the balance of interest, $\$.408\frac{4}{10}$, are on the debit side. Were, then, Blake to settle at that date by paying the balance, he evidently would lose the use of $120 for as many days as would be equivalent to $.408$\frac{4}{10}$ interest. Hence, the balance is not equitably due, or subject to interest, till as many days after May 8 as would allow $120 to earn $.408$\frac{4}{10}$ interest. This we find to be 20 days, and May 8 + 20 days gives May 28 as the required time.

Had the balance of account and of interest been on opposite sides of the account, then the balance would have been due, or subject to interest, earlier than that date as many days as would allow the balance of account to earn the balance of interest.

Or, reckoning from the earliest date of maturity of any of the items, we find that the balance of account, which is $120, and of credit, which is that of $2450 for 1 day, are both on the debit side. Hence, the debtor, by paying $120 May 8, would lose the credit of $2450 for 1 day, which is the same as the credit of $120 for as many days as $120 is contained times in $2450, or for 20 days.

Hence, the average time of payment is May 8 + 20 days, or May 28.

585. Rule.—*Select the earliest date at which any of the items mature, and, reckoning from it, find the interest of each item from that date to its maturity.*

Divide the balance of interest by the interest of the balance of items for 1 day, and the quotient will be the average term of credit, which must be counted forward from the selected date, for the average time, when the balance of interest is on the larger side of the account, but backward when the balance of interest is on the smaller side.

PROBLEMS.

1. When will the balance of the following account average due, allowing 3 days' grace on each merchandise item?

Dr. J. B. HALSTEAD. *Cr.*

1870.					1870.				
Mar.	10	To Mdse., on 30 days,	$100	00	Mar.	4	By Mdse., on 30 days,	$200	00
April	15	" Cash	100	00	"	12	" " "	150	00
"	20	" "	100	00					

Ans. March 3, 1870.

2. What should be the date of a note drawing interest for the balance of the following account, allowing 3 days' grace on each item?

Dr. R. B. ALFRED. *Cr.*

1870.					*1870.*				
May	*1*	*To Mdse., on 30 days,*	*$120*	*00*	*May*	*21*	*By Mdse., on 30 days,*	*$180*	*00*
"	*18*	" " "	*100*	*00*	"	"	" " "	*220*	*00*
"	*30*	" " "	*100*	*00*	"	"	" " "	*50*	*00*

Ans. July 7, 1870.

CASH BALANCE OF ACCOUNTS.

586. The **Balance of an Account**, in equity, is entitled to interest from the time of its becoming due. Hence,

If settlement of an account is made after the average time, the *Cash Balance* is the balance of items plus the interest from the average time to the time of settlement.

If settlement is before the average time, the *Cash Balance* is the balance of items minus the interest from time of settlement to the average time.

EXERCISES.

587.—Ex. 1. The balance of items of an account, which is $50, is due by average, March 3, 1870. What should be its cash value, April 20, interest at 6%? *Ans.* $50.40.

2. The balance of items of an account, which is $130, is due by average, June 17, 1871. What was the cash balance June 2, interest at 6%? *Ans.* $129.67½.

3. What is the cash balance of the following account, Oct. 16, at 7%?

Dr. HALL, WESTON & CO. *Cr.*

1871.					*1871.*				
May	*5*	*To Mdse., on 3 mo.,*	*$19*	*83*	*June*	*20*	*By Mdse., on 4 mo.,*	*$25*	*00*
July	*15*	" " *on 4 mo.,*	*40*	*00*	*Aug.*	*10*	" " *on 3 mo.,*	*100*	*00*
Sept.	*25*	" " *on 2 mo.,*	*60*	*17*	*Oct.*	*5*	" " *on 2 mo.,*	*120*	*00*

TEST QUESTIONS.

588.—1. What is AVERAGE OF PAYMENTS? What is the average time? The term of credit? The average term of credit?

2. What are the RULES for finding the average term of credit when the terms of credit begin at the same time? What will be the average time?

3. When the terms of credit begin at different times, how is the average term found by the product method? By the interest method? What is the rule for finding the average time?

4. What method is generally preferred by accountants in finding the average time of payment of a debit and credit account? Why? What is the rule?

5. To what is the BALANCE OF AN ACCOUNT entitled in equity from the time of becoming due? What is the cash balance of an account when the settlement is made after the average time? When the settlement is made before the average time?

SECTION LX.

STOCKS AND INVESTMENTS.

589. **Stock** is money or property employed in business, or invested in a company or in a public debt.

590. The **Par Value** of stock is its original value.

591. The **Market Value** of stock is the sum it will bring when sold.

592. A stock is *at par* when it sells for its original or face value, or 100%; *above par*, or at a *premium*, when it sells for more than its face value, or above 100%; and *below par*, or at a *discount*, when it sells for less than its face value, or less than 100%.

Thus, when stock is at par, it is quoted at 100; when at 6% above par, at 106; and when at 6% below par, at 94.

593. When paper money is depreciated, **Gold** ceases to be a circulating medium, and, like stocks, becomes an object of investment.

Thus, when gold is quoted at 109, $1 of gold is of the same value as $1.09 of currency.

594. A **Broker** is a dealer in stocks, bonds, gold, etc.

The usual rate of brokerage for buying or selling is from $\frac{1}{16}\%$ to $\frac{1}{4}\%$ of the par value.

CORPORATE STOCKS.

595. A **Corporation** is a company, or an association of persons, authorized by law to transact business jointly.

596. A **Corporate Stock** is the property invested in a corporation.

597. A **Share** is one of the equal parts into which a corporate stock is divided.

The Par Value of a share is usually $100.

598. An **Installment** is a certain part of the stock of an incorporated company paid at a particular time.

599. An **Assessment** is a sum which stockholders are called upon to pay, on each share held, to meet losses or to make up deficiencies.

600. The *Gross Earnings* of a company are its entire receipts; the *Net Earnings* of a company are what remains after deducting expenses; and the *Dividend* is the sum paid to the stockholders from the profits of the business.

601. Dividends and assessments are usually reckoned at a certain per cent. of the par value.

GOVERNMENT SECURITIES.

602. **Bonds** are obligations securing the payment of a certain sum of money on or before a specified time.

These, when issued by Government or corporations, bear interest payable at fixed dates.

603. **Coupons** are interest certificates attached to bonds. They are to be cut off and presented for payment when the interest is due.

604. Treasury Notes are notes issued by the Government, payable on demand, without interest, or payable at a specified time, with interest.

605. United States Government Securities consist of bonds and Treasury notes.

6's of 1881 are bonds which are payable in 1881. The interest on them is at the rate of 6% in gold, and is payable semi-annually.

5-20's are bonds which are payable after 20 years, and are redeemable after 5 years, from their issue. The interest on them is at the rate of 6% in gold, and is payable semi-annually.

10-40's are bonds which are payable after 40 years, and redeemable after 10 years, from their issue. The interest on them is at the rate of 5% in gold, and is payable semi-annually on \$500 and \$1000 coupon bonds, and annually on registered bonds, and on \$100 and \$50 coupon bonds.

5's of 1881 are bonds which are payable after 1881. The interest on them is at the rate of 5% in gold, and is payable quarterly.

$4\frac{1}{2}$'s of 1886 are bonds which are payable after 1886. The interest on them is at the rate of $4\frac{1}{2}$% in gold, and is payable quarterly.

4's of 1901 are bonds which are payable after 1901. The interest on them is at the rate of 4% in gold, and is payable quarterly.

606. Bonds issued by cities, counties, States and corporations are usually named according to the rate of interest they bear.

Thus, Virginia 6's are bonds bearing interest at 6%, issued by the State of Virginia.

WRITTEN EXERCISES.

607.—Ex. 1. What is the cost, including brokerage, of 400 shares of railroad stock, at 95%?

SOLUTION.

$$(95\% + \tfrac{1}{4}\%) \textit{ of } \$100 = \$95.25, \textit{ cost of 1 share.}$$

$$\$95.25 \times 400 = \$38100, \textit{ cost of 400 shares.}$$

2. What is the market value of 50 shares of National Bank stock, at 115?

3. How much, including brokerage, must be paid for \$1000 Maine 6's at 101?

SOLUTION.

$(101\% + \frac{1}{4}\%)$ *of* $\$1 = \$1.01\frac{1}{4}$, *amount paid for* $\$1$.

$\$1.01\frac{1}{4} \times 1000 = \$1012.50$, " " $\$1000$.

4. When gold is quoted at $112\frac{1}{2}$, what is the value in currency of \$5000 in gold? *Ans.* \$5625.

5. I bought Government securities of the par value of \$3000 at $100\frac{3}{4}$, and sold them at $109\frac{1}{4}$. How much did I gain? *Ans.* \$255.

6. When gold is worth $112\frac{1}{2}$, what is the value in gold of \$5625 in currency?

SOLUTION.

At $112\frac{1}{2}\%$, $\$1.12\frac{1}{2}$ *in currency* $= \$1$ *in gold.*

$\$5625 \div \$1.12\frac{1}{2} = 5000$, *the number of dollars in gold.*

7. When gold is at 150, what is the value in gold of \$1 in currency? *Ans.* $\$.66\frac{2}{3}$.

8. When gold is at 125, what is the value in currency of \$1000 in gold?

9. When the cost of 400 shares of railroad stock, including brokerage, is \$38100, what is the market value per share?

SOLUTION.

$\frac{1}{4}\%$ *of* $\$100 \times 400 = \$100$, *the brokerage.*

$\$38100 - \$100 = \$38000$, *market value of* 400 *shares.*

$\$38000 \div 400 = \$95$, *market value of* 1 *share.*

10. For how many shares of telegraph stock, including brokerage, will \$1524 pay when the stock is selling at 95? *Ans.* 16.

11. For how many shares of telegraph stock, including the brokerage, will \$1524 pay, the stock being at 5% discount?

12. I bought 120 shares of stock at 98, and paid the brokerage. How much did I gain by selling the same at 103?

13. I bought 50 shares of stock at 95, and sold them at par, receiving in the mean time a dividend of 4%. What was the profit? *Ans.* $450.

14. When a certain 5% stock is at 85, how much must be invested in it to yield an annual income of $650?

SOLUTION.

$100 × .05 = $5, income of 1 share of 5% stock.

$650 ÷ $5 = 130, number of shares to yield $650.

$85 × 130 = $11050, the sum which must be invested.

15. What sum must be invested in U. S. $4\frac{1}{2}$% bonds, at 96, brokerage at $\frac{1}{4}$%, to secure an annual income of $900?

Ans. $19250.

16. When U. S. 10-40's are quoted at 106, what sum must be invested in them to secure an annual income of $400?

17. How much must be invested in Central Railroad 7's, at 90, to produce an annual income of $350? *Ans.* $4500.

18. What per cent. on the investment will an 8% stock, at 120, yield?

SOLUTION.

$1 of stock at 100 yields 8%.

$1 at 120 must yield $\frac{100}{120}$ of 8%, or $6\frac{2}{3}$%.

19. What per cent. will U. S. 4's, purchased at 80, yield?

20. What is the rate of income upon money invested in railroad 7's, at $87\frac{1}{2}$?

21. Which will pay the better rate of income—U. S. $4\frac{1}{2}$'s, at $89\frac{3}{4}$, or a 6% stock, at $119\frac{3}{4}$, the brokerage of each being $\frac{1}{4}$%?

SOLUTION.

$$\textit{Income of } 4\tfrac{1}{2}\text{'s} = \frac{100}{89\frac{3}{4}+\frac{1}{4}}, \text{ or } \frac{100}{90} \text{ of } 4\tfrac{1}{2}\% = 5\%.$$

$$6\text{'s} = \frac{100}{119\frac{3}{4}+\frac{1}{4}}, \text{ or } \frac{100}{120} \text{ of } 6\% = 5\%.$$

Ans. Income same from both.

22. At what rate must I buy a 5% stock that I may receive 8% on my investment?

23. For how much above par must a 6% stock sell to pay an interest of 5% on the investment? *Ans.* 20%.

24. For how much below par must a 7% stock be bought to pay an interest of 8% on the investment?

25. At what rate must U. S. 5's be bought to secure $4\frac{1}{2}$% on the investment?

26. What rate of interest does an investment in U. S. $4\frac{1}{2}$'s, at 96, pay?

27. When gold is at 120, what must be the price of U. S. 5-20's to yield $5\frac{1}{2}$% in currency? *Ans.* $130\frac{1}{11}$.

SECTION LXI.

EXCHANGE.

608. A **Draft** is a written order by one party to another to pay a certain sum of money to a third party or to his order.

609. The **Drawer** is the maker of the draft.

610. The **Drawee** is the party ordered to pay.

611. The **Payee** is the party to whom, or to whose order, the draft is payable.

The drawee *accepts* a draft by writing his name across the face of it. This denotes his agreement to pay the draft.

The payee may *assign* a draft by writing his name upon the back of it. This makes him *security* for its payment.

612. A **Sight Draft** is a draft payable when presented.

613. A **Time Draft** is a draft payable at a time named in the draft.

Time drafts are entitled to three days of grace, but grace is not usually allowed on sight drafts.

When a draft is drawn at *usance*, it is entitled to the time allowed by custom or by the law of the place where it is payable.

614. **Exchange** is the process of remitting value from one place to another by means of drafts.

FORM OF A DRAFT.

	$\$565\frac{75}{100}$. WILMINGTON, *Aug.* 4, 1871. ———*Ten days after Sight, pay to the order of Moore, Merrill & Co.,*——— *Five Hundred Sixty-Five*——$\frac{75}{100}$ *Dollars.* *Value received, and charge to account of* *Henry Hartwell.* *To Cook & Farnham,* *Washington.*

Omit in the above the words "ten days after sight," or insert in their stead "at sight," and the form is that of a *Sight Draft.*

615. The **Course of Exchange** is the variation between the face of a draft, or bill, and its cost.

Exchange is at *par* when a draft sells for its face; at a *premium* when it sells for more than its face; and at a *discount* when it sells for less than its face.

DOMESTIC OR INLAND EXCHANGE.

616. Domestic or **Inland Bills** are drafts payable in the country in which they are drawn.

617. In the **Computation** of the cost of an inland bill, the *Base* is the face of the bill, and the *Rate* is the rate per cent. of premium or discount.

The cost of a bill at par is the face of the bill; at a premium, the face plus the premium; and at a discount, the face minus the discount. Hence the following—

618. Rules for Inland Exchange.—*1. To find the cost of a bill, multiply the cost of \$1 of exchange by the number denoting the face of the bill.*

2. To find the face of a bill that can be bought for a given sum, divide the given sum by the number denoting the cost of \$1 of exchange.

PROBLEMS.

1. What will be the cost of a sight draft on New York for \$5850, at $\frac{1}{2}\%$ premium? *Ans.* \$5879.25.

2. When exchange is at $\frac{1}{2}\%$ premium, what is the face of a sight draft that costs \$5879.25?

3. How much must be paid in Pottsville for a draft of \$750 on Pittsburg, exchange being at $1\frac{1}{2}\%$ discount? *Ans.* \$738.75.

4. What is the face of a draft that can be purchased for \$4301, when exchange is at $2\frac{1}{4}\%$ discount? *Ans.* \$4400.

5. What must be paid in Vicksburg for a draft of \$600 on St. Louis, at 60 days, exchange being at 101, and interest at 9%?

SOLUTION.

Bank discount of \$1 for 63 days, at 9% = \$.01575.

\$1 − \$.01575 = \$.98425, cost of \$1 at par.

\$.98425 + \$.01 = \$.99425, cost of \$1 at 101.

\$.99425 × 600 = \$596.55, cost of the draft.

Or,

Bank discount of \$600 for 63 days, at 9% = \$9.45.

\$600 − \$9.45 = \$590.55, cost of draft at par.

\$600 × .01 = \$6, premium of draft at 101.

\$590.55 + \$6 = \$596.55, cost of the draft.

6. What will be the face of a draft payable 60 days after sight, that can be bought for \$596.55, exchange being 1% premium, and interest 9%?

7. How much must be paid for a draft of \$750, payable 10 days after sight, exchange being at $\frac{1}{2}\%$ discount, and interest at 6%? *Ans.* \$744.62½.

8. I owe a note, in New York, of \$3000, with interest for 1 year at 7%. What must be the face of a sight draft, exchange at $101\frac{1}{4}$, which I can remit, and thus exactly discharge the note and interest? *Ans.* \$3250.12½.

FOREIGN EXCHANGE.

619. Foreign Bills are drafts drawn in one country and payable in another.

620. The **Par of Exchange** between two countries is the value of the currency of one country estimated in the currency of the other.

621. Bills of Exchange between the United States and foreign countries for convenience are generally drawn and negotiated on London or Paris.

Foreign bills are drawn in sets of three, of the same tenor and date, and called, respectively, *First, Second* and *Third* of Exchange. They are forwarded differently, to prevent delay by accident. When one is paid, the others are cancelled.

BILLS ON ENGLAND.

622. Bills on England, or London, are drawn in English, or sterling money, the denominations of which are shown in the following—

TABLE.

4 farthings (qr. or far.)	*are*	*1 penny*	*. . . d.*
12 pence	"	*1 shilling*	*. . s.*
2 shillings	"	*1 florin*	*. . . fl.*
10 florins, or 20 shillings,	"	*1 pound*	*. . . £.*

623. The **Value** of a pound sterling, which is represented by the English gold coin called the *Sovereign*, previous to the change in the United States coinage in 1834, was $\$4\frac{4}{9}$, or $\$4.44\frac{4}{9}$.

In the present gold coinage of the United States a sovereign of standard weight is equal to \$4.8634. Allowing for the wear of coins, we have what the Government has established as the Custom-house or legal value of the pound, which is \$4.84.

624. In the **Computation** of Exchange the old value of $\$4\frac{4}{9}$ is usually considered the *Base.*

Hence, when English exchange is quoted at 109, it is at the Custom-house value, and when at $109\frac{1}{2}$, it is at about the intrinsic value, or true par.

WRITTEN EXERCISES.

625.—Ex. 1. What is the cost in New York of a bill of exchange on London of £400 10 s. 6 d., at 109¾, including brokerage at ¼%?

£400 10 s. 6 d. = £400.525

$\frac{\$40 \times 1.10}{9} \times 400.525 = \$1958.12+$

SOLUTION.—£400 10 s. 6 d., decimally expressed, is £400.525. £1, at 109¾, and brokerage at ¼%, will cost $\frac{\$40 \times 1.10}{9}$. Hence, £400.525 will cost 400.525 times $\frac{\$40 \times 1.10}{9}$, or \$1958.12 +.

2. What is the face of a bill on London which, at 109¾, and brokerage at ¼%, can be bought for \$1958.12?

$\$1958.12 \div \frac{\$40 \times 1.10}{9} = 400.525$

£400.525 = £400 10 s. 6 d.

SOLUTION.—£1 of face of exchange, at 9¾ premium, can be bought for $\frac{\$40 \times 1.10}{9}$. Hence, as many pounds of face can be bought for \$1958.12 as is denoted by the quotient of $1958.12 \div \frac{\$40 \times 1.10}{9}$, or £400.525 = £400 10 s. 6 d.

3. What will a bill on London for £2000 cost, at 108?

626. Rules for Exchange on England.—*1. To find the cost of a bill, multiply the cost of £1 at the given rate by the number denoting the face of the bill, expressed in pounds and the decimal of a pound.*

2. To find the face of a bill that can be bought for a given sum, divide the given sum by the cost of £1 of exchange at the given rate.

PROBLEMS.

1. What will be the cost in United States money of a bill on England of £7000, exchange at 111? *Ans.* \$34533.33⅓.

2. When exchange is at 108½, how much English money will \$4822⅔ purchase? *Ans.* £1000.

3. When exchange is at 111, what will be the face of a bill on England that can be purchased for \$34533.33⅓?

4. How much must be paid in United States currency for a sterling bill of £505 15 s. 6 d., exchange at 108¾, brokerage ¼%, and gold at 110? *Ans.* $2695.22.

BILLS ON FRANCE.

627. **Bills on France,** or **Paris,** are drawn in the money of France, the denominations of which are as in the following—

TABLE.

100 centimes are 1 franc . . . fr.

628. Francs and centimes may be written together after the manner of dollars and cents.

Thus, 25.20 fr. expresses twenty-five francs twenty centimes.

629. The **Franc** in silver, which is the standard in France, is valued at the United States Custom-house at $.186.

The intrinsic value of a franc in silver, if new, is $.196, so that about 5.10¼ francs are equal to $1.

630. Exchange on France is quoted at a certain number of francs to a dollar.

Thus, when exchange on Paris is quoted at 5.16, it is at 5.16 francs to a dollar, and is above par.

The value in the United States of some of the Foreign Coins often named in quotations of foreign markets is given in the following—

TABLE.

PLACES.	NAME OF COIN.	VALUE.
Great Britain	*1 pound = 20 shillings*	*$4.8634*
France	*1 franc = 100 centimes*	*.196*
Prussia	*1 thaler (new)*	*.7289*
Germany	*1 florin or guilder*	*.4165*
Russia	*1 rouble*	*.7944*
Portugal	*1 milrei*	*1.18*
Spain	*1 real*	*.05*
Turkey	*1 piastre*	*.0439*
Mexico	*1 dollar (new)*	*1.0622*
East Indies	*1 rupee*	*.4662*
China	*1 tael*	*1.48*

WRITTEN EXERCISES.

631.—Ex. 1. How much must be paid for a bill on Paris for 2565 francs, exchange being at 5.13? *Ans.* $500.

2. What will a bill cost in currency on Paris for 10300 francs, exchange being at 5.15, and gold at 120?

3. What is the face of a bill on Paris that can be purchased for $2400 in currency, when exchange is at 5.15, and gold at 120?

4. How much French exchange at 5.18 can be bought for $600? *Ans.* 3108 francs.

TEST QUESTIONS.

632.—1. What is STOCK? What is the par value of stock? The market value? When does gold become an object of investment? What is a broker?

2. What is a CORPORATION? What is a corporate stock? A share? An installment? An assessment? What are the gross earnings of a company? The net earnings? What is a dividend?

3. Of what do United States GOVERNMENT SECURITIES consist? What are bonds? Coupons? Treasury notes? According to what are the bonds issued by cities, counties, states and corporations named?

4. What is EXCHANGE? What is a draft? A sight draft? A time draft? Who is the drawer of a draft? The drawee? The payee?

5. How is a draft accepted? How may a draft be assigned? What kind of a draft is entitled to grace? What is the course of exchange?

6. What are INLAND BILLS? What is the base of a bill? The rate? What are the rules for inland exchange?

7. What are FOREIGN BILLS? What is the par of exchange between two countries? How are foreign bills drawn and negotiated?

8. In what are BILLS ON ENGLAND drawn? By what is the value of a pound sterling represented? To what, in the present gold coinage of the United States, is a sovereign equal?

9. In the COMPUTATION of exchange on England, what value of a pound is considered the base? What is the value of exchange when quoted at 109? When quoted at 109½? What are the rules for exchange on England?

10. In what are BILLS ON FRANCE drawn? How may francs and centimes be written? What is the value of a franc in silver? How is exchange on France quoted?

SECTION LXII.

GENERAL TAXES.

633. Real Estate is such property as houses, lands, mills and mines.

634. Personal Property is movable property, as money, stocks, cattle and ships.

635. A **Tax** is a sum of money imposed on persons or property for public purposes.

636. A **Property Tax** is a tax on property, and is reckoned at a certain rate per cent. on the estimated value of the property.

637. An **Income Tax** is a tax on an income, and is reckoned in the same manner as a property tax.

638. A **Poll Tax** is a tax on the person of all male citizens not exempt by law.

In some States no poll or capitation tax is imposed, and in others, as in Vermont, each taxable poll is reckoned as a certain amount of property. In Massachusetts, one-sixth part of the tax to be raised is assessed on the polls, provided the poll tax of one individual, except highway taxes separately assessed, for the year shall not exceed \$2.

639. Assessors are officers appointed to estimate the value of the taxable property, to make a list of taxable polls, if required, and to apportion the tax to be raised among the taxpayers.

640. General Taxes are such as are imposed for city, town, district, county or state purposes.

WRITTEN EXERCISES.

641.—Ex. 1. A tax of \$21900 is to be imposed on a certain town. The taxable property and incomes amount to \$1500000. There are 450 taxable polls, each to be assessed \$2. Find the tax of S. A. Potter, W. B. Rice, and D. J. Snyder, according to the following—

INVENTORY.

NAMES.	No. POLLS.	REAL ESTATE.	PER'L ESTATE.	INCOME.	TOTAL.
S. A. Potter,	*2*	*$15000*	*$31000*	*$3160*	*$49160*
W. B. Rice,	*1*		*12550*	*850*	*13400*
D. J. Snyder,	*1*	*9300*			*9300*

SOLUTION.

$2 × 450 = $900, sum to be assessed on the polls.
$21900 − $900 = $21000, sum to be assessed on valuation.
$21000 ÷ $1500000 = .014, rate of taxation on valuation.
$49160 × .014 = $688.24, S. A. Potter's tax on valuation.
$688.24 + $4 = $692.24, S. A. Potter's entire tax.
$13400 × .014 = $187.60, W. B. Rice's tax on valuation.
$187.60 + $2 = $189.60, W. B. Rice's entire tax.
$9300 × .014 = $130.20, D. J. Snyder's tax on valuation.
$130.20 + $2 = $132.20, D. J. Snyder's entire tax.

2. A tax of $31000 is imposed on a certain town. The taxable property amounts to $3720000. There are 658 taxable polls, each of which is to be assessed $1.50. What will be the tax on each $1 of valuation? What is S. M. Allen's tax, whose valuation is $12500, and who pays for one poll?

642. Rule for General Taxes.—*Subtract the amount of the poll taxes, if any, from the whole tax, and the remainder will be the tax on valuation.*

Apportion the tax on valuation among the property owners according to the valuation of their property, and add their poll tax, if any.

The computation of taxes may be facilitated by constructing a table showing the tax on $1, $2, $3, etc., from which to compute the individual taxes.

TABLE,

For a Rate of 14 Mills on a Dollar.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
$1	$.014	$7	$.098	$40	$.56	$100	$1.40	$800	$11.20
2	.028	8	.112	50	.70	200	2.80	900	12.60
3	.042	9	.126	60	.84	300	4.20	1000	14.
4	.056	10	.14	70	.98	500	7.00	2000	28.
5	.070	20	.28	80	1.12	600	8.40	3000	42.
6	.084	30	.42	90	1.26	700	9.80	4000	56.

PROBLEMS.

1. Find J. Holt's tax, whose valuation is $3060, and who pays for 2 polls, at $1.50.

SOLUTION.

Tax by Table on $3000 = $42.00
" " 60 = .84
" " $3060 = $42.84
Tax on 2 polls @ $1.50 = 3.00
Entire tax = $45.84.

2. Find Walter Hartman's tax, whose valuation is $5350, and who pays for 1 poll, at $1.25.

3. A's personal estate is valued $5600, and his real estate $5000. What is his tax, if the rate of taxation on the real estate is .015, and the personal estate is taxed twice as high as the real estate? *Ans.* $243.00.

4. A tax of $6000 is to be assessed on a town having 800 polls and a valuation of $500000. If one sixth of the tax be laid on the polls, what will be the tax on $1, and how much will be each poll tax?

5. C. Washburn is assessed on $5760 of property and on a taxable income of $1240. What is his tax, if the tax on $1 of valuation is $.016, and his poll tax is $2? *Ans.* $114.00.

SECTION LXIII.

NATIONAL TAXES.

643. CUSTOMS, or DUTIES, are taxes upon imported goods and upon the tonnage of vessels.

644. An **Ad Valorem Duty** is a certain per cent. on the net invoice value of an imported article.

645. A **Specific Duty** is a uniform tax on certain imported articles.

646. **Tare** is an allowance of the weight of a cask, bag or case containing a commodity, and which has been weighed with it.

647. **Breakage** is an allowance for such breakage of bottles which contained liquors as is actually ascertained and certified by a custom-house appraiser.

All allowances, in general, for waste, as by leakage, are such as may be actually ascertained. But no allowance is made for the loss by decay of such fruits as oranges, lemons and bananas, except when in excess of 25 per cent. of the whole quantity.

648. **Internal Revenue** is the revenue of Government derived from tax on incomes, business licenses, stamps, imposts on manufactured products, etc.

649. **National Taxes** are the taxes imposed by the National Government.

PROBLEMS.

1. What is the duty, at 2 cents per pound, on 5400 pounds of ginger-root, tare allowed being 5%? *Ans.* $102.60.

2. Lombard, Brainard & Co. imported from Havana 80 hogsheads of molasses, each containing 63 gallons, duty 5 cents per gallon; 60 boxes of sugar, each weighing 450 pounds, duty $2\frac{1}{4}$ cents per pound, and 500 boxes of oranges, invoiced at $1.50 per box, duty 20% ad valorem. Required the amount of duty on the whole. *Ans.* $1144.50.

3. The income of David Welch for the year 1870 was

\$6360. Deductions were made on \$2000 exempted by law, \$500 for house rent, \$125 for insurance, \$425 for repairs, \$960 for labor and \$200 for taxes. Required the national tax on the balance at $2\frac{1}{2}\%$.

4. What is the duty on 500 dozen bottles of champagne wine, each bottle containing 1 quart 1 pint, the duty at \$6 per dozen, and at the rate of \$2 per gallon on the quantity in excess of 1 quart per bottle, the breakage being 19 bottles?

Ans. \$4485.75.

5. Stockton & Bradley have imported 600 pounds of prunes, and 400 pounds of Zante currants, duty on each $2\frac{1}{2}$ cents per pound; and 750 boxes of Sicily oranges, invoiced at \$4 per box, duty 20% ad valorem. Required the duty on the whole, provided the loss of the oranges by decay was 35% of the whole quantity.

SECTION LXIV.

REVIEW PROBLEMS.

MENTAL EXERCISES.

650.—Ex. 1. What is the ratio of $\frac{3}{4}$ to $\frac{3}{12}$? Of $\frac{2}{5}$ to $\frac{1}{20}$?

2. Divide 42 into two parts that shall be in the ratio of $\frac{2}{7}$ to $\frac{3}{14}$.

3. Two men purchase 80 pounds of sugar. One pays $\$2\frac{1}{2}$, and the other $\$7\frac{1}{2}$. What is each man's share of it?

4. The means of a proportion are 6 and 8. One of the extremes is 7. What is the other extreme?

5. Three men hired a pasture for \$108. A put in 3 oxen, B, 4 oxen, and C, 5 oxen. How much should each man pay?

6. A furnished 2 loaves for dinner, and B furnished 3, while C contributed 30 cents to be divided between A and B. How much should each receive?

7. A man wishing to contribute money to an equal number of poor men and women, gave to each man 7 dimes, and to each woman 5 dimes. If he gave them in all \$6, how many men and women were there respectively?

8. Divide \$108 among A, B and C, so that B may get 3 times as much as A, and C 4 times as much as A.

9. A and B hired a field for \$96. A put in 2 horses for 3 months, and B put in 2 horses for 5 months. How much should each man pay?

10. Two men, A and B, trade together. The contribution of A to that of B is as $\frac{1}{2}$ to $\frac{1}{4}$, and A's money was in the business 3 months, and B's 4 months. They gained \$400. What were their respective shares of it?

11. Bryant owes Forster \$50 payable in 6 months, and \$100 payable in 3 months. What is the average time for paying the whole by one payment?

12. A friend has loaned me \$500 for 6 months. How long should I loan him \$300 to requite the favor?

13. The rate of taxation in a certain town is 9 mills on a dollar of valuation. What is the rate for \$100 of valuation, and what is Johnson's entire tax, whose valuation of property is \$2000, and his poll tax \$2?

14. I bought 50 shares of stock at 97, and sold it at 101. What did I gain, allowing $\frac{1}{4}\%$ brokerage on each transaction?

15. If you gain \$200 by selling 50 shares of stock for 101, at what rate did you buy the shares?

16. Considering the value of a pound sterling to be \$4.86, and the value of 5 francs to be \$.98, how much less do 25 francs differ in value from \$5, than does £1?

WRITTEN EXERCISES.

651.—Ex. 1. Which is the greater ratio, 7 : 22 or 113 : 355?

Ans. The latter.

2. What is the ratio of 39 to 52, expressed in its lowest terms?

3. The second, third and fourth terms of a proportion are 17, 11 and $93\frac{1}{2}$. What is the first term?

4. A man purchased a horse, a cow and a sheep for \$205. The cow was valued at as much as 10 sheep, and the horse at as much as 3 cows. What was the price of each?

Ans. Sheep, \$5; cow, \$50; horse, \$150.

5. If $\frac{12}{25}$ of a dollar will pay for $\frac{4}{5}$ of a bushel of apples, for what part of a bushel will $\frac{7}{20}$ of a dollar pay? *Ans.* $\frac{7}{12}$.

6. A surveyor having measured a line by a chain, finds it to be 1584 yards, but on examining his chain, sees it deficient two entire links, besides several not straight—the whole error amounting to 18 inches. What is the true length of the line, a chain measuring 66 feet? *Ans.* 1548 yards.

7. If 18 men can construct 150 rods of road in 25 days, how many men will be required to construct 120 rods in 15 days?

8. If 15 bushels of corn be worth as much as 2 barrels of flour, and 4 barrels of flour as much as 5 tons of coal, how many bushels of corn should be given for 27 tons of coal?

$$15\ bu. = 2\ bbl.$$
$$4\ bbl. = 5\ T.$$
$$27\ T. = x\ bu.$$
$$\frac{\overset{3}{\cancel{15}} \times \overset{2}{\cancel{4}} \times 27}{\cancel{2} \times \cancel{5}} = 162\ bu.$$

SOLUTION.—By arranging the terms of equal value so that denominations of the same kind shall stand on opposite sides of the sign of equality, it is evident, if the value of 27 tons were known, that the product of the values on one side of the sign would be equal to the product of those on the other.

Hence, to find the required term, divide the product of the terms on the other side by the product of the given terms on its own side.

This method is sometimes called CONJOINED PROPORTION.

9. How many pounds of tea must be given for 28 pounds of rice, if 4 pounds of sugar are worth 1 pound of coffee, 15 pounds of sugar are worth 14 pounds of rice, and 30 pounds of coffee are worth 7 pounds of tea? *Ans.* $1\frac{3}{4}$.

10. If 15 women, working 12 hours daily, can gather 60 bushels of cranberries in 16 days, in how many days can 20 boys, working 10 hours daily, gather 98 bushels, 7 women being able to do as much as 8 boys in the same time?

Ans. $26\frac{22}{25}$.

11. If \$1 is worth 4 s. $1\frac{1}{2}$ d., and is also worth 5 francs 17 centimes, what is the value of a franc in sterling money?

12. William Hunt owes me \$3000, payable in 6 months. Should he, to accommodate me, pay one third of the amount down, how long in equity should he be permitted to retain the remainder? *Ans.* 9 months.

13. A and B are in partnership; A's stock of \$560 continued in trade 15 months, and drew a profit of \$36; B's stock of \$320 was in trade 21 months. What should be the amount of B's gain? *Ans.* \$28.80.

14. Bought of Messrs. Bancroft, Wright & Co., Sept. 1, 1870, a bill of \$756 on 4 months; Sept. 20, a bill of \$144 on 2 months; Oct. 3, a bill of \$567 on 5 months; and Oct. 16, a bill of \$128 on 6 months. What would be the discount, at 6%, on a note for the balance, dated September 1, and maturing at the average time of payment?

15. An American in London bought £16000 consols, an English government security, at $93\frac{5}{8}$, and sold out at $94\frac{1}{8}$. What was his profit in United States money, allowing for brokerage $\frac{1}{8}$% on each transaction, and \$4.86 as the value of a pound sterling? *Ans.* \$194.40.

16. What will be the cost of a draft of \$575 at 60 days, exchange at $101\frac{3}{4}$, and interest at 7%?

17. Which is the more profitable stock for investment—the U. S. 4's, at 85, or the French Rentes 3's, at 65?

Ans. U. S. 4's.

18. How many francs are worth £1, if $49\frac{1}{2}$ d. buy \$1, and \$200 bring 1034 francs?

19. At what must gold sell, that an investment in U. S. 5's, at 95, may yield an interest of 6% in currency?

Ans. 114.

20. For what price must a 10% stock sell to pay an interest of 8% on the investment?

21. Sixty gallons of alcohol are mixed with 14 gallons of water. What weight of alcohol is there in every pound of the mixture, the weights of equal measures of alcohol and water being in the ratio 6 : 5?

22. The assessment rolls of a town show the value of the taxable property to be \$1000500, and the number of polls to be 600. A tax of \$15207 is to be raised. What will be the rate on property, allowing that each poll shall pay \$2? What is A's entire tax, who has 1 poll, and whose property is valued at \$6500? *Ans.* Rate on property, .014; A's tax, \$93.

SECTION LXV.

INVOLUTION.

652.—Ex. 1. What is the result of taking 3 twice as a factor?

2. What is the result of taking 5 twice as a factor?

3. What is the result of taking 2 three times as a factor?

4. What is the result of taking 2 four times as a factor?

5. What number will be produced by taking 5 three times as a factor?

DEFINITIONS.

653. A **Power** of a number is the result obtained by using it a certain number of times as a factor.

Thus, 16 is a power of 4, since it is obtained by using 4 twice as a factor.

654. The *Powers* of a number are named from the number of times it is used as a factor.

Thus, 4 is the *second power* of 2, 8 the *third power*, and 16 the *fourth power*, since 2 is used *twice* in obtaining the first number, *three* times in obtaining the second, and *four* times in obtaining the third.

655. The second power is frequently called the **Square**, from the method of finding the area of a square; and,

The third power is frequently called the **Cube**, from the method of finding the contents of a cube.

656. The **Exponent** of a power is a figure or figures placed at the right and above the number, to show the number of times the given number is to be used as a factor.

Thus, in the expression 3^2, 2 is the exponent, and indicates that the square of 3 is to be found.

The powers of numbers are found by using each number as many times as a factor as is indicated by the exponent.

Any number that is the product of equal factors is called a *Perfect Power*.

Any number that is not the product of equal factors is called an *Imperfect Power*.

657. **Involution** is the process of finding a power of a number.

WRITTEN EXERCISES.

658.—Ex. 1. Find the third power of 25.

$$\begin{array}{rl} 25 & = \text{1st power.} \\ \underline{25} & \\ 125 & \\ \underline{50} & \\ 625 & = \text{2d power.} \\ \underline{25} & \\ 3125 & \\ \underline{1250} & \\ 15625 & = \text{3d power.} \end{array}$$

SOLUTION.—The third power of 25, or 25^3, is equal to the product of 25 taken 3 times as a factor. Hence, 15625, the equal of $25 \times 25 \times 25$, is the required power.

2. What is the square of 31?
3. What is the cube of 15? *Ans.* 3375.

659. Rule for Involution.—*Use the given number as many times as a factor as is indicated by the exponent of the required power.*

PROBLEMS.

1. What is the third power of 57? *Ans.* 185193.
2. Raise 302 to the second power.
3. What is the square of 962? *Ans.* 925444.
4. Raise 407 to the fifth power. *Ans.* 11167913618807.
5. Raise $\frac{2}{3}$ to the fourth power.

SOLUTION.

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}, \text{ the required power.}$$

6. Raise $\frac{9}{84}$ to the third power. *Ans.* $\frac{729}{592704}$.
7. What is the value of $(\frac{3}{4})^6$? *Ans.* $\frac{729}{4096}$.
8. What is the fourth power of 1.05 to four orders of decimals? *Ans.* 1.2155.

Here all the figures of the process are retained, but the final result is expressed approximately only to the nearest decimal of the fourth order.

9. What is the value of 1.06^5 to three orders of decimals? *Ans.* 1.338.

SECTION LXVI.

EVOLUTION.

660.—Ex. 1. What are equal factors of 25? Of 49? Of 343?

2. Sixteen is the result of taking 2 how many times as a factor?

3. One hundred twenty-five is the result of taking 5 how many times as a factor?

DEFINITIONS.

661. The **Root** of a number is one of the equal factors of that number.

Thus, 8 is a root of 64, since it is one of the two equal factors of that number.

662. If a number is used *twice* as a factor to produce a certain number, it is called the *second* or *square root* of that number; if *three* times, the *third* or *cube root;* and so on.

Thus, 4 is the second or square root of 16, since it is used twice as a factor in producing 16; the third or cube root of 64, since it is used three times as a factor in producing this number.

663. The roots of numbers are indicated by the character, $\sqrt{\ }$, called the **Radical Sign.**

If no figure is written in the opening of the sign, the square root is indicated; if the figure 3 is placed there, as $\sqrt[3]{\ }$, the cube root; if 4, the fourth root; and so on.

664. An **Index** of a root is the figure or figures which indicate the root.

No imperfect power can have an exact root; and a root that can only be approximately obtained is called a *Surd Root,* or *Surd.*

665. **Evolution** is the process of finding the roots of numbers.

The roots of small numbers which are not imperfect powers may be readily found by factoring.

WRITTEN EXERCISES.

666.—Ex. 1. Find the cube root of 3375.

$$\begin{array}{r} 3)\overline{3375} \\ 3)\overline{1125} \\ 3)\overline{375} \\ 5)\overline{125} \\ 5)\overline{25} \\ 5 \end{array}$$

$5 \times 3 = 15$

SOLUTION.—The prime factors of 3375 are 3, 3, 3, 5, 5 and 5.

Since the cube root of a number is one of its three equal factors, the cube root of 3375 must be 5×3, or 15.

2. Find the square root of 4096. *Ans.* 64.
3. Find the cube root of 74088. *Ans.* 42.
4. Find the fourth root of 1296. *Ans.* 6.
5. Find the fifth root of 537824. *Ans.* 14.

GENERAL METHOD FOR SQUARE ROOT.

667. A **General Method** of finding the square root of numbers may be deduced from raising numbers of different orders of units to the second power, and observing the various relations existing between the roots and their squares.

668. The square of 1 unit of units, or 1, is 1; of 1 unit of tens, or 10, is 100; of 1 unit of hundreds, or 100, is 10,000; of 1 unit of thousands, or 1000, is 1000000; and so on, from which it may be seen that

The square of every number between 1 and 10 must be a number between 1 and 100; of every number between 10 and 100 must be a number between 100 and 10000; of every number between 100 and 1000 must be a number between 10000 and 1000000; and so on. Hence,

669. The square of a number of units must be found either in the order of units, or in the orders of tens and units; the square of the tens, in the order of hundreds, or in the orders of thousands and hundreds; of the hundreds, in the order of ten-thousands, or in the orders of hundred-thousands and ten-thousands; and so on. Or,

Numbers with	*1* figure	have in their	squares	*1* or *2*	figures,
"	*2* figures	"	"	*3* or *4*	"
"	*3* "	"	"	*5* or *6*	"
"	*4* "	"	"	*7* or *8*	"

and so on.

670. Take any number composed of tens and units, as 37, and squaring it, as in actual multiplication, notice of what parts the square is composed and the order in which these parts are found. Thus,

$$\begin{array}{r} 37 = 3 \text{ tens} + 7 \text{ units} \\ 3 \text{ tens} + 7 \text{ units} \\ \hline 3 \text{ tens} \times 7 \text{ units} + 7 \text{ units}^2 \\ 3 \text{ tens}^2 + 3 \text{ tens} \times 7 \text{ units} \qquad\qquad \\ \hline 3 \text{ tens}^2 + 2 \times 3 \text{ tens} \times 7 \text{ units} + 7 \text{ units}^2. \end{array}$$

Or, $$37^2 = 30^2 + 2 \times 30 \times 7 + 7^2;$$

hence, $$37^2 = 900 + 420 + 49 = 1369.$$

Writing t for tens and u for units, and omitting the figures indicating the number of the tens and units, we have

$$(t + u)^2 = t^2 + 2 \times t \times u + u^2.$$

Or, more concisely,

$$(t + u)^2 = t^2 + 2tu + u^2.$$

671. Principles.—1. *The number of figures in the square root of a number is one half of the number of figures in the square, or one half of one more than the number of figures in the square.*

2. *If the square of an integer be expressed by two or more figures, the first two orders must contain the square of the units; the next two higher orders, the square of the tens; the next two higher orders, the square of the hundreds; and so on. Hence,*

3. *If a square be separated into periods of two figures each, commencing with the units, the number of figures in the root is*

indicated, and also the orders of the square in which are to be found the squares of the numbers of the orders of the root.

4. *Every square consisting of more than two orders of figures is equal to the square of the tens in its root, plus twice the product of the tens by the units, plus the square of the units.*

NOTE.—The square of a fraction cannot be integral, or, which is the same thing, an integer cannot have a fractional square root.

No exact square can end in 2, 3, 7, 8, or an odd number of ciphers.

Terminating decimals can only be squares if the number of decimal orders be even.

WRITTEN EXERCISES.

672.—Ex. 1. What is the square root of 2209?

$$
\begin{array}{lrll}
 & & & \quad t \;\; u \\
 & \dot{2}2\dot{0}9 = & t^2 + 2tu + u^2 & (4\;7 \\
 & 4^2 = 16 = & t^2 & \\
2t = 2 \times 4 \text{ tens} = 80 &)609 = & 2tu + u^2 = (2t+u)u & \\
u \qquad\quad = 7 & & & \\
2t + u \qquad = 87 & & & \\
 & 87 \times 7 = 609 = & (2t+u)u & \\
 & 0 & 0 &
\end{array}
$$

SOLUTION.—Since the given number is expressed by 4 orders of figures, it may be separated into two periods, and its square root will consist of 2 figures, and will express tens and units.

Representing the tens by t and the units by u, we have $2209 = t^2 + 2tu + u^2$.

In reversing the process of involution, we must first find the t^2. As this must be a number of hundreds, we find the greatest number contained in the left-hand period that is a square of tens. This number is 16 hundreds, whose root, 4 tens, we write for the tens of the root.

Subtracting the 16 hundreds from the given number, and its equal t^2 from the expression above, 609 remains, which is equal to $2tu + u^2$; or, since u is a common factor of these terms, equal to $(2t + u)\,u$.

Of these two factors, $2t + u$ and u, $2t$ is the only term whose value we can find; if the whole of either could be found, we should use that factor in finding the other. Since $2t$ of the factor $2t + u$ is large compared with the other term, we use it as a trial divisor in finding the value of u.

Now, $2t = 2 \times 4$ tens $= 8$ tens; and dividing the 60 tens of the remainder by the 8 tens, we have 7, which we write for the units of the required root.

Adding to 80, the equal of $2t$, 7, the equal of u, we have 87, and multiplying this by 7, the equal of u, we obtain 609. Subtracting this product and its equal $(2t + u)\,u$ from the quantities above them, we find no remainder. Hence, the square root of 2209 is 47.

2. What is the square root of 55225?

SOLUTION.

$$
\begin{array}{lrcr}
 & \dot{5}5\dot{2}|\dot{2}5 & = t^2+2tu+u^2 & (\overbrace{\overset{t}{2}\,\overset{u}{3}}^{t}\,\overset{u}{5} \\
2^2 = & 4 & = t^2 & \\
\hline
2t = 2\times 2 \text{ tens} = 40) & 152 & = & 2tu+u^2 = (2t+u)u \\
u \qquad\qquad = 3 & & & \\
\hline
2t+u \qquad\qquad = 43 & & & \\
43\times 3 = & 129 & = & (2t+u)u \\
\hline
2t = 2\times 23 \text{ tens} = 460) & 2325 & = & 2tu+u^2 = (2t+u)u \\
u \qquad\qquad = 5 & & & \\
\hline
2t+u \qquad\qquad = 465 & & & \\
465\times 5 = & 2325 & = & (2t+u)u \\
\hline
 & 0 & & 0
\end{array}
$$

Since the given number is expressed by 5 orders of figures, it may be separated into three periods, and its square root will consist of 3 figures, and will express hundreds, tens and units.

Representing the hundreds, or tens of tens, by t, and the units of the tens by u, the square of the root thus represented will equal $t^2 + 2tu + u^2$. This root, then, will be found in 552 units of hundreds.

The greatest number contained in the left-hand period that is a square of tens is 4 hundreds, whose root, 2 tens, we write for the tens of tens of the root. Subtracting 4 hundreds and its equal t^2 from the expression above, 152 hundreds remain, and $2tu + u^2$, which is equal to $(2t + u)u$.

Since $2t$ of the factor $2t + u$ is large compared with the other term, we use it as a trial divisor in finding the value of u. Now, $2t = 2 \times 2$ tens $= 4$ tens, and dividing the 15 tens of the remainder by the 4 tens, we have 3, which we write for the units of the tens of the root.

Adding to $2t$ or 40, 3, the equal of u, we have 43, and multiplying this by 3, the equal of u, we obtain 129. Subtracting 129 from the dividend, and its equal $(2t + u)u$ from the expression above it, and bringing down the remaining period of the power, we have as a remainder 2325.

We now consider 55225 as the square of 23 tens and some number of units of units. Representing the 23 tens of the root, which have been found, by t, and the units of the root to be found, by u, the square of the root is represented by $t^2 + 2tu + u^2$. The equal of t^2, or 230^2, has been subtracted; hence, the remainder, 2325, is equal to $2tu + u^2$, or $(2t + u)u$.

We find the value of $2t$ to be 46 tens, or 460, and making use of it as a trial divisor for finding the value of u, we have for that value 5, which we write for the units of the root.

Adding to 460, the equal of $2t$, 5, the equal of u, and multiplying this by 5, the equal of u, we obtain 2325. Subtracting 2325 from the dividend, and its equal $(2t + u)u$ from the expression above it, we find no remainder. Hence, the square root of 55225 is 235.

If the root had consisted of four or more figures, the explanation of the process of solution would have been similar. Thus, if the root had consisted of four figures, expressing thousands, hundreds, tens and units, we should represent the thousands or tens of hundreds by t, and the units of hundreds by u, and having found their values, should regard the part of the root found, as tens of tens, and represent it by t, and the units of tens by u, and so on.

If the given number had been a decimal fraction, its periods would have been pointed off to the right, beginning with units; for the square of .1 is .01, the square of .09 is .0081, etc.

3. Find the square root of 4096. *Ans.* 64.
4. Find the square root of 64516. *Ans.* 254.

673. Rule for Square Root.—*Point off the given number into periods of two orders each, beginning with the units, and proceeding toward the left and right.*

Find the greatest square in the highest period, considered as units, and place its root at the right for the first figure of the required root. Subtract this square from the highest period, and to the remainder bring down the next period for a dividend.

Take for a trial divisor twice the root already found, considered as tens; divide the dividend, omitting its right-hand order, by the trial divisor, and write the quotient for the second figure of the required root.

To the trial divisor add the part of the root found by it, multiply the result by that part of the root, and subtract the product from the dividend.

Continue the process, if there are other periods, as before.

The trial divisor being less than the true divisor, the probable root found by it may prove too large; if so, diminish by 1 or more, and renew the process.

When 0 occurs in the root, instead of indicating the multiplication by 0 and subtracting, it is simpler to annex a cipher to the trial divisor, and to the dividend bring down another period.

If there be a remainder after all the periods have been used, periods of decimals may be formed by annexing ciphers, and the work continued.

PROBLEMS.

What is the square root—

1. Of 9216? *Ans.* 96.
2. Of 4096?
3. Of .0676? *Ans.* .26.
4. Of 717409? *Ans.* 847.
5. Of 46656? *Ans.* 216.
6. Of 7569?
7. Of 62504836? *Ans.* 7906.
8. Of 21.16? *Ans.* 4.6.

9. Extract the square root of 5 to three decimal orders, or to within less than $\frac{1}{1000}$. *Ans.* 2.236.

10. Extract the square root of .5 to four decimal orders, or to within less than $\frac{1}{10000}$. *Ans.* .7071.

11. A general has 11664 men. How many must he place in rank and file to form them into a square? *Ans.* 108.

12. A square lot contains 18225 square feet. What is the length of its equal sides? *Ans.* 135 feet.

13. What is the value of $\sqrt{.000625}$? *Ans.* .025.

14. A circular garden contains 6561 square feet. What is the length of one side of a square containing the same number of square feet?

15. A man wishes to lay out a farm in a square containing exactly 140 acres 100 square rods. What must be the length of each side? *Ans.* 150 rods.

674. Since the square of a fraction is found by squaring the numerator and denominator, *the square root of a fraction is found by taking the square root of the numerator and denominator.*

16. What is the square root of $\frac{224}{350}$?

SOLUTION.—$\frac{224}{350} = \frac{16}{25}$; $\sqrt{\frac{16}{25}} = \frac{4}{5}$, *Ans.*

17. Find the square root of $\frac{961}{5476}$. *Ans.* $\frac{31}{74}$.

18. Find the square root of $\frac{450}{2048}$. *Ans.* $\frac{15}{32}$.

19. Find the square root of $11\frac{25}{64}$. *Ans.* $\frac{27}{8}$, or $3\frac{3}{8}$.

20. What is the value of $\sqrt{30\frac{1}{4}}$? *Ans.* $\frac{11}{2}$, or $5\frac{1}{2}$.

675. When the numerator and denominator of a fraction are not both squares, to find the approximate square root,

First reduce the fraction to a decimal, and then take the root. Or,

Multiply the numerator by the denominator, and divide the root of the product by the denominator.

21. What is the square root of $\frac{3}{7}$, to within less than $\frac{1}{100}$?

$$\text{Solution.}—\frac{3}{7} = .4285+; \ \sqrt{.4285} = .65+.$$

Or,
$$\sqrt{\frac{3}{7}} = \sqrt{\frac{3 \times 7}{7^2}} = \sqrt{\frac{21}{49}} = \frac{4.58}{7} = .65+.$$

22. Find the square root of $\frac{15}{28}$, to within less than $\frac{1}{1000}$.
Ans. .807.

23. What is the square root of $\frac{357}{476}$, to within less than $\frac{1}{1000}$?

24. What is the square root of $6\frac{2}{5}$, to within less than $\frac{1}{10000}$?

GENERAL METHOD FOR CUBE ROOT.

676. A **General Method** of finding the cube root of numbers may be deduced from raising numbers of different orders of units to the third power, and noticing the various relations existing between the roots and their cubes.

677. The cube of 1 unit of units, or 1, is 1; of 1 unit of tens, or 10, is 1000; of 1 unit of hundreds, or 100, is 1000000; of 1 unit of thousands, or 1000, is 1000000000; and so on. Hence,

The cube of units must be found in the orders of units, tens and hundreds, or in the units' period; of tens, in the orders thousands, ten-thousands and hundred-thousands, or in the thousands' period; of hundreds, in the millions' period; of thousands, in the billions' period, and so on. Or,

Numbers with	*1* figure	have in	their cubes	*1, 2* or *3*	figures.
"	*2* figures	"	"	*4, 5* or *6*	"
"	*3* figures	"	"	*7, 8* or *9*	"
"	*4* figures	"	"	*10, 11* or *12*	"

and so on.

678. Take any number composed of tens and units, as 47, and cubing it as in actual multiplication, notice of what parts the cube is composed, and the orders in which these parts are found. Thus,

$$\begin{array}{r}
47 = 4\ tens + 7\ units \\
4\ tens + 7\ units \\
\hline
4\ tens \times 7\ units + 7\ units^2 \\
4\ tens^2 + 4\ tens \times 7\ units \qquad\qquad\qquad \\
\hline
4\ tens^2 + 2 \times 4\ tens \times 7\ units + 7\ units^2 \\
4\ tens + 7\ units \\
\hline
4\ tens^2 \times 7\ units + 2 \times 4\ tens \times 7\ units^2 + 7\ units^3 \\
4\ tens^3 + 2 \times 4\ tens^2 \times 7\ units + 4\ tens \times 7\ units^2 \qquad\qquad \\
\hline
4\ tens^3 + 3 \times 4\ tens^2 \times 7\ units + 3 \times 4\ tens \times 7\ units^2 + 7\ units^3
\end{array}$$

Writing t for tens, and u for units, and omitting the figures indicating the number of the tens and units, we have

$$(t + u)^3 = t^3 + 3 \times t^2 \times u + 3 \times t \times u^2 + u^3.$$

Or, more concisely,

$$(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3.$$

679. Principles.—1. *The number of figures in the cube root of a number is one third of the number of figures in the cube, or one third of one or two more than the number of the figures in the cube.*

2. *If the cube of an integer be expressed by three or more figures, the first three orders must contain the cube of the units; the next three higher orders, the cube of the tens, the next three higher orders, the cube of the hundreds, and so on. Hence,*

3. *If a cube be separated into periods of three figures each, commencing with the units, the number of figures in the root is indicated, and also the orders of the cube in which are to be found the cubes of the numbers of the given orders of the root.*

4. *Every cube consisting of more than three orders of figures is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

NOTE.—The cube of a fraction cannot be integral.

A cube may be an expression ending in any figure, but if the expression end in 0, it must end in a number of ciphers divisible by 3.

Terminating decimals can only be cubes if the number of decimal orders be divisible by 3.

WRITTEN EXERCISES.

680.—Ex. 1. What is the cube root of 300763?

SOLUTION.

$$
\begin{array}{llrcl}
 & & 30\dot{0}76\dot{3} & = & t^3 + 3t^2u + 3tu^2 + u^3\,(\overset{t}{6}\;\overset{u}{7} \\
 & 6^3 = & 216 & = & t^3 \\ \hline
3t^2 = 3\times 6\text{ tens}^2 = 10800 & &)84763 & = & 3t^2u + 3tu^2 + u^3 = \\
3tu = 3\times 6\text{ tens}\times 7 = 1260 & & & & (3t^2 + 3tu + u^2)u \\
u^2 = 7^2 = 49 & & & & \\ \hline
3t^2 + 3tu + u^2 = 12109 & & & & \\
 & 12109\times 7 = & 84763 & = & (3t^2 + 3tu + u^2)u \\ \hline
 & & 0 & & 0
\end{array}
$$

Since the given number is expressed by 6 orders of figures, it may be separated into 2 periods, and its cube root must consist of 2 figures, expressing tens and units.

Representing the tens by t, and the units by u, 300763 is represented by $t^3 + 3t^2u + 3tu^2 + u^3$.

In reversing the process of involution, we must first find t^3, and as this must be a number of thousands, we find the greatest number contained in the left-hand period that is a cube of tens. This number is 216 thousands, whose root is 6 tens, which we write for the tens of the required root.

Subtracting 216 thousands from the cube, and its equal, t^3, from the expression above, 84763 remains, equal to $3t^2u + 3tu^2 + u^3$, or, since u is a common factor of these terms, equal to $(3t^2 + 3tu + u^2)u$.

Of the two factors of the latter expression, one term, $3t^2$, of the factor within the parentheses can be found; and since this, compared with the other terms, is large, we find and make use of its equal as a trial divisor for finding the equal of the factor u. Now, $3t^2 = 3$ times 6 tens square, or 108 hundreds; and dividing the 847 hundreds of the remainder by the 108 hundreds, we have 7, which we write for the units of the required root.

Adding to the 108 hundreds, 126 tens, the equal of $3tu$, and 49, the equal of u^2, we have 12109; and multiplying this by 7, the equal of u, we obtain 84763, which we write equal to $(3t^2 + 3tu + u^2)u$. Subtracting these quantities from those above them, we find no remainder. Hence, the cube root of 300763 is 67.

2. What is the cube root of 658503?
3. What is the cube root of 970299? *Ans.* 99.

4. Find the cube root of 24137569.

SOLUTION.

$$\dot{2}\dot{4}13\dot{7}|56\dot{9} = t^3 + 3t^2u + 3tu^2 + u^3\,(\overbrace{\overset{t}{2}\;\overset{u}{8}}^{t}\;\overset{u}{9}$$

$$2^3 = 8 \qquad = t^3$$

$$\begin{array}{lr|rcl}
3t^2 = 3 \times 2\ tens^2 & = 1200 &)16137 & = & 3t^2u + 3tu^2 + u^3 = \\
3tu = 3 \times 2\ tens \times 8 = & 480 & & & (3t^2 + 3tu + u^2)u \\
u^2 = 8^2 & = 64 & & & \\
\hline
3t^2 + 3tu + u^2 & = 1744 & & & \\
 & 1744 \times 8 = & 13952 & = & (3t^2 + 3tu + u^2)u \\
\hline
3t^2 = 3 \times 28\ tens^2 & = 235200 &)\ 2185569 & = & (3t^2 + 3tu + u^2)u \\
3tu = 3 \times 28\ tens \times 9 = & 7560 & & & \\
u^2 = 9^2 & = 81 & & & \\
\hline
3t^2 + 3tu = u^2 & = 242841 & & & \\
 & 242841 \times 9 = & 2185569 & = & (3t^2 + 3tu + u^2)u \\
\hline
 & & 0 & & 0
\end{array}$$

Since the given number is expressed by 8 figures, it may be separated into 3 periods, and its cube root must consist of 3 figures, expressing hundreds, tens and units.

Represent the hundreds, or tens of tens of the root, by t, and the units of tens by u. The cube of the root thus represented must equal $t^3 + 3t^2u + 3tu^2 + u^3$. This root, then, will be found in 24137 units of thousands.

The greatest cube of any number of tens contained in 24 thousands is 8 thousands, whose root, 2 tens, we write for the tens of tens of the required root.

Subtracting 8 thousands from the 24 thousands, and its equal, t^3, from the expression above, 16137 remains, which is equal to $3t^2u + 3tu^2 + u^3$, or, since u is a common factor of these terms, 16137 must contain the product of $(3t^2 + 3tu + u^2)$ by u.

We find the value of $3t^2$ to be 12 hundreds, and make use of it as a trial divisor for finding the value of u. Dividing the 161 hundreds of the remainder by the 12 hundreds, we have 13, or 1 ten and 3 units. But as we have already found and subtracted the cube of the greatest number of tens, we know that the quotient is too large (as may often be the case, since the trial divisor is much smaller than the factor of which it is only a term), and therefore diminish it to 9. Taking that as the probable number of units of tens of the required root, add to the trial divisor 54 tens and 81 units, the equals of $3tu$ and u^2, and we have 1821. But 1821 multiplied by 9 equals 16389, which is greater than the dividend. The quotient is therefore still too large, and we diminish it by 1, and have 8, which we write for the units of tens of the required root.

Adding to the trial divisor 48 tens and 64, the equals of $3tu$ and u^2, the sum is 1744, and this multiplied by 8 equals 13952. Subtracting 13952 from the dividend, and its equal $(3t^2 + 3tu + u^2)u$ from the expression above it, and bringing down the remaining period of the cube, we have a remainder of 2185569.

Regarding the tens and units of tens already found as 28 tens of the order of units, we represent them by t, and the units which are to be found by u. The cube of the root now represented must equal $t^3 + 3t^2u + 3tu^2 + u^3$. The cube of the 28 tens here represented by t^3 has been already subtracted from the given cube; hence the remainder, 2185569, must equal $3t^2u + 3tu^2 + u^3$, or $(3t^2 + 3tu + u^2)u$.

We find the value of $3t^2$ to be 2352 hundreds, and make use of it as a trial divisor for finding the value of u. Dividing the 21855 hundreds of the remainder by the 2352 hundreds, we have 9, which we write as the units of the required root.

Adding to the trial divisor 756 tens and 81, the equals of $3tu$ and u^2, we have 242841; and multiplying this by 9, the equal of u, we obtain 2185569, which we write equal to $(3t^2 + 3tu + u^2)u$. Subtracting these equals from the equals above, we have no remainder. Hence, the cube root of 24137569 is 289.

If the given number had been a decimal fraction, its periods would have been pointed off, beginning with units, to the right; for the cube of .1 is .001, the cube of .09 is .000729, etc.

5. Find the cube root of 91125000. *Ans.* 450.
6. Find the cube root of 34012.224. *Ans.* 32.4.

681. Rule for Cube Root.—*Point off the given number into periods of three orders each, beginning with the units, and proceeding toward the left and right.*

Find the greatest cube in the highest period, considered as units, and place its root at the right for the first figure of the root. Subtract this cube from the highest period, and to the remainder bring down the next period for a dividend.

Take for a trial divisor three times the square of the part of the root already found; divide the dividend, omitting its two right-hand orders, and write the quotient for the second figure of the required root.

To the trial divisor add three times the product of the preceding part of the root (considered as tens) by the last root figure, and also the square of the last figure; multiply the sum by the last root figure, and subtract the product from the dividend.

Continue the process, if there are other periods, as before.

The quotient found by the trial divisor, since that divisor is less than the true divisor, must often be diminished by 1 or more.

When 0 occurs in the root, annex two ciphers to the trial divisor, and to the dividend another period.

If there be a remainder after all the periods have been used, periods of decimals may be formed by annexing ciphers, and the work continued.

PROBLEMS.

What is the cube root—

1. Of 110592? *Ans.* 48.
2. Of 91125?
3. Of 178453.547? *Ans.* 56.3.
4. Of 373248? *Ans.* 72.
5. Of 157464?
6. Of 1003003000?

7. Extract the cube root of 379, to within less than $\frac{1}{1000}$. *Ans.* 7.236.

8. Extract the root of .08, to within less than $\frac{1}{10000}$. *Ans.* .4308.

9. What is the length of one side of a cubical box containing 42875 cubic inches? *Ans.* 35 inches.

10. What is the side of a cube which will contain as much as a bin 8 feet 3 inches long, 3 feet wide, and 2 feet 7 inches deep? *Ans.* 47.98 + inches.

11. The capacity of a cubical cistern is 74088 cubic inches. What is the area of its bottom? *Ans.* 1764 square inches.

12. Required the dimensions of a cube that shall have the capacity of a chest 2 feet 8 inches long, 2 feet 3 inches wide, and 1 foot 4 inches thick. *Ans.* 24 inches.

682. Since the cube of a fraction is found by cubing the numerator and denominator, *the cube root of a fraction is found by taking the cube root of the numerator and denominator.*

13. What is the cube root of $\frac{27}{64}$?

SOLUTION.—$\sqrt[3]{\frac{27}{64}}=\frac{\sqrt[3]{27}}{\sqrt[3]{64}}=\frac{3}{4}$.

14. What is the cube root of $\frac{343}{4096}$? *Ans.* $\frac{7}{16}$.

15. What is the value of $\sqrt[3]{\frac{4913}{9261}}$? *Ans.* $\frac{17}{21}$.

16. What is the cube root of $49\frac{8}{27}$? *Ans.* $3\frac{2}{3}$.

683. When the numerator and denominator of a fraction are not both cubes, to find the approximate cube root,

First reduce the fraction to a decimal, and then take the root.

17. What is the cube root of $\frac{4}{7}$, to within less than $\frac{1}{1000}$?

SOLUTION.

$\frac{4}{7}=.571428571+$; $\sqrt[3]{.571428571}=.829$, *Ans.*

18. What is the cube root of $\frac{1}{50}$, to within less than $\frac{1}{10000}$? *Ans.* .2714.

19. What is the cube root of $7\frac{1}{5}$, to within less than $\frac{1}{100}$? *Ans.* 1.93.

20. What is the length of the edge of a cubical block of marble, to within $\frac{1}{1000}$, if the block contains $9\frac{1}{6}$ cubic feet? *Ans.* 2.092 feet.

TEST QUESTIONS.

684.—1. What is a POWER? From what are the powers of a number named? What is the second power frequently called? The third power? What is the exponent of a power?

2. What is INVOLUTION? By what is involution performed? Explain how a number is raised to the third power. What is the rule for involution?

3. What is EVOLUTION? What is a root of a number? What is the second or square root of a number? The third or cube root? What is the sign called by which the roots of numbers are indicated? What is the index of the root? What is a surd? How may the roots of small numbers, which are not imperfect powers, be readily found?

4. What is the RULE for Square root? For Cube root?

SECTION LXVII.

MENSURATION.

685. A **Point** is that which has only position.

686. A **Plane** is a surface (Art. 302) in which, any two points being taken, the straight line joining them will be wholly in the surface.

687. A **Curved Line** is one that has no part in a straight line (Art. 308).

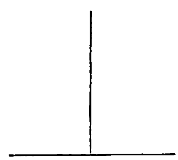
688. **Parallel Lines** are such as are wholly in the same plane and have the same direction.

A CURVED LINE.

PARALLEL LINES.

689. A **Right Angle** is an angle formed by one straight line meeting another, making the adjacent angles equal.

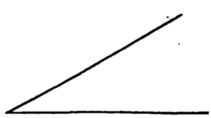
The straight line meeting the other is said to be a *perpendicular* to it.



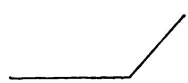
TWO RIGHT ANGLES.

690. An **Acute Angle** is less than a right angle.

691. An **Obtuse Angle** is greater than a right angle.



AN ACUTE ANGLE.



AN OBTUSE ANGLE.

692. **Mensuration** treats of the measurement of lines, surfaces and volumes (Art. 303).

POLYGONS.

693. A **Triangle** is a plane bounded by three straight lines.

694. An **Equilateral Triangle** has the three sides equal.

695. An **Isosceles Triangle** has two equal sides.

696. A **Scalene Triangle** has three unequal sides.

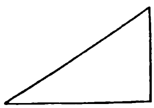
697. A **Right-angled Triangle** has one right angle.

The side opposite the right angle is called the *hypotenuse.*

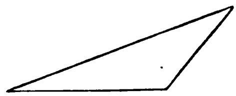
The side meeting another side and forming the right angle is called the *perpendicular.*

698. An **Acute-angled Triangle** has three acute angles.

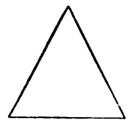
699. An **Obtuse-angled Triangle** has one obtuse angle.



A Right-angled Triangle.



An Obtuse-angled Triangle.



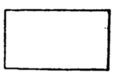
An Acute-angled Triangle.

700. A **Quadrilateral** is a plane bounded by four straight lines.

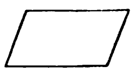
701. A **Parallelogram** is a quadrilateral having its opposite sides parallel.

702. A **Rhomboid** is a parallelogram having no right angles.

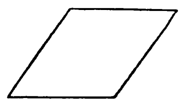
703. A **Rhombus** is a rhomboid having all its sides equal.



A Parallelogram.



A Rhomboid.



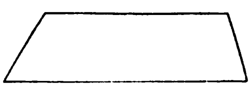
A Rhombus.

704. A **Trapezoid** is a quadrilateral having only two of its sides parallel.

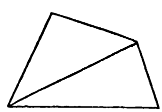
705. A **Trapezium** is a quadrilateral having no two of its sides parallel.

706. A **Diagonal** is a straight line which joins two angles of a plane figure which are not adjacent.

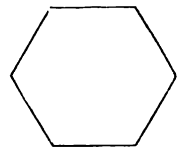
707. The **Base** of a figure is the side upon which it is supposed to stand.



A Trapezoid.



A Trapezium.



A REGULAR POLYGON.

708. A **Polygon** is a plane figure bounded by straight lines.

709. A **Regular Polygon** has all its angles and all its sides equal.

Of regular polygons, the—

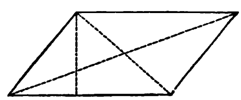
Pentagon has 5 sides.	*Octagon* has 8 sides.
Hexagon has 6 sides.	*Nonagon* has 9 sides.
Heptagon has 7 sides.	*Decagon* has 10 sides.

710. A **Perimeter** is the boundary-line of a plane figure.

711. The **Altitude** of a figure is the line which, being drawn perpendicular to the base, measures the height of the figure.

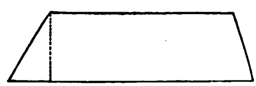
712. By Geometry may be established the following—

713. Principles.—1. *The area* (Art. 395) *of a parallelogram is equal to the product of the base by the altitude.*



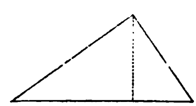
That is, if the altitude of a parallelogram be 4 inches, and the base be 8 inches, the area will be as many square inches as the product of 8 by 4, or 32 square inches.

2. *The area of a trapezoid is equal to the product of half of the sum of the parallel sides by the altitude.*



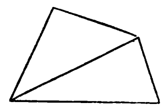
That is, if the altitude of a trapezoid be 4 inches, the longer of the parallel sides be 14 inches, and the shorter of the parallel sides 10 inches, the area will be as many square inches as the product of $\frac{14+10}{2}$ by 4, or 48 square inches.

3. *The area of a triangle is equal to the product of half the base by the altitude.*



That is, if the altitude of a triangle be 4 inches, and the base 8 inches, the area will be as many square inches as the product of $\frac{8}{2}$ by 4, or 16 square inches.

4. *The area of a trapezium, or of any polygon, is equal to the sum of the areas of any set of triangles into which it may be resolved.*



That is, if the trapezium be separated into two triangles, the sum of the areas of these triangles will be the area of the trapezium.

NOTE.—The area of a regular polygon is equal to the square of its side multiplied by the area standing against its name in the following table:

NAME.	AREA.	NAME.	AREA.	NAME.	AREA.
Triangle,	.433013	Heptagon,	3.633913	Decagon,	7.694209
Pentagon,	1.720477	Octagon,	4.828427	Undecagon,	9.36564
Hexagon,	2.598076	Nonagon,	6.181824	Dodecagon,	11.196152

Also, the area of any triangle is equal to the square root of the product of half the sum of the three sides by the three remainders found by subtracting each side separately from half the sum of the three sides.

PROBLEMS.

1. What is the area of a right-angled triangle whose base is 340 feet, and perpendicular 120 yards?

Ans. 20400 square feet.

2. How many acres in a triangular lot whose base is 517 yards, and perpendicular 341 yards?

Ans. 18 acres 34 square rods.

3. Required the area of a parallelogram whose length is 36 feet, and altitude 15 feet. *Ans.* 540 square feet.

4. The parallel sides of a field in the form of a trapezoid are 8 chains 15 links, and 10 chains 45 links, and the perpendicular distance between the parallel sides is 6 chains 24 links. What is its area in acres?

Ans. 5 acres 128.512 square rods.

5. How many square yards in a garden in the form of a trapezium whose diagonal is 20 yards, and the perpendiculars on it to opposite angles are 4.2 yards and 3.8 yards?

Ans. 80 square yards.

6. What is the area of a triangular field whose sides are 2569, 5025 and 4900 links? *Ans.* 61 acres 79.68 square rods.

7. How many square yards of surface in the gable of a house whose breadth is 20 feet 6 inches, and perpendicular height 10 feet 4 inches?

Ans. 11 square yards $6\frac{1}{12}$ square feet.

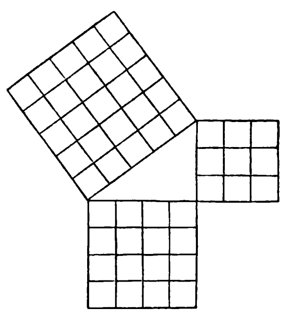
8. How many square yards of painting are there in the ceiling of a room whose length is 24 feet and breadth 15 feet 6 inches? *Ans.* 41 sq. yd. 3 sq. ft.

9. How many acres are contained in a farm of the form of a regular decagon whose side is 2050 links?

Ans. 323 acres 55.86 square rods.

RIGHT-ANGLED TRIANGLES.

714. By Geometry may be established the following



Principles.—1. *The square of the hypotenuse is equal to the sum of the squares of the other two sides.*

That is, if the base be 4 and the perpendicular 3, the square of the hypotenuse will equal $4^2 + 3^2$, or $16 + 9$, and the hypotenuse itself will equal $\sqrt{16+9} = 5$.

2. *The square of either of the sides forming the right angle of the triangle is equal to the square of the hypotenuse minus the square of the other side.*

That is, if the base be 4 and the hypotenuse be 5, the square of the perpendicular will equal $5^2 - 4^2$, or $25 - 16$; and the perpendicular will equal $\sqrt{25-16} = 3$. Also, if the perpendicular be 3 and the hypotenuse be 5, the square of the base will equal $5^2 - 3^2$, or $25 - 9$, and the base itself will equal $\sqrt{25-9} = 4$.

NOTE.—For practical purposes, such as squaring the foundations of a building, a right-angled triangle is readily constructed by so fastening two pieces of wood together, one 8 feet long and the other 6 feet long, that the distance between the outer sides of the two ends shall be 10 feet.

PROBLEMS.

1. The base of a right-angled triangle is 85 yards, and the perpendicular 132 yards. Find the hypotenuse.

SOLUTION.

$$85^2 = 7225, \text{ and } 132^2 = 17424; \quad 7225 + 17424 = 24649;$$
$$\text{and } \sqrt{24649} = 157, \textit{ Ans.}$$

2. A ladder 40 feet long standing upon a level street reaches to a window in a building 32 feet from the ground. Required the distance of the foot of the ladder from the building.

3. A carpenter building a house 32 feet wide wishes to have the gable end 16 feet high. How long must the rafters be made?

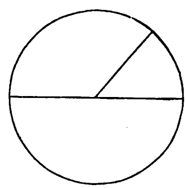
4. A park whose length is 105 rods and width 88 rods has a walk running diagonally across it. What is the length of the walk? *Ans.* 137 rods.

5. At $1.10 per rod, what will be the cost of fencing a lot, in the form of a right-angled triangle, whose sides forming the right angle are 264 rods and 23 rods?

6. A boy flying his kite found that he had let out 210 feet of the string when the kite lodged upon the top of a flag-staff whose height was 91.53 feet. Required the distance of the boy from the foot of the staff. *Ans.* 189 feet.

CIRCLES.

715. By Geometry may be established the following



Principles.—1. *The ratio of the circumference to the diameter is expressed approximately by* $3\frac{1}{7}$, *or by* 3.1416; *and of the diameter to the circumference, by* $\frac{7}{22}$, *or* .3183.

2. *The area of a circle is equal to the product of half the circumference by half the diameter.*

3. *The area of a circle is also equal to the product of the square of the diameter by* .7854, *or to the product of the square of the circumference by* .07958.

4. *The side of the largest square that can be inscribed in a given circle is equal to the square root of half the square of the diameter, or to* .7071 *times the diameter.*

PROBLEMS.

1. The diameter of a circular flower-plat is 17.5 feet. What is its circumference? *Ans.* 55 feet.

2. The circumference of a circular pond is 506 yards. How many feet is its diameter? *Ans.* 483 feet.

3. How many acres in a circular lot whose diameter is 476 links? *Ans.* 1 acre $124\frac{524}{625}$ square rods.

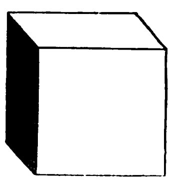
4. What is the side of the largest square that can be laid out in a circular enclosure whose diameter is 8 rods? *Ans.* 5.6568 rods.

VOLUMES.

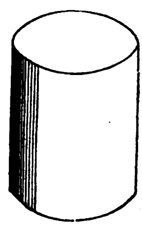
716. A **Prism** is a volume whose ends are equal and parallel polygons, and whose sides are parallelograms.

A prism is *triangular, rectangular,* etc., according as its ends are triangles, rectangles (Arts. 313, 693), etc.

717. A **Cylinder** is a volume of uniform diameter, bounded by a curved surface and two equal and parallel circles.



A RECTANGULAR PRISM.



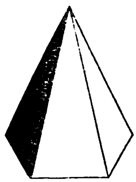
CYLINDER.

718. A **Pyramid** is a volume whose base is a polygon and whose sides are triangles meeting in a point, called the vertex.

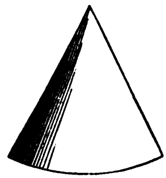
A pyramid is *triangular, quadrangular,* etc., according as its base is a triangle, quadrilateral, etc., and its taper surface is that of the sum of its triangular faces.

719. A **Cone** is a volume whose base is a circle, from which the remaining surface tapers uniformly to a point or vertex.

720. The **Slant Height** of a pyramid or cone is the shortest line that can be drawn from the vertex to the perimeter or circumference of the base.



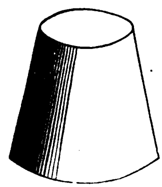
A PYRAMID.



A CONE.

The curved surface of a cone is equal to the surface of a triangle whose base is the circumference of the cone and whose altitude is the slant height.

721. A **Frustum** of a pyramid or cone is that which remains after cutting off the upper part of it by a plane parallel to the base.



THE FRUSTUM OF A CONE.

722. A **Sphere** is a volume bounded by a surface, all points of which are equally distant from a point within, called the center.

A SPHERE.

723. By Geometry may be established the following

724. Principles.—1. *The cubic contents of a prism or cylinder are equal to the product of the area of the base by the altitude.*

2. *The cubic contents of a pyramid or cone are equal to one third of the product of the area of the base by the altitude.*

3. *The cubic contents of a frustum of a pyramid or cone equal the product of the sum of the areas of the two ends and the square root of their product, by one third of the altitude.*

4. *The surface of a sphere is equal to the product of the circumference by the diameter, or to the product of the square of the diameter by* 3.1416.

5. *The cubic contents of a sphere are equal to the product of the surface by one sixth of the diameter, or to the product of the cube of the diameter by* .5236.

PROBLEMS.

1. Required the cubic contents of a squared beam whose length is 32 feet, and whose breadth and thickness are each 1 foot 6 inches. *Ans.* 72 cubic feet.

2. A pentagonal stone, whose side is 2 feet, has an altitude of 6 feet. What are its cubic contents?
Ans. 41.291448 cubic feet.

3. The circumference of a cylindrical log is 5 feet 6 inches, and its length 20 feet. What are its cubic contents?
Ans. 48.1458 cubic feet.

4. The sides of the base of a triangular pyramid are 11, 13, 20 feet, and the altitude is 36 feet. What are the cubic contents? *Ans.* 792 cubic feet.

5. What are the cubic contents of a conical tower whose circumference is 13.20 yards, and altitude 27 yards?
Ans. 124.74 cubic yards.

6. What is the curved surface of a cone whose circumference is 9.4248 yards, and slant height 15 yards?

7. What is the weight of a marble monument in the form of a square pyramid, each side of the base being 3 feet and its altitude 10 feet, at 166 pounds per cubic foot?
Ans. 4980 pounds.

8. What will it cost to paint a spire, in the form of a quadrangular pyramid, whose slant height is 80 feet, and each side of the base 18 feet, at $.50 per square yard? *Ans.* $160.

9. Required the surface in square inches of a sphere whose diameter is 4 feet 8 inches. *Ans.* 9856 square inches.

10. Each side of the greater end of a piece of squared timber is 28 inches, each side of the lesser end 14 inches, and its

length 18 feet 9 inches. How many cubic feet does it contain? *Ans.* 59.5486+ cubic feet.

11. What are the cubic contents of a sphere whose diameter is 10 feet 6 inches? *Ans.* 606.375 cubic feet.

12. How many gallons of water will a boiler hold whose shape is that of a half sphere 25 inches in diameter?

Ans. 17.7154+ gallons.

SIMILAR FIGURES.

725. Similar Figures are such as are of the same form, and differ from each other only in size.

726. Similar figures have such a relation to each other that—

1. *The like dimensions of similar figures are proportional.*

2. *The areas of similar figures are to each other as the squares of their corresponding dimensions.*

3. *The cubic contents of similar volumes are to each other as the cubes of their corresponding dimensions.*

PROBLEMS.

1. The sides of a triangular lot are 15, 28 and 41 rods, and the area is 126 square rods. What are the sides of a similar lot whose area is 56 square rods?

SOLUTION.

By Art. 726—2, $126 : 56 :: 15^2 : 100.$

Hence, the corresponding sides of the similar lot,

$\sqrt{100} = 10$, *the one side.*

Then by Art. 726—1, $15 : 10 :: 28 : 18\frac{2}{3}$,

and $15 : 10 :: 41 : 27\frac{1}{3}$.

2. The area of a triangle whose base is 17 chains is 3 acres 71 square rods. What is the area of a similar triangle whose base is 57 chains? *Ans.* 38 acres 114.46 square rods.

3. If the side of a square field containing 10 acres is 40 rods, what must be the side of a similar field containing one fourth as many acres? *Ans.* 20 rods.

4. The area of a circle whose diameter is 100 feet, is 7854 square feet; what is the area of a circle whose diameter is 10 feet?

5. What are the dimensions of a rectangular garden containing 180 square rods, whose length is to its breadth as 5 to 4?

SOLUTION.

By Art. 398, $5 \times 4 = 20$, *the area of a rectangle 5 by 4.*

Then, $20 : 180 :: 5^2 : 225$;

and, $\sqrt{225} = 15$, *the greater side.*

Also, $5 : 4 :: 15 : 12$, *the smaller side.*

6. The three edges of a rectangular volume are 3, 4, 7 inches; what are sides of a similar volume whose contents are 777924 cubic inches? *Ans.* 63, 84 and 147 inches.

7. If a ball 3 inches in diameter weigh 4 pounds, how much will a ball of like density weigh whose diameter is 6 inches? *Ans.* 32 pounds.

8. What are the dimensions of a rectangular field containing 30 acres, whose length is to its breadth as 4 to 3?

9. A stack of hay 5 feet high weighs 100 pounds; how high must be a similar stack to weigh 3 tons 400 pounds?

10. If the edge of a cube of Quincy granite weighing 165.75 pounds is 12 inches, what is the edge of a cube weighing 500 pounds?

11. If a ball of thread 4 inches in diameter should be reduced to half that diameter, what part of the thread will remain? *Ans.* $\frac{1}{8}$.

TEST QUESTIONS.

727.—1. Of what does MENSURATION treat? What is a point? What is a line? (Art. 301.) A curved line? What are parallel lines?

2. What is an ANGLE? (Art. 309.) How is a right angle formed? What name is given to an angle less than a right angle? To an angle greater than a right angle?

3. What is a SURFACE? (Art. 302.) What is a plane? The area of a figure? The base of a figure? The altitude of a figure? The perimeter? A diagonal?

4. What is a TRIANGLE? An equilateral triangle? An isosceles triangle? A scalene triangle? A right-angled triangle? An acute-angled triangle? An obtuse-angled triangle? How is the area of a triangle found?

5. What is a Quadrilateral? A square? (Art. 312.) A rectangle? (Art. 313.) A parallelogram? When is a parallelogram a rhomboid? When a rhombus? When is a quadrilateral a trapezoid? When a trapezium?

6. How is the AREA of a rectangle found? (Art. 398.) How is the area of a parallelogram found? How is the area of a trapezoid found? How is the area of a trapezium found?

7. What is a POLYGON? A regular polygon? How many sides has a pentagon? A hexagon? A heptagon? An octagon? A nonagon? A decagon? How is the area of a polygon found?

8. What is the longest side of a right-angled triangle called? To what is the square of the hypotenuse equal? When any two sides of a right-angled triangle are given, how is the other side found?

9. What is a CIRCLE? The diameter of a circle? (Art. 347.) The circumference of a circle? (Art. 345.) What is the ratio of the circumference to the diameter? Of the diameter to the circumference? How is the area of a circle found?

10. What is a VOLUME? (Art. 303.) What is a cube? (Art. 317.) A rectangular volume? What are the cubic contents of a volume? How are the cubic contents of a rectangular volume found? (Art. 405.)

11. What is a PRISM? A cylinder? How are the cubic contents of a prism or of a cylinder found?

12. What is a PYRAMID? A cone? The slant height of a pyramid or a cone? A frustum of a pyramid or cone? How are the cubic contents of a pyramid or a cone found? The cubic contents of a frustum of a pyramid or a cone?

13. What is a SPHERE? To what is the surface of a sphere equal? To what are the cubic contents equal?

14. What are SIMILAR FIGURES? What relations have like dimensions of similar figures? The areas of similar figures? The cubic contents of similar figures?

15. How does a curved line differ from a straight line? An acute angle from an obtuse angle? What is the perimeter of a triangle? Of a square? Of a circle?

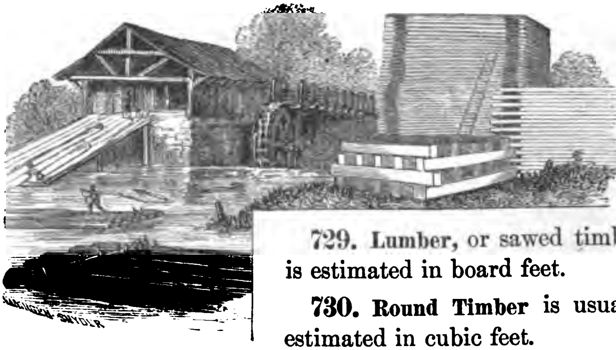
16. What kind of a prism is a rectangular solid? What kind of a pyramid is one having a four-sided base? How does a cone differ from a pyramid? How does a sphere differ from a cylinder?

SECTION LXVIII.

MEASUREMENTS OF BOARDS AND TIMBER.

728. A **Board Foot** is 1 foot long, 1 foot broad and 1 inch thick; hence,

12 board feet are 1 cubic foot.



729. Lumber, or sawed timber, is estimated in board feet.

730. Round Timber is usually estimated in cubic feet.

731. Squared or **Hewn Timber** is estimated either in board feet or cubic feet.

732. The **Mean Girt** of a tapering log is the circumference, clear of bark, one-third the distance from the larger to the smaller end of the log.

733. The **Mean Breadth** and **Thickness** of tapering squared timber is the breadth and thickness of the timber measured at the middle of its length.

CASE I.

Dimensions of Lumber, or Squared Timber, given, to find the Contents.

734.—Ex. 1. What is the number of board feet in a board 24 feet long, 18 inches wide and 1 inch thick?

SOLUTION.—18 inches $= \frac{18}{12}$ feet. $24 \times \frac{18}{12} \times 1 = \frac{24 \times 18}{12} = 36$, the number of board feet required.

2. How many cubic feet in a piece of hewn timber 45 feet long, whose breadth and thickness are 15 and 14 inches?

SOLUTION.—15 and 14 inches $= \frac{15}{12}$ and $\frac{14}{12}$ feet. $45 \times \frac{15}{12} \times \frac{14}{12} = \frac{45 \times 15 \times 14}{144} = 65.625$ cubic feet.

3. What are the contents in board feet of a joist 16 feet long, 4 inches thick, and tapering in breadth from 6 inches to 4 inches? *Ans.* $26\frac{2}{3}$.

The mean of the tapering breadth is 5 inches, which is used in the computation.

735. Rule for Finding the Contents of Lumber and Squared Timber.—*Multiply the length in feet by the breadth and thickness in inches, and divide by 12 for board feet, or by 144 for cubic feet.*

If the lumber or squared timber tapers, the mean breadth and thickness must be used in the computation.

A common rule for estimating tapering squared timber is—

Add together the areas of the two ends in square inches, and multiply half the sum by the length in feet, and divide by 12 for board feet, or by 144 for cubic feet.

PROBLEMS.

1. What are the contents of a plank whose length is 20 feet, breadth 16 inches and thickness 3 inches?

Ans. 80 board feet.

2. A beam is $30\frac{2}{3}$ feet long, 22 inches broad and 15 inches thick. What are its contents? *Ans.* $843\frac{1}{3}$ board feet.

3. The length of a piece of timber is 9.8 feet, and its mean breadth and thickness 2.6 and 1.5 feet. What are the cubic contents? *Ans.* 38.22 cubic feet.

4. Bought 20 joists, each 18 feet long, 5 inches wide and 3 inches thick, at \$30 a thousand feet, board measure. What did they cost me? *Ans.* \$13.50.

5. The breadth and thickness of one end of a stick of timber, whose length is 17 feet 3 inches, are 36 and 20 inches,

and of the other end 18 and 10 inches. What are its cubic contents by the rule, allowing the mean breadth and thickness to be 27 and 15 inches; and what the true contents, measured as a frustum of a pyramid?

Ans. By the rule, 48.5156 + cubic feet; true contents, 50.3125 cubic feet.

CASE II.

Length and Mean Girt of Round Timber given, to find the Contents.

736.—Ex. 1. What are the cubic contents of a piece of round timber whose mean girt is 100 inches and length 18 feet?

SOLUTION.—$\frac{1}{4}$ of mean girt $= \frac{1}{4}$ of $100 = 25$; square of $\frac{1}{4}$ of mean girt $= 25^2 = 625$; $625 \times 18 = 11250$; and $11250 \div 144 = 78.125$, the number of cubic feet required.

2. The mean girt of a log is 88 inches, and the length of the log is 40 feet. What are the cubic contents?

Ans. 134.44 cubic feet.

737. Rule for Finding the Cubic Contents of Round Timber.—*Multiply the square of one fourth of the mean girt in inches by the length in feet, and divide by 144.*

NOTE.—The rule gives about one fifth less than the exact quantity, one fifth being allowed for crooks and waste in working.

The exact cubic contents may be found very nearly by *multiplying the square of one fifth of the mean girt in feet by twice the length in feet.*

PROBLEMS.

1. The length of a log is 32 feet 6 inches, and its mean girt, after allowing for the bark, is 60 inches. What are the contents by the rule, and what by the note under the rule?

Ans. By the rule, 50.78125 cubic feet; by the note, 65 cubic feet.

2. What is the value of a pine log 30 feet long, and whose mean girt is 10 feet, at $20 per ton of 40 cubic feet?

Ans. $93.75.

3. The circumference of a piece of round timber is 6 feet 8 inches, and its length 24 feet. What are its contents by the rule, and what as a cylinder? (Art. 724—1.)

SECTION LXIX.

MEASUREMENTS OF STONE, AND BRICK-WORK.

738. Stone Masonry is usually estimated by the cubic foot or by the perch.

739. Brick-Laying is generally estimated by the thousand bricks.

740. A **Perch** of stone-work is $16\frac{1}{2}$ feet long, 1 foot deep and $1\frac{1}{2}$ feet thick, and is equivalent to $24\frac{3}{4}$ cubic feet.

741. Bricks are of various dimensions.

Philadelphia or Baltimore front bricks are $8\frac{1}{4}$, $4\frac{1}{8}$ and $2\frac{3}{8}$ inches; North River bricks, 8, $3\frac{1}{2}$ and $2\frac{1}{4}$ inches; Maine bricks, $7\frac{1}{2}$, $3\frac{3}{8}$ and $2\frac{3}{8}$; and Milwaukee bricks, $8\frac{1}{2}$, $4\frac{1}{8}$ and $2\frac{3}{8}$ inches.

CASE I.

Dimensions of Stone-work given, to find the Number of Perches.

742.—Ex. 1. How many perches of stone-work in a wall 66 feet long, 4 feet high and 3 feet wide?

SOLUTION.

$66 \times 4 \times 3 = 792$, *number of cubic feet.*

$792 \div 24.75 = 32$, *number of perches.*

743. Rule for Finding the Number of Perches of Stone-work.—*Find the contents in cubic feet, and divide by 24.75.*

PROBLEMS.

1. What are the contents in perches of a stone wall whose dimensions are 24 feet 3 inches, 10 feet 9 inches and 2 feet? *Ans.* 21.065 + perches.

2. What will it cost, at \$3.25 per perch, for the stone- and mason-work of a cellar 8 feet deep, under a house whose length and width are $41\frac{1}{4}$ and 33 feet, the wall of the cellar to be $1\frac{1}{2}$ feet thick, and no allowance to be made for corners or openings? *Ans.* \$234.

CASE II.

Dimensions of Bricks and Thickness of Mortar of Brick-work given, to find the Number of Bricks.

744.—Ex. 1. The width of a wall is $10\frac{1}{2}$ inches, laid of Maine bricks, in courses of mortar $\frac{1}{4}$ of an inch thick. How many bricks has it in a cubic foot?

SOLUTION.

$7.5 + (.25 \times 2) \div 2 = 7.75$, *length of brick and joint.*

$2.375 + (.25 \times 2) \div 2 = 2.625$, *width of brick and joint.*

$7.75 \times 2.625 = 20.34375$, *area of face.*

$10.5 \div 3 = 3.5$; $20.34375 \times 3.5 = 71.2 +$ *cubic inches.*

$1728 \div 71.2 = 24.269 +$, *the number of bricks.*

745. Rule for Finding the Number of Bricks in Brick-work.—*To the face dimensions of the kind of bricks used, add half the thickness of the mortar in which they are laid, and find the area. Multiply this area by the quotient obtained by dividing the width of the wall by the number of bricks of which it is composed, and the product will be the contents in cubic inches. Divide 1728 cubic inches by these contents, and the quotient will be the number of bricks in a cubic foot.*

PROBLEMS.

1. A wall of Milwaukee bricks is 40 feet long, 7 feet high and $21\frac{3}{4}$ inches thick, and the courses of mortar in which the bricks are laid are $\frac{1}{4}$ of an inch thick. How many thousand bricks are there in the wall?

2. The width of a wall is $11\frac{3}{4}$ inches, laid of North River bricks, in courses of mortar $\frac{1}{4}$ of an inch thick. How many bricks has it in a cubic foot?

3. A wall of Philadelphia bricks is 20 feet long, 5 feet high and $12\frac{3}{4}$ inches thick, and the courses of mortar in which the bricks are laid are $\frac{1}{4}$ of an inch thick. What must have been the cost of the bricks at \$12 per thousand? *Ans.* \$23.23.

SECTION LXX.

MEASUREMENTS OF GRAIN AND HAY.

746. Grain is usually estimated in this country by the bushel or by the cental. (Art. 332.)

747. The **Standard Bushel** in the United States contains 2150.4 cubic inches. Hence, a cubic foot is nearly .8 of a bushel.

748. Hay is usually bought and sold by the ton.

About 550 cubic feet of clover, or 450 feet of meadow-hay, well settled, as an average, in large mows, make a ton.

749. Rules for Estimating Grain.—*1. To find the quantity, in bushels, of grain in a bin or wagon, multiply the contents in cubic feet by .8.*

2. To find the quantity, in bushels, of grain when heaped upon a floor, make the heap in the form of a cone, and multiply the area of the base by one-third of the altitude, and the result by .8.

If the grain be heaped against the side of a wall in the form of a half cone, take half the result by Rule 2, or, if heaped against an inner corner, take one fourth.

750. Rule for Estimating Hay.—*1. To find the quantity, in tons, of hay, in mows, well settled, divide the contents by 550 for clover, or by 450 for meadow-hay.*

PROBLEMS.

1. How many bushels of wheat will a bin hold that is 6 feet long, 4 feet wide and 5 feet deep? *Ans.* 96.

2. A wagon 9 feet long, $3\frac{1}{3}$ feet wide and 3 feet deep is two thirds full of shelled corn. How many bushels does it contain?

3. A farmer had a heap of oats, which when made in a conical form, measured in the circumference of the base 22

feet, and in altitude 6 feet. How many bushels does it contain? *Ans.* 61.6.

4. In the inner corner of a building corn is heaped in the form of a quarter cone, whose altitude is 6 feet and slant height 10 feet. What is the quantity of corn?

Ans. $80\frac{16}{35}$ bushels.

5. A heap of grain, piled against a wall, measures a height of 9 feet and a semi-circumference of 33 feet. How many bushels does it contain? *Ans.* 415.8.

6. A bin 6 feet long, 4 feet wide and 5 feet deep is filled with rye. What is its value at $2 per cental?

Ans. $107.52.

7. How many tons of meadow-hay in a portion of a stack, dry and settled, which is 10 feet long, 9 feet wide and 6 feet high? *Ans.* $2\frac{1}{5}$.

8. What is the value, at $30 a ton, of a mow of clover-hay that is 24 feet long, 12 feet wide and 9 feet high?

SECTION LXXI.

GAUGING.

751. Gauging is the process of finding the capacity of casks and other vessels.

752. The **Mean Diameter** of a cask is the diameter of an equivalent cylinder having the same length as the cask.

It is nearly equal to the head diameter plus two thirds of the difference between that and the long diameter, or three fifths when the staves are but slightly curved.

753. The **Ullage,** or wantage, of a cask is the quantity it lacks of being full.

754. Rules for Gauging.—*1. Multiply the product of the square of the mean diameter and the length or depth of the cask, expressed in inches, by .0034, and the result will be its capacity in gallons.*

2. Multiply the square of one third of the sum of the head, mean and bung diameters, expressed in inches, by the height of the liquid in inches, and that product by .0034, and the result will be the contents of an ullage cask.

PROBLEMS.

1. How many gallons is the capacity of a cask whose length is 40 inches and mean diameter 25 inches? *Ans.* 85 gallons.

2. Required the quantity of vinegar in a cask whose bung and head diameters are 37 and 28 inches, and the height of the liquid 10 inches. *Ans.* 37.026 gallons.

3. How much will a cask of molasses cost whose mean diameter is 30 inches and length 36 inches, at $.55 per gallon? *Ans.* 60.58\frac{8}{10}$.

TEST QUESTIONS.

755.—1. What are the dimensions of a BOARD FOOT? How many board feet are one cubic foot?

2. What kind of timber is LUMBER? In what is squared or hewn timber estimated? What are the rules for finding the contents of lumber and squared timber?

3. In what is ROUND TIMBER estimated? What is the mean girt of a tapering log? What is the mean breadth and thickness of tapering squared timber? What is the rule for finding the cubic contents of round timber?

4. By what is STONE MASONRY estimated? What is a perch of stone-work? What is the rule for finding the number of perches of stone-work?

5. How is BRICKLAYING usually estimated? What is the rule for finding the number of bricks in brick-work?

6. How is GRAIN usually estimated? How much does the standard bushel contain? What is the rule for finding the quantity, in bushels, of grain in a bin or wagon? For finding the quantity of grain when heaped upon a floor?

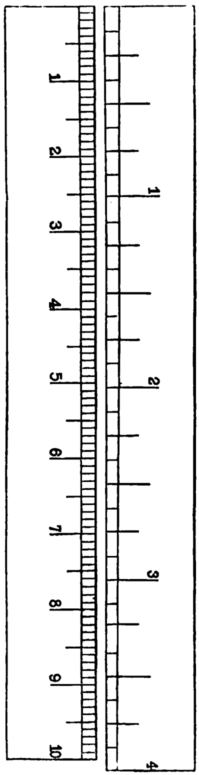
7. How is hay bought and sold? About how many cubic feet of clover make a ton? About how many cubic feet of meadow-hay make a ton? What is the rule for finding the quantity of hay in tons?

8. What is GAUGING? What are the rules for gauging?

SECTION LXXII.

METRIC SYSTEM.

756. The **Metric System** is a system of weights and measures based upon a unit called a meter.



757. The **Meter** is one ten-millionth part of the distance from the equator to either pole, measured on the earth's surface at the level of the sea.

758. The **Names** of derived metric denominations are formed by prefixing to the name of the primary unit of a measure—

Milli (mill'e), a thousandth; **Centi** (sent'e), a hundredth; **Deci** (des'e), a tenth;

Deka (dek'a), ten; **Hecto** (hek'to), one hundred; **Kilo** (kil'o), a thousand; **Myria** (mir'ea), ten thousand.

This system, first adopted by France, has been extensively adopted by other countries, and is much used in the sciences and the arts. It was legalized in 1866 by Congress to be used in the United States, and is already employed by the Coast Survey, and to some extent by the Mint and the General Post-Office.

The illustration adjoining shows the length of 10 centimeters, or *a tenth of a meter*, compared with 4 inches, or *a third of a foot.*

The nickel 5-cent pieces are each two hundredths of a meter in diameter; hence, 50 of them placed side by side in a straight line will measure 1 meter.

The simplicity and utility of the system, recognized now by all civilized nations, is likely to lead to its general adoption, and to the great advantage of home and foreign trade.

In the tables the units most used are denoted by CAPITALS or by plain Roman type.

LINEAR MEASURES.

759. The **Meter** is the primary unit of lengths.

TABLE.

10 millimeters (*mm.*)	*are*	*1* centimeter (*cm.*)	=	*.3937 in.*
10 centimeters	"	*1 decimeter*	=	*3.937* "
10 decimeters	"	*1* METER (*m.*)	=	*39.37* "
10 meters	"	*1 dekameter*	=	*393.7* "
10 dekameters	"	*1 hectometer*	=	*328 ft. 1* "
10 hectometers	"	*1* KILOMETER (*km.*)	=	*.62137 mi.*
10 kilometers	"	*1 myriameter*	=	*6.2137* "

The **Meter** is used in ordinary measurements; the **Centimeter,** or **Millimeter,** in reckoning very small distances; and the **Kilometer,** for roads or great distances.

A *Centimeter* is about $\frac{3}{8}$ of an inch; a *Meter* is about 3 feet 3 inches and $\frac{3}{8}$ of an inch; a *Kilometer* is about 200 rods, or $\frac{5}{8}$ of a mile.

SURFACE MEASURES.

760. The **Square Meter** is the primary unit of ordinary surfaces; and,

761. The **Are** (air), a square each of whose sides is ten meters, is the unit of land measures.

TABLE.

100 sq. millimeters (*sq. mm.*)	*are*	*1 sq.* centimeter (*sq. cm.*)	=	*.155 sq. in.*
100 sq. centimeters	"	*1 sq. decimeter*	=	*15.5 sq. in.*
100 sq. decimeters	"	*1 sq.* METER (*sq. m.*)	=	*1550 sq. in.*, or *1.196 sq. yd.*

Also,

100 centiares, or sq. meters,	*are*	*1* ARE (*ar.*)	=	*119.6 sq. yd.*
100 ares	"	*1 hectare* (*ha.*)	=	*2.471 acres.*

A *Square Meter*, or *1 Centiare*, is about $10\frac{3}{4}$ square feet, or $1\frac{1}{5}$ square yards, and a *Hectare* is about $2\frac{1}{2}$ acres.

CUBIC MEASURES.

762. The Cubic Meter, or Stere (stair), is the primary unit of a volume.

TABLE.

1000 cu. millimeters (cu. mm.)	*are*	*1 cu. centimeter (cu. cm.)*	=	*.061 cu. in.*
1000 cu. centimeters	"	*1 cu. decimeter*	=	*61.022 cu. in.*
1000 cu. decimeters	"	*1* CU. METER (*cu. m.*)	=	*35.314 cu. ft.*

The *Stere* (stair) is the name given to the cubic meter in measuring wood and timber. A tenth of a stere is a *Decistere*, and ten steres are a *Dekastere*.

A *Cubic Meter*, or *Stere*, is about $1\frac{1}{3}$ cubic yards, or about $2\frac{1}{4}$ cord feet.

LIQUID AND DRY MEASURES.

763. The **Liter** (leeter) is the primary unit of measures of capacity, and is a cube, each of whose edges is a tenth of a meter in length; and,

764. The **Hectoliter** is the unit in measuring large quantities of grain, fruits roots and liquids.

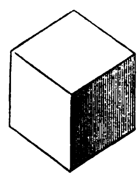
TABLE.

10 milliliters (*ml.*)	*are 1* centiliter (*cl.*)	=	*.338 fl. oz.*
10 centiliters	" *1 deciliter*	=	*.845 liq. gill.*
10 deciliters	" *1* LITER (*l.*)	=	*1.0567 liq. qt.*
10 liters	" *1 dekaliter*	=	*2.6417 gal.*
10 dekaliters	" *1* HECTOLITER (*hl.*)	=	*2 bu. 3.35 pk.*
10 hectoliters	" *1 kiloliter*	=	*28 bu.* $1\frac{1}{8}$ *pk.*

A *Centiliter* is about $\frac{1}{3}$ of a fluid ounce; a *Liter* is about $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart; a *Hectoliter* is about $2\frac{5}{6}$ bushels; and a *Kiloliter* is 1 cubic meter or stere.

WEIGHTS.

765. The **Gram** is the primary unit of weights, and is the weight in a vacuum of a cubic centimeter of distilled water, at the temperature of 39.2 degrees Fahrenheit.



A CUBIC CENTIMETER.

TABLE.

10 milligrams (*mg.*)	*are 1 centigram*	=	*.1543 gr. T.*
10 centigrams	" *1 decigram*	=	*1.543* "
10 decigrams	" *1* GRAM (*g.*)	=	*15.432* "
10 grams	" *1 dekagram*	=	*.3527 av. oz.*
10 dekagrams	" *1 hectogram*	=	*3.5274* "
10 hectograms	" *1* KILOGRAM (*k.*)	=	*2.2046 av. lb.*
10 kilograms	" *1 myriagram*	=	*22.046* "
10 myriagrams	" *1 quintal*	=	*220.46* "
10 quintals	" *1* TONNEAU (*t.*)	=	*2204.6* "

The *Gram* is used in weighing gold, jewels, letters and small quantities of things. The *Kilogram*, or, for brevity, *Kilo*, is used by grocers; and the *Tonneau* (tonno), or *Metric Ton*, is used in finding the weight of very heavy articles.

A *Gram* is about 15½ grains Troy; the *Kilo*, about 2⅕ pounds avoirdupois; and the *Metric Ton*, about 2205 pounds.

A *Kilo* is the weight of a liter of water at its greatest density, and the *Metric Ton* of a cubic meter of water.

766. Metric Numbers are written with the decimal point (.) at the right of the figures denoting the unit.

Thus, 15 meters 3 centimeters are written 15.03 m.

767. When metric numbers are expressed by figures, the part of the expression at the left of the decimal point is read as the number of the unit, and the part at the right, if any, as a number of the lowest denomination indicated, or as a decimal part of the unit.

Thus, 46.525 m. is read 46 metres and 525 millimeters, or 46 and 525 thousandths meters.

768. In writing and reading metric numbers, according as the scale is 10, 100 or 1000, each denomination should be allowed one, two or three orders of figures.

WRITTEN EXERCISES.

769.—Ex. 1. Express by figures two kilometers one hundred sixty-nine meters seventy-five centimeters as meters.
Ans. 2169.75 m.

2. Express nine hundred sixteen millimeters as a decimal of a meter. *Ans.* .916 m.

3. Express four hundred fifty kilometers three hundred twelve meters as kilometers. *Ans.* 450.312 km.

4. Express thirty-eight hectares three ares ninety-four centiares as hectares. *Ans.* 38.0394 ha.

5. Express twenty-five square meters seventy-one square centimeters as square meters. *Ans.* 25.71 sq. m.

6. Express five cubic meters one thousand seventy-six cubic centimeters as cubic meters. *Ans.* 5.001076 cu. m.

7. Express four hundred twenty-two kilos thirty-five grams as kilos. *Ans* 422.035 k.

8. Express one hundred one tonneaux nine hundred nine kilos as tonneaux. *Ans.* 101.909 t.

9. Express fifty-five liters five centiliters as liters. *Ans.* 55.05 l.

10. Express one thousand thirty-seven hectoliters twenty-five liters as hectoliters. *Ans.* 1037.25 hl.

Write and read—

11. 2169.75 m.	15. 104.147 cu. cm.	19. 67.305 l.
12. 195.007 km.	16. 106.07 st.	20. 6.005 gr.
13. 31.9 cm.	17. 51.001001 cu. m.	21. 316.08 k.
14. 8.0394 ha.	18. 31.15 hl.	22. 163.455 t.

COMPUTATIONS.

770. The **Computations** in metric numbers are similar to those in United States money.

From the nature of the scales, the numbers are operated with in like manner as are simple integers and decimals.

Thus, 16.55 m., or 16 meters 55 centimeters, may be changed to centimeters by removing the decimal point two orders to the left, in the same manner as 16.55 dollars may be changed to cents; and 1634 millimeters may be changed to meters by pointing off three orders from the right, in the same manner as 1634 mills may be changed to dollars.

771. **Units** of the common system may be readily changed to units of the metric system by aid of the following

TABLE.

1 inch	*= 2.54 centimeters.*	*1 cu. inch*	*= 16.39 cu. centimeters.*
1 foot	*= 30.48 centimeters.*	*1 cu. foot*	*= 28320 cu. centimeters.*
1 yard	*= .9144 meter.*	*1 cu. yard*	*= .7646 cu. meter.*
1 rod	*= 5.029 meters.*	*1 cord*	*= 3.625 steres.*
1 mile	*= 1.6093 kilometers.*	*1 fl. ounce*	*= 2.958 centiliters.*
1 sq. inch	*= 6.4528 sq. centimeters.*	*1 gallon*	*= 3.786 liters.*
1 sq. foot	*= 929 sq. centimeters.*	*1 bushel*	*= .3524 hectoliter.*
1 sq. yard	*= .8361 sq. meter.*	*1 grain Troy*	*= 64.8 milligrams.*
1 sq. rod	*= 25.29 centiares.*	*1 pound Troy*	*= .373 kilo.*
1 acre	*= 40.47 ares.*	*1 pound av.*	*= .4536 kilo.*
1 sq. mile	*= 259 hectares.*	*1 ton*	*= .907 tonneau.*

PROBLEMS.

1. Reduce 14937 meters to kilometers. *Ans.* 14.937 k.
2. Reduce 160000 square meters to hectares. *Ans.* 16 ha.
3. What decimal of a tonneau are 83000 grams?
4. How many centiliters are 56.55 hectoliters?
5. Reduce 20 miles 40 rods to kilometers.

$$\begin{array}{rcrl} 1.6093 \text{ km.} \times 20 & = & 32.186 & \text{km.} \\ .005029 \text{ “} \quad \times 40 & = & .20116 & \text{km.} \\ \hline & & 32.38716 & \text{“} \end{array}$$

SOLUTION.—Since 1 mile is 1.6093 km., 20 miles are 20 times 1.6093 km., or 32.186 km. Since 1 rod is 5.029 m., or .005029 km., 40 rods are 40 times .005029 km., or .20116 km. 32.186 km. + .20116 km. are 32.38716 km., the result required.

6. How many miles are 32.3871 kilometers?

7. The length of the tunnel through Mt. Cenis is about 12.22 kilometers; what is its length in miles?

8. The French post-office allows 7.5 grams for a single postage—the United States, ½ of an ounce avoirdupois. How many grains does the latter exceed the former? *Ans.* 103.01.

9. How many cubic centimetres are 31.631 cubic meters? *Ans.* 31631000.

10. How many hectares in a rectangular farm whose length is 1500 meters and width 800 meters; and what is its value at $80 per acre? *Ans.* 120 ha.; value, $23721.60.

11. A square mile is how many hectares?

12. If a sack of flour of 150 kilos be sold at 54.17 francs, what would be the corresponding price of a cental in United States money, allowing 5.14 francs to a dollar? *Ans.* $3.18+.

13. A bin is 3.75 meters long, 2.50 wide and 1.80 deep. How many hectoliters will it contain?

14. What must be the height of a range of wood which is 25 meters long, 1.12 meters wide, to contain 35 steres? *Ans.* 1.25 meters.

15. When wine is at 2 francs a liter, what is it a gallon in United States money, the value of a franc being $.18$\frac{6}{10}$? *Ans.* $1.40+.

SECTION LXXIII.

SERIES OR PROGRESSION.

772. A **Series,** or Progression, is a succession of numbers in which each succeeding number is formed from the preceding one by adding or subtracting the same quantity, or by the multiplication by a constant factor.

The Terms of a series are its numbers; the first and last terms are its *Extremes*, and the other terms its *Means*.

774. A series or progression is *ascending* when the terms increase regularly from the first, and *descending* when the terms decrease regularly from the first.

ARITHMETICAL PROGRESSION.

775. An **Arithmetical Progression** is a series whose terms increase or decrease by a common difference.

Thus, 3, 5, 7, 9, 11, and 16, 14, 12, 10, are arithmetical progressions in which the common difference is 2.

776. The first term, the last term, the common difference, the number of the terms, and the sum of the terms, are the elements of a series.

The relations of these are such that when three of them are known the others may be determined.

WRITTEN EXERCISES.

777.—Ex. 1. The first term of an ascending arithmetical series is 5, and the common difference 2. What is the 4th term?

SOLUTION.

1st term =			5
2d "	$= 5 + 2$	$= 5 + 2 \times 1 =$ *1st term* + *com. diff.* $\times 1 =$	7
3d "	$= 5 + 2 + 2$	$= 5 + 2 \times 2 =$ *1st term* + *com. diff.* $\times 2 =$	9
4th "	$= 5 + 2 + 2 + 2$	$= 5 + 2 \times 3 =$ *1st term* + *com. diff.* $\times 3 =$	11

2. The first term of a descending arithmetical series is 11, and the common difference 2. What is the 4th term?

SOLUTION.

1st term =			*11*
2d "	$= 11 - 2$	$= 11 - (2 \times 1) =$ *1st term* $-$ *com. diff.* $\times 1 =$	*9*
3d "	$= 11 - 2 - 2$	$= 11 - (2 \times 2) =$ *1st term* $-$ *com. diff.* $\times 2 =$	*7*
4th "	$= 11 - 2 - 2 - 2$	$= 11 - (2 \times 3) =$ *1st term* $-$ *com. diff.* $\times 3 =$	*5*

3. What is the sum of the arithmetical series 3, 7, 11, 15, 19?

SOLUTION.

3, 7, 11, 15, 19, is the arithmetical series.

19, 15, 11, 7, 3, is the series inverted.

$22 + 22 + 22 + 22 + 22 = 110$, *the sum of twice the series.*

$11 + 11 + 11 + 11 + 11 = 55$, *the sum of the series.*

Or, since the sum of the extremes, or of any two terms equally distant from them, is the average of the several terms of the series, $\frac{3+19}{2} \times 5 = 55$, is the sum of the series.

778. Rules for Arithmetical Progression.—*1. Multiply the common difference by the number of terms less one; add the product to the smaller extreme, and the sum will be the greater; or, subtract the product from the greater extreme, and the remainder will be the smaller.*

2. Multiply half the sum of the extremes by the number of terms, and the result will be the sum of the series.

PROBLEMS.

1. A man being asked the age of his eldest child, replied that his youngest child was 2 years old, the number of his children was 6, and the common difference in their ages was 3 years. What was the age of the eldest child? *Ans.* 17 years.

2. A man travelled 10 days, increasing the distance gone over $1\frac{1}{2}$ miles each day compared with the day preceding, so that he travelled the tenth day 24 miles. How far did he travel the first day? *Ans.* $10\frac{1}{2}$ miles.

3. What will \$400, at 7% simple interest, amount to in 44 years? *Ans.* \$1632.

4. There are a number of rows of corn, the first of which contains 3 hills, the second 7, the third 11, and so on to the last, which has 43 hills. How many hills are there in all the rows? *Ans.* 253.

5. If a stone fall through 16.1 feet in the first second, 48.3 feet in the second second, 80.5 feet in the third second, and so on, how deep will be the shaft of a mine where a stone takes 7 seconds to reach the bottom?

6. If you should begin with a capital of $3500, and decrease every year $60, what would remain of your capital at the end of 10 years?

GEOMETRICAL PROGRESSION.

779. A **Geometrical Progression** is a series whose terms increase or decrease by a constant factor.

780. The **Rate,** or **Ratio,** of a series is the constant factor.

Thus, 3, 6, 12, 24, 48, is a geometrical series whose rate is 2; and 75, 15, 3, is a geometrical series whose rate is $\frac{1}{5}$.

781. An **Infinite Series** is a descending series of an infinite number of terms.

Of such a series the last term must be smaller than any assignable quantity; hence, it may be considered 0.

782. As in an arithmetical series, the relation of the elements of a geometrical series are such that when any three of them are known the others may be determined.

WRITTEN EXERCISES.

783.—Ex. 1. The first term of a geometrical series is 5, and the common rate 2. What is the fourth term of the series?

SOLUTION.

1st term	$=$			5
2d "	$= 5 \times 2$	$= 5 \times 2^1 =$	*1st term* $\times$ *rate*	$= 10$
3d "	$= 5 \times 2 \times 2$	$= 5 \times 2^2 =$	*1st term* $\times$ *square of the rate*	$= 20$
4th "	$= 5 \times 2 \times 2 \times 2$	$= 5 \times 2^3 =$	*1st term* $\times$ *cube of the rate*	$= 40$

2. What is the sum of a geometrical series whose first term is 5, last term 135, and rate 3?

SOLUTION.

$$\begin{array}{rl} 15 + 45 + 135 + 405 & = 3 \text{ times the sum of the series.} \\ 5 + 15 + 45 + 135 \qquad & = \text{once the sum of the series.} \\ \hline -5 \qquad\qquad\qquad + 405 & = \text{twice the sum of the series.} \end{array}$$

Hence, $\frac{405 - 5}{2} = 200 =$ *the sum of the series.*

Had the series been descending, the first term 135, and rate $\frac{1}{3}$, by inverting the series, making the first term the last, and the rate 3, the solution would then be the same as now given.

3. What is the sum of an infinite series whose first term is 4, and rate $\frac{1}{4}$?

SOLUTION.

The series extended $= 4,\ 1,\ \frac{1}{4},\ \frac{1}{16},\ \ldots\ 0.$

The series inverted $= 0\ \ldots\ \frac{1}{16},\ \frac{1}{4},\ 1,\ 4,$

whose rate is 4. Then,

$$\begin{array}{ll} 4 \text{ times the sum of the series} & = 0 \ldots + \frac{1}{16} + \frac{1}{4} + 1 + 4 + 16 \\ \text{Once the sum of the series} & = 0 \ldots + \frac{1}{16} + \frac{1}{4} + 1 + 4 \\ \hline 3 \text{ times the sum of the series} = & \qquad\qquad\qquad\qquad 16 \end{array}$$

Hence, the sum of the series $= \frac{16}{3} = 5\frac{1}{3}.$

784. Rules for Geometrical Progression.—*1. Multiply the first term by the rate raised to a power whose exponent is one less than the number of terms, and the product will be the required term.*

2. Multiply the last term by the rate; subtract the first term from the product, and divide the difference by the rate less one, and the result will be the sum of the series.

If the series is descending, use the series inverted, making the first term the last, and the rate greater than one; and,

If the series is infinite, multiply the larger term by the rate of the series inverted, and divide the quotient by the rate less one.

PROBLEMS.

1. A person, travelling, goes 5 miles the first day, 10 miles the second, 20 miles the third, and so on. If he travel 7 days, how far will he go the last day? *Ans.* 320 miles.

2. What is the amount of $100 for 9 years, at 5% compound interest? *Ans.* $155.13.

Here $100 is the first term, 10, or 1 more than the number of years, is the number of terms, and 1.05 is the rate. Required the last term.

3. What is the sum of the series 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, etc., to infinity? *Ans.* 4.

4. If a ball be put in motion by a force which would move it 10 rods the first minute, 8 rods the second, 6.4 the third, and so on in the ratio of .8, how far would it move?

5. If a farmer should sow 5 grains of wheat, and its produce every year for 9 years, how many bushels would there be in the last harvest, supposing that each harvest amounts to 10 times the quantity sowed, and that 8000 grains make 1 pint? *Ans.* 9765 bu. $2\frac{1}{2}$ pk.

SECTION LXXIV.

LIFE INTERESTS AND REVERSIONS.

785. An **Annuity** is a sum of money to be paid annually, or at regular intervals of time.

786. A **Life Interest** is an annuity to continue for life or lives.

Thus, a pension for life and a widow's dower, or life estate, are each a life interest.

787. A **Reversionary Interest** is an interest which does not commence until after a certain period, or until after a certain event.

788. The United States Treasury Department, in the computation of life interests, uses the following tables, known as the Carlisle Tables:

CARLISLE TABLES,

Of the Expectancy of Life, and of the Present Value of Life Annuities.

Age.	Expectancy of Life in years.	Present Value of Annuity of $1 for the years in 2d column, interest at 6 per cent.	Present Value of $1 to be received at the end of the years in 2d column, interest at 6 per cent.	Age.	Expectancy of Life in years.	Present Value of Annuity of $1 for the years in 2d column, interest at 6 per cent.	Present Value of $1 to be received at the end of the years in 2d column, interest at 6 per cent.
0	38.72	14.9202	.104788	41	26.97	13.2043	.207741
1	44.68	15.4325	.074065	42	26.34	13.0737	.215580
2	47.55	15.6225	.062645	43	25.71	12.9395	.223635
3	49.82	15.7521	.054874	44	25.09	12.8032	.231812
4	50.76	15.8008	.051953	45	24.46	12.6576	.240548
5	51.25	15.8252	.050490	46	23.82	12.5059	.249646
6	51.17	15.8213	.050722	47	23.17	12.3454	.259278
7	50.80	15.8029	.051830	48	22.51	12.1751	.269494
8	50.24	15.7742	.053551	49	21.81	11.9889	.280669
9	49.57	15.7385	.055689	50	21.11	11.7946	.292324
10	48.82	15.6972	.058168	51	20.39	11.5846	.304922
11	48.04	15.6523	.060860	52	19.68	11.3701	.317792
12	47.27	15.6055	.063670	53	18.97	11.1481	.331108
13	46.51	15.5573	.066559	54	18.28	10.9201	.344791
14	45.75	15.5072	.069566	55	17.58	10.6805	.359172
15	45.00	15.4558	.072650	56	16.89	10.4364	.373815
16	44.27	15.4028	.075832	57	16.21	10.1839	.388767
17	43.57	15.3501	.078996	58	15.55	9.9287	.404275
18	42.87	15.2956	.082267	59	14.92	9.6788	.419268
19	42.17	15.2384	.085695	60	14.34	9.4368	.433789
20	41.46	15.1778	.089331	61	13.82	9.2154	.447078
21	40.75	15.1151	.093095	62	13.31	8.9900	.460612
22	40.04	15.0500	.097002	63	12.81	8.7636	.474183
23	39.31	14.9972	.101248	64	12.30	8.5245	.488530
24	38.59	14.9068	.105592	65	11.79	8.2793	.503231
25	37.86	14.8307	.110157	66	11.27	8.0211	.518737
26	37.14	14.7521	.114876	67	10.75	7.7552	.534690
27	36.41	14.6685	.119892	68	10.23	7.4813	.551125
28	35.69	14.5829	.125024	69	9.70	7.1926	.568446
29	35.00	14.4982	.130105	70	9.18	6.9022	.585867
30	34.34	14.4123	.135258	71	8.65	6.5945	.604328
31	33.68	14.3240	.140560	72	8.16	6.3045	.621730
32	33.03	14.2343	.145938	73	7.72	6.0341	.637953
33	32.36	14.1366	.151789	74	7.33	5.7894	.652634
34	31.68	14.0344	.157932	75	7.01	5.5887	.664681
35	31.00	13.9291	.164255	76	6.69	5.3762	.677427
36	30.32	13.8174	.170957	77	6.40	5.1833	.688999
37	29.64	13.7021	.180796	78	6.12	4.9971	.700193
38	28.96	13.5833	.185000	79	5.80	4.7763	.713420
39	28.28	13.4579	.192530	80	5.51	4.5719	.725687
40	27.61	13.3289	.200208	81	5.21	4.3604	.738376

PROBLEMS.

1. What is the present value of a widow's dower whose yearly rent is $520, and whose age is 49 years?

SOLUTION.

Expectancy of life at 49 years of age = *21.81 years.*
Present value of $1 annuity for 21.81 years = *$11.9889.*
Present value of $520 annuity for 21.81 years = *$11.9889 × 520* = *$6234.23.*

2. What is the ready-money value of a legacy of $1000, to be received after 29 years?

SOLUTION.

Present value of $1, to be received after 29 years = *$.130105.*
Present value of $1000, to be received after 29 years = *$.130105 × 1000* = *$130.10$\frac{1}{2}$.*

3. A person 62 years old has a yearly pension of $96; what is its present value? *Ans* $863.04.

4. A widow aged 51 has set off, as her dower, property whose appraised value is $3800. What is the present value of the reversionary interest? *Ans.* $1158.70.

5. Smith, who is 70 years old, has a life annuity of $700 per annum. What is its present value?

6. What should be the present value of a legacy of $4000, to be received after 10 years 9 months? *Ans.* $2138.76.

TEST QUESTIONS.

789.—1. What is the METRIC SYSTEM? A meter? How are the names of derived metric denominations formed?

2. What is the PRIMARY UNIT of lengths? Of ordinary surfaces? Of land measures? Of volumes? Of capacity? Of large quantities of grains, fruits, etc.?

3. How are METRIC NUMBERS written? How many orders of figures are allowed to each metric denomination? How is the part of a metric expression at the left of the decimal point read? At the right of the decimal point?

4. What is a SERIES or a PROGRESSION? What are the terms of a series? The extremes? The means? When is a series ascending? When descending?

5. What is an ARITHMETICAL PROGRESSION? What is the rule for finding either extreme of an arithmetical series? For finding the sum of the series?

6. What is a GEOMETRICAL PROGRESSION? The rate or ratio of a geometrical series? What is an infinite series? What is the rule for finding the last term of a geometrical series? For finding the sum of an ascending series? The sum of a descending series? The sum of an infinite series?

7. What is an ANNUITY? A life interest? A reversionary interest? What is used by the United States Treasury Department in the computation of life interests?

8. How do the metric measures differ from the measures in common use? How do arithmetical and geometrical progressions differ? How does a reversionary interest differ from a life interest? How does a root differ from a power? The square root from the cube root? Is the root of a proper fraction smaller or larger than its corresponding power?

SECTION LXXV.

PROBLEMS FOR ANALYSIS.

MENTAL EXERCISES.

790.—1. Two boys on counting their marbles found that one had 9 more than the other, and that together they had 49. How many had each of the boys?

2. The greater of two numbers is $7\frac{3}{8}$, and their difference is $3\frac{1}{8}$. What are the numbers?

3. A man being asked how many cows he had, answered that if he had 2 more, twice the number would be 26. How many cows had he?

4. If John were 3 years younger, twice his age would be 18 years. How old is he?

5. A man wishing to contribute money to an equal number of poor men and women, gave to each man 9 dimes and to each woman 3 dimes. If he gave them in all $120, how many men and women were there respectively?

6. If 9 be taken from the sum of two numbers of which 7 is one, there will be 13 left. What is the larger number?

7. A and B by working together can do a piece of work in 2 days. B can do it alone in 5 days. How much of it can both together do in 1 day? What part of it can A alone do in 1 day? In what time can A alone do it?

8. Edward and Philip start from the same place and travel the same road. Edward starts 4 days before Philip, and travels 20 miles a day. Philip follows, travelling 25 miles a day. In what time will Philip overtake Edward?

9. A thief, having 50 steps the start of an officer, takes 4 steps while the officer takes 3; and 2 steps of the officer are equal to 3 of the thief. How many steps can the thief take before the officer can catch him?

SOLUTION.—2 steps of the officer are equal to 3 of the thief; hence, 6 steps of the officer are equal to 9 steps of the thief.

While the officer takes 6 steps, the thief takes 8; hence, while the thief takes 8 steps, the officer gains upon him 1 step of the thief.

Then while the officer is gaining 50 of the steps of the thief, the thief can take 8 times 50 steps, or 400 steps.

10. An officer is in pursuit of a thief who has some miles the start. The thief goes 20 miles a day, and the officer 25. If it take the officer 8 days to overtake the thief, how many days had the latter the start?

11. If 4 men can earn $16 in 2 days, how long will it take 6 men to earn $48?

12. If 6 men can do a piece of work in 8 days, what number of men can do $\frac{1}{2}$ of it in $\frac{3}{4}$ of the time?

13. Arthur is 16 years old, and Albert is 4. In how many years will Arthur be only twice as old as Albert?

SOLUTION.—4 years ago Arthur was 12 years old; hence in 12 years from that time, or in 12 years less 4 from this time—that is, in 8 years—Arthur will be 24 years old, or twice as old as Albert.

14. Mary is 15 years old, and her mother 36. In what time will Mary be only $\frac{1}{2}$ as old as her mother?

15. A father is 35 years old, and his son is 5. In what time will the son be $\frac{1}{3}$ as old as his father?

16. John gave each of his brothers 4 apples, giving all he had. If he had had 12 more apples he could have given each of his brothers 7 apples. How many brothers had he?

SOLUTION.—To give 7 apples to each brother he would have required 3 apples more for each; but he would have required 12 more for all. Hence, there were as many brothers as 3 apples are contained times in 12 apples, which are 4.

17. Jane wishes to purchase a certain number of oranges. If she pays 6 cents each, she will have 10 cents left; but if she pays 8 cents each, it will take all her money. How many oranges does she want?

18. James wishes to divide some peaches among his friends. If he gives each of them 3 he will have 9 left; but if he tries to give each of them 5, he will not have enough by 5. How many friends has he?

19. Eggs are sold at the rate of 4 for 5 cents. At what rate were they bought if the profit is 25 per cent.?

20. A has 5 times as much money as B has, and the sum of the interest received by both for 2 years, at 7 per cent., is $70. What sum has each?

21. If a wagon cost $80, what would be the cost of a harness if $\frac{7}{16}$ of the cost of the wagon were $\frac{7}{8}$ of the cost of the harness?

22. A has 4 times as much money as B has, and the sum of the interest received by both for 2 years, at 7 per cent., is $70. What sum has each?

23. A and B have $12, and $\frac{1}{2}$ of A's money equals $\frac{1}{4}$ of B's. How many dollars has each?

SOLUTION.—$\frac{1}{2}$ of A's money equals $\frac{1}{4}$ of B's; hence, $\frac{2}{2}$, or the whole, of A's money must equal $\frac{2}{4}$ of B's.

If A's money equals $\frac{2}{4}$ of B's, and B's must equal $\frac{4}{4}$ of itself, $12 must equal $\frac{2}{4}$ plus $\frac{4}{4}$, or $\frac{6}{4}$, of B's.

If $\frac{6}{4}$ of B's money is $12, $\frac{1}{4}$ is $\frac{1}{6}$ of $12, or $2, and $\frac{2}{4}$, or A's money, is $4, and $\frac{4}{4}$, or B's money, is $8.

24. A pole 60 feet long broke into two parts, one of which was $\frac{4}{9}$ of the other. What was the length of each part?

25. Edward says to Thomas, "$\frac{2}{3}$ of my age lacks 2 years

of being $\frac{2}{3}$ of yours, and the sum of our ages is 33 years." What is the age of each?

26. The time between three and four o'clock is such that $\frac{4}{5}$ of the minutes past three are equal to $\frac{2}{3}$ of the minutes before four. What is the time?

27. What is the time in the afternoon when the time past noon is equal to $\frac{1}{3}$ of the time to midnight?

WRITTEN EXERCISES.

791.—Ex. 1. Two travelers leave the same place at the same time. One goes 20 miles a day and the other $23\frac{1}{2}$. How far apart will they be at the end of 28 days, if they both travel in the same direction? How far if they travel in opposite directions? *Ans.* 98 miles; 1218 miles.

2. A merchant, who commences business with a capital of \$12000, gains at the rate of \$9000 in 4 years, by trading in flour, and at the rate of \$9000 in 6 years, by trading in grain. If his annual expenses are \$4500, in what time will he have lost all? *Ans.* 16 years.

3. I have a stick of squared timber 20 feet 6 inches long, 16 inches wide and 12 inches thick. If $3\frac{1}{2}$ solid feet should be sawed off at one end, how long would the stick then be?

$$\frac{1728\ in. \times 3\frac{1}{2}}{16 \times 12} = \frac{6048\ in.}{192} = 31.5\ in. = 2\ ft.\ 7\frac{1}{2}\ in.$$

$$20\ ft.\ 6\ in. - 2\ ft.\ 7\frac{1}{2}\ in. = 17\ ft.\ 10\frac{1}{2}\ in.$$

SOLUTION.—1728 solid inches are 1 solid foot; hence, the piece sawed off must contain $3\frac{1}{2}$ times 1728 solid inches, or 6048 solid inches. The 6048 solid inches must be the product of the numbers, in inches, which denote the dimensions of the piece sawed off.

The product of the width by the thickness, or 16×12, is 192; hence, the quotient of 6048 divided by 192, which is 31.5, must denote the number of inches of the piece sawed off.

The stick with 31.5 inches, or 2 ft. $7\frac{1}{2}$ in., sawed off must be 20 ft. 6 in. — 2 ft. $7\frac{1}{2}$ in., or 17 ft. $10\frac{1}{2}$ in. long.

4. From a plank which is 16 feet 5 inches long I wish to cut off a strip containing a square yard. At what distance from the edge must the line be drawn? *Ans.* $6\frac{114}{197}$ inches.

5. My bushel and half-peck measures, which are of a cylindrical form, are respectively $18\frac{1}{2}$ and $9\frac{1}{4}$ inches in diameter. What must be the depth of each?

6. I have a range, 56 feet long, of firewood, cut 4 feet long. When such wood is worth \$6 per cord, how high must the range be piled to be worth \$52.50? *Ans.* 5 feet.

7. A laborer agreed to work 12 weeks upon the conditions that he should receive \$18 per week for every week he worked, and for every week he was idle he should pay \$3.50 for his board. At the expiration of the time he received \$151.50. How many weeks did he work?

$$\$18 \times 12 = \$216$$
$$\$216 - \$151.50 = \$64.50$$
$$\$18 + \$3.50 = \$21.50$$
$$\$64.50 \div \$21.50 = 3$$
$$12 \text{ weeks} - 3 \text{ weeks} = 9 \text{ weeks.}$$

SOLUTION.—Had he labored 12 weeks, he would have received 12 times \$18, or \$216; as he received but \$151.50, he lost by idleness \$216 — \$151.50, or \$64.50. Each week he was idle he lost his wages and \$3 50, amounting to \$21.50; hence he was idle as many weeks as \$21.50 is contained times in \$64.50, or 3 weeks. Since he was idle 3 weeks, he worked 12 weeks less 3 weeks, or 9 weeks.

8. James received \$1 a day for his work, and paid \$.25 for every day he was idle. At the end of 18 days he received \$6.75. How many days was he idle?

9. A man agreed to carry 28 packages to a certain place on the conditions that for every one promptly delivered he should receive 30 cents, and for every one delayed he should forfeit 50 cents. He forfeited \$2.80 more than he received. How many packages did he deliver promptly? *Ans.* 14.

10. How many cows can be kept on a farm of 48 acres if for every 5 cows there must be 2 acres of meadow, and for every 3 cows 2 acres of pasture-land?

$$\frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$
$$48 \div \frac{16}{15} = 45$$

SOLUTION.—Since for every 5 cows there must be 2 acres of meadow, and for every 3 cows there must be 2 acres of pasture, for 1 cow there must be $\frac{2}{5}$ of an acre of meadow and $\frac{2}{3}$ of an acre of pasture, or $\frac{16}{15}$ of an acre. Hence, as many cows can be kept on a farm of 48 acres as 48 contains times $\frac{16}{15}$, or 45.

11. A farmer has 150 acres. He cultivates 5 acres for every 3 horses he has, and allows 10 acres of pasture for every 4 horses. How many horses can he keep? *Ans.* 36.

12. A certain field will furnish pasturage for 3 horses, or 4 cows, 56 days. For what time will it furnish pasturage for 1 horse and 1 cow grazing together? *Ans.* 96 days.

13. A bankrupt's stock was sold for $1660, at a loss of 17% on the cost price. Had it been sold in the course of trade, it would have realized a profit of 20%. How much was it sold below the trade price? *Ans.* $740.

14. A merchant sells tea to a trader at a profit of 60%; but the trader becomes bankrupt and only pays 75 cents on a dollar. How much per cent. does the merchant gain or lose? *Ans.* He gains 20%.

15. The head of a fish is 28 inches long; the tail is as long as the head and $\frac{1}{2}$ of the body; and the body is as long as the head and tail. What is the length of the fish?

28 in. + 28 in. = 56 in. = $\frac{1}{2}$ the length of body.

56 in. × 4 = 224 in = the length of the fish.

SOLUTION. — Since the tail is as long as the head and $\frac{1}{2}$ of the body, and the body is as long as both the head and tail, 28 inches plus 28 inches, or 56 inches, must equal $\frac{1}{2}$ the length of the body. Since the body is $\frac{1}{2}$ the length of the fish, 56 inches must equal $\frac{1}{4}$ the length of the fish, and 56 inches × 4, or 224 inches, must equal the length of the fish.

16. A sum of money was divided among 3 men. The first received $96, the second received half as much as the third, and the third received as much as the other two. How much did the second and the third receive?

17. A, B and C, dividing a plantation consisting of 120 acres, agreed that B should have a third part more than A, and C a fourth part more than B. What number of acres will each have? *Ans.* A, 30; B, 40; C, 50.

18. If a steamboat, running uniformly at the rate of 12 miles per hour in still water, were to run 4 hours with a current of 4 miles per hour, then to return against that current, what

length of time from the time she started would she require to reach the place whence she started? *Ans.* 12 hours.

19. A merchant purchased goods for $1200, and sold them at a loss of $12\frac{1}{2}\%$. He then purchased more goods with the proceeds, and sold them at a gain of 14%. Did he gain or lose by these transactions, and how much?

Ans. Lost $3.

20. I wish to plant 5292 trees equally distant in straight rows, and to make the length of the grove 3 times the width. How many of the shorter rows shall I have?

$$\sqrt{\frac{5292}{3}} = 42$$

$$42 \times 3 = 126$$

SOLUTION. — Since the length is 3 times the width, one third of the trees are to form an exact square. The square root of one third of the number 5292 is 42, which must denote the number of trees in a side of the square. Since there must be 3 such squares, there must be 3 times 42, or 126, short rows.

21. A farmer has 2 10-acre lots; one is a square, and the other is a parallelogram 4 times as long as it is wide. How many rods of fence will each lot require to exactly enclose it?

Ans. The square, 160 rods; the parallelogram, 200 rods.

22. What is the mean proportional between 4 and 9?

$$4 \times 9 = 36$$

$$\sqrt{36} = 6$$

SOLUTION.—Since the mean proportional between the extremes of a proportion is one of the equal means of the proportion (Art. 547), the mean proportional between two numbers is equal to the square root of the product of those numbers. The product of 4 by 9 is 36, and the square root of 36 is 6, the mean proportional required.

23. What is the mean proportional between 7 and 252?

Ans. 42.

24. A cheese, when put into one scale of an incorrect balance, was found to weigh $31\frac{1}{4}$ pounds, but when put into the other it weighed only 20 pounds. What was its true weight?

Ans. 25 pounds.

25. I have corn of four different qualities, worth respectively 65, 72, 80 and 90 cents per bushel. In what proportions may

these kinds be taken to form a quantity worth 75 cents per bushel?

SOLUTION.

$$\begin{array}{llll}
\text{At } 65\text{ c. to gain } 1\text{ c. take } \frac{1}{10}\text{ bu.}; & \frac{1}{10}\text{ bu.} \times 10 = 1\text{ bu.} \\
\text{" } 72\text{ c. " } 1\text{ c. " } \frac{1}{3}\text{ bu.}; & \frac{1}{3}\text{ bu.} \times 15 = 5\text{ bu.} \\
\text{" } 80\text{ c. to lose } 1\text{ c. " } \frac{1}{5}\text{ bu.}; & \frac{1}{5}\text{ bu.} \times 10 = 2\text{ bu.} \\
\text{" } 90\text{ c. " } 1\text{ c. " } \frac{1}{15}\text{ bu.}; & \frac{1}{15}\text{ bu.} \times 15 = 1\text{ bu.}
\end{array}$$

On 1 bushel, worth 65 cents, taken at 75 cents, there is a gain of 10 cents; hence, to gain 1 cent we take $\frac{1}{10}$ of a bushel. On 1 bushel at 80 cents there is a loss of 5 cents; hence, to lose 1 cent, we take $\frac{1}{5}$ of a bushel. Therefore, we take $\frac{1}{10}$ of a bushel at 65 cents as often as we take $\frac{1}{5}$ of a bushel at 80 cents; or, multiplying these fractions by the least common multiple of their denominators, we find we may, also, take 1 bushel at 65 cents as often as we take 2 bushels at 80 cents.

In like manner we take $\frac{1}{3}$ of a bushel at 72 cents as often as we take $\frac{1}{15}$ of a bushel at 90 cents; or we may take 5 bushels at 72 cents as often as we take 1 bushel at 90 cents.

It is also evident that any number of times these proportions may be taken.

Hence, the different qualities of corn may be taken in the proportions of $\frac{1}{10}$, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{15}$; 1, 5, 2 and 1; etc.

26. In what proportions may coffees, at 35, 40, 50 and 55 cents a pound, be mixed to produce a quantity worth 45 cents per pound?

Ans. In the proportions of 1 lb. at 35 c., 2 lb. at 40 c., 2 lb. at 50 c., and 1 lb. at 55 c.

27. How much gold, at 20, 21 and 23 carats fine, must be mixed with 12 ounces 20 carats fine, so that the mixture may be 22 carats fine?

SOLUTION.

$$\begin{array}{ll}
\text{At } 20\text{ ca. to gain } 1\text{ ca. take } \frac{1}{2}\text{ oz.}; \ldots & \frac{1}{2}\text{ oz.} \times 24 = 12\text{ oz.} \\
\text{" } 21\text{ ca. " } 1\text{ ca. " } 1\text{ oz.}; \ldots & 1\text{ oz.} \times 24 = 24\text{ oz.} \\
\text{" } 23\text{ ca. to lose } 1\text{ ca} + 1\text{ ca. take } 1\text{ oz} + 1\text{ oz.}; & 2\text{ oz.} \times 24 = 48\text{ oz.}
\end{array}$$

$$12\text{ oz.} \div \frac{1}{2}\text{ oz.} = 24\text{ oz.}$$

We find the proportions, without regard to any of the quantities being limited, to be $\frac{1}{2}$ oz. at 20 carats fine, 1 oz. at 21 carats, and 2 oz. at 23 carats.

But of the 20 carats fine it is required to take 12 oz., or 24 times $\frac{1}{2}$ oz.; hence the other proportions must also be taken 24 times as large. We must then take for the required mixture 12 oz. at 20 carats fine, 24 oz. at 21 carats, and 48 oz. at 23 carats.

28. A grocer mixed 20 pounds of sugar, worth 15 cents a pound, with others at 16, 18 and 22 cents. How many pounds of each were taken to make a mixture worth 17 cents per pound?

Ans. 20 lb. at 15 c., 4 lb. at 16 c., 4 lb. at 18 c., 8 lb. at 22 c.

29. A farmer has oats worth 46, 48, 51 and 54 cents a bushel. What quantity of each of these kinds must be taken to make 8 bushels worth 50 cents a bushel?

SOLUTION.

At 46 c.	*to gain*	*1 c.*	*take*	$\frac{1}{4}$ *bu.*;	$\frac{1}{4}$ *bu.* $\times 4 = 1$ *bu.*
" *48 c.*	"	*1 c.*	"	$\frac{1}{2}$ *bu.*;	$\frac{1}{2}$ *bu.* $\times 4 = 2$ *bu.*
" *51 c.*	*to lose*	*1 c.*	"	*1 bu.*;	*1 bu.* $\times 4 = 4$ *bu.*
" *54 c.*	"	*1 c.*	"	$\frac{1}{4}$ *bu.*;	$\frac{1}{4}$ *bu.* $\times 4 = 1$ *bu.*
				2 bu.	*8 bu.*

8 bu. $\div$ *2 bu.* $= 4$.

We find the proportions, without regard to the total of the quantities being limited, to be $\frac{1}{4}$ bu. at 46 cents, $\frac{1}{2}$ bu. at 48 cents, 1 bu. at 51 cents, and $\frac{1}{4}$ bu. at 54 cents.

The sum of these quantities is 2 bushels; but the total of the mixture must be 8 bushels, or 4 times as large; hence, each of the quantities found must be taken 4 times as large. We, therefore, must take for the required mixture, 1 bu. at 46 cents, 2 bu. at 48 cents, 1 bu. at 51 cents, and 1 bu. at 54 cents.

30. What quantities of sugars, worth \$.12, \$.11 and \$.08 per pound respectively, must be taken to form a mixture containing 35 pounds, at 9 cents per pound?

31. A grocer requires a chest of tea containing 75 pounds, worth 66 cents per pound. What quantities of several kinds, worth 42, 48, 72 and 78 cents per pound, must he mix to form it?

Ans. 9 lb. at 42 cents, 12 lb. at 48 cents, 36 lb. at 72 cents, and 18 lb. at 78 cents.

Note.—The last seven problems are examples in what is called *Alligation Alternate*, which is the process of finding the proportions of several articles of different values that may form a quantity of a given average value.

32. Express as a series .135135 +, in which the figures 135 continually repeat in the same order.

Solution.—The decimal .135135 +, or .135 . . . , may be regarded as a geometrical progression, in which the rate is $\frac{1}{1000}$. Marking the repeating figures by placing a dot over the first and last of the set, and we have $.\dot{1}3\dot{5} = \frac{135}{1000} + \frac{135}{1000000} + \ldots$, the series required.

Note.—A decimal in which a figure or a set of figures is repeated in the same order indefinitely is called a *Circulate*, and the figure or set of figures repeated is called a *Repetend*.

33. Express as a common fraction in its lowest terms the circulate $.\dot{2}\dot{7}$.

Solution.

$$\begin{array}{lr} \textit{100 times the circulate } .\dot{2}\dot{7} & = 27.27\ldots \\ \underline{\textit{1 time} \quad \text{"} \quad \text{"}} & \underline{.27\ldots} \\ \textit{99 times} \quad \text{"} \quad \text{"} & = 27. \end{array}$$

Hence, once the circulate, or $.\dot{2}\dot{7} = \frac{27}{99} = \frac{3}{11}$, *the fraction required.*

That is, a repetend is equal to a common fraction having for its denominator as many nines as there are figures in the repetend, and for the numerator the figures of the repetend.

34. Express as a mixed decimal $.2\dot{2}5\dot{9}$. *Ans.* $.2\frac{259}{999}$.

35. Express as equivalent common fractions $.\dot{7}$, $.\dot{9}\dot{0}$ and $.\dot{7}0\dot{2}$.

36. Express as common fractions in their lowest terms $7.\dot{9}3\dot{6}$, $8.9\dot{3}\dot{6}$ and $32.71\dot{5}$. *Ans.* $\frac{881}{111}$, $\frac{983}{110}$, $\frac{7361}{225}$.

37. Express as common fractions in their lowest terms $.\dot{0}7\dot{4}$, $.\dot{8}14\dot{5}$ and $.13\dot{8}$. *Ans.* $\frac{2}{27}$, $\frac{905}{1111}$, $\frac{5}{36}$.

SECTION LXXVI.

GENERAL REVIEW.

792.—1. Add thirty-five million eight hundred forty thousand three hundred fifty ten-thousandths, four hundred sixty-three thousand nine hundred and eight-hundredths, and three hundred four thousands and three hundred four millionths.

2. What number divided by 417 will give the quotient 105 and the remainder 113?

3. If $\frac{11}{12}$ of a ton of hay cost $18.50, how much will two loads cost, one weighing $\frac{5}{6}$ of a ton and the other $\frac{13}{24}$ of a ton?
Ans. $27.75.

4. $20.004 + (20.104 \times 5.07) - (6.44 \div .0005) =$ what?

5. Which is the greater—a garden 40 rods square, or one containing 40 square rods, and how much?

6. From 95 mi. subtract 57 mi. 192 rd. 4 yd. 3 ft. 18 in.

7. What part of $2\frac{2}{5}$ is $\frac{3}{4}$ of $\frac{2}{3}$ of $\dfrac{\frac{2}{5}}{\frac{3}{8}}$? *Ans.* $\frac{2}{9}$.

8. Find the difference between 7 thousand and 7 thousandths, and divide the remainder by 7 millionths.

9. What is the value in compound numbers of .3945 of a day? *Ans.* 9 h. 28 m. 4.8 sec.

10. How much greater is the quotient of $\frac{3}{4} \div \frac{2}{3}$ than the product of $\frac{3}{4} \times \frac{2}{3}$?

11. A hall, 50 feet long and 30 feet wide, has around it a mop-board 9 inches high. The hall has one door 6 feet wide and 2 doors 3 feet wide. How many square feet are there in the surface of the mop-board?

12. If a merchantman, sailing $9\frac{1}{2}$ knots an hour, is chased by a gun-boat steaming $10\frac{3}{4}$ knots, how far ahead must the sailing vessel be to escape 3 knots ahead into a port from which she is $15\frac{1}{2}$ knots at the commencement of the chase?

13. If by selling a horse at $80 I lose $12\frac{1}{2}\%$ of the first cost, shall I gain or lose, and what per cent., by selling him at $90?

14. How many times has February 29th occurred since the year 1799?

15. A merchant bought $\frac{8}{9}$ of a hogshead of molasses at $\frac{5}{8}$ of a dollar a gallon. At what price per gallon must he sell it to make $4.20? *Ans.* $.70.

16. I bought apples at $5 per barrel, and lost one fourth of them. At what price must I sell the rest that I may gain 10% on the whole cost? *Ans.* 7.33\frac{1}{3}$.

17. If 6 men in 8 hours thresh 30 bushels of wheat, in how many hours can 5 men thresh 50 bushels?

18. A has a farm $\frac{1}{8}$ of a mile square, and B has one containing $\frac{1}{16}$ of a square mile. How do the farms compare in size?

19. An agent received $67.50 for collecting $4500. What was the rate of his commission?

20. What is the least number which, being divided by 3, by 5, by 7, by 9 and by 10, leaves in each case a remainder of 2?

21. How much shall I gain by borrowing $3560 for 1 year 6 months 10 days, at 6%, and lending it at 7%?

22. I obtained a discount at a bank at 7%, and left $\frac{1}{8}$ of the proceeds in the bank until the note was paid. At what rate did I get the money I used? *Ans.* 8$\frac{56}{93}$%.

23. If 14 men can perform a piece of work in 36 days, in how many days can they perform the same labor with the assistance of 7 more men?

24. How much more time will it require a sum of money to double itself at 6% interest than at 7%?

25. What is the amount of $1450.40 from April 19, 1871, to August 3, 1872, at 6%?

26. What is the difference between the simple and the compound interest on $5000 for 3$\frac{1}{2}$ years, at 7%? *Ans.* $114.60.

27. A man bequeathed $\frac{1}{4}$ of his estate to his wife, $\frac{1}{5}$ to a college and $\frac{1}{6}$ to his eldest son; and these three legacies amounted to $18500. How much did each receive?

Ans. Wife, $7500; college, $6000; son, $5000.

28. A merchant paid $4200 for cotton, and sold it at 10% advance, taking his pay in prints, which he sold at a loss of 10%. Did he gain or lose, and how much?

29. What is the present worth of $770, due in 1 year 8 months, at 6%? *Ans.* $700.

30. A and B traded in company. A put in \$950, and B \$800. They gained \$300. What was each partner's share of it? *Ans.* A's, \$162$\frac{6}{7}$; B's, \$137$\frac{1}{7}$.

31. A person has a field measuring 3 acres 75 square rods, which he wishes to exchange for a square one of inferior quality, but $3\frac{1}{2}$ times as large. How many rods is the length of its side? *Ans.* 44.0738.

32. Smith and Doland trade in company, Smith contributing \$800 for 9 months, and Doland \$600 for 8 months. They gain \$450. What should each receive?

33. What is the area of a triangle whose base is $5\frac{1}{2}$ yards, and whose altitude is $8\frac{1}{2}$ yards? *Ans.* $23\frac{3}{8}$ sq. yd.

34. Gold is selling at 112. Find the interest in currency on 7 \$1000 U. S. 5-40 bonds from June 1, 1870, to July 1, 1871.

35. A person being asked the time of day, replied that it was between 5 and 6 o'clock, and that the hour- and minute-hands of the watch were together. What was the time?

Ans. $27\frac{3}{11}$ minutes past 5 o'clock.

36. If the diameter of a 9-pound cannon-ball be 5 inches, what must be the diameter of a 28-pound ball? *Ans.* 7.3 in.

37. A merchant bought goods to the amount of \$1400, on a credit of 6 months. At the end of 3 months he paid \$600, and 1 month later \$400. What extension ought he to have on the balance of the debt? *Ans.* $6\frac{1}{2}$ months.

38. Wishing to find the distance between two trees, which cannot be directly measured on account of an intervening pond, I measure due west 50 rods from the foot of one of them; then, turning north, measure 34 rods, when I find that I am just 20 rods west of the other tree. How far are the trees apart? *Ans.* 45.34 + rods.

39. A gentleman selling a mortgage of \$4410, for which he received 5% interest, invested the proceeds in Government $3\frac{1}{2}$% bonds at 70. After receiving the interest for 5 years, on the bonds rising to 75, he sold out. What was his gain upon the whole transaction over what he would have received had he continued the mortgage? *Ans.* \$315.

40. After the outbreak of the Prusso-French war in 1870, the Prussian Government issued a 5% war loan at 88. The French 3 per cents. stood at $65\frac{1}{2}$. State the ratio of the two rates of interest. *Ans.* $5\frac{15}{22}$ to $4\frac{76}{131}$.

41. A joist is $7\frac{1}{2}$ inches wide and $2\frac{1}{2}$ thick, but I want one just twice as large, which shall be $3\frac{3}{4}$ inches thick. What must be the width? *Ans.* 10 inches.

42. A and B can do a piece of work alone in 12 and 16 days respectively. They labor together on the work for 3 days, when A leaves it, but B continues, and after 2 days is joined by C, and they finish it together in 3 days. In what time would C do it alone? *Ans.* 12 days.

43. How many cubic yards of gravel will be required for a walk surrounding a rectangular lawn 200 yards long and 100 yards wide, the walk to be 3 yards wide, and the gravel 3 inches deep?

44. A saves $\frac{1}{3}$ of his income; but B, who has the same income, spends twice as fast as A, and thereby contracts a debt of $120 annually. What is the income of each?

Ans. $360.

45. A stole a horse from B, and made off with him. Five days afterward, B gets intelligence of A, and follows him at the rate of 60 miles a day, by which he gains 20% upon A. How far must B ride to overtake A, and how many days?

Ans. 1500 miles; 25 days.

46. A steamer, working with a given force, can run down the river at the rate of $12\frac{1}{2}$ miles per hour. Of this speed, $\frac{2}{7}$ is due to the current. How long would the steamer require to go 15 miles up the stream?

47. Subtract the square root of $\frac{64}{4096}$ from the cube root of the same.

48. The plan of a town is $17\frac{1}{2}$ inches long and 14 inches broad, and the scale annexed to it is just $2\frac{5}{8}$ inches to 1100 yards. What is the length of a mile upon this scale, and what will be the length and breadth of the plan if it be enlarged to a scale of 6 inches to a mile?

Ans. $4\frac{1}{5}$ inches; 25 inches by 20 inches.

APPENDIX.

ROMAN NOTATION.

793. Roman Notation uses seven letters: I, V, X, L, C, D and M, which express, respectively, *one, five, ten, fifty, one hundred, five hundred* and *one thousand.*

All numbers can be expressed by these letters, used singly, or combined according to the following

794. Principles.—1. *When a letter is repeated, the number which it expresses is repeated.*

Thus, II $= 1 + 1 = 2$; XXX $= 10 + 10 + 10 = 30$; CC $= 100 + 100 = 200$.

2. *When a letter expressing a certain number stands after one expressing a greater number, the sum of the numbers is denoted.*

Thus, VI $= 5 + 1 = 6$; XI $= 10 + 1 = 11$; LX $= 50 + 10 = 60$.

3. *When a letter expressing a certain number stands before one expressing a greater number, the difference of the numbers is denoted.*

Thus, IV $= 5 - 1 = 4$; IX $= 10 - 1 = 9$; XL $= 50 - 10 = 40$.

4. *When a letter expressing a certain number stands between two letters expressing greater numbers, the least number is to be subtracted from the sum of the other two.*

Thus, XIV $= (10 + 5) - 1 = 14$; CXL $= (100 + 50) - 10 = 140$.

5. *A bar, —, placed over a letter makes it denote thousands.*

Thus, $\overline{\text{V}} = 5000$; $\overline{\text{D}} = 500{,}000$; $\overline{\text{M}} = 1{,}000{,}000$.

EXERCISES.

Write and read—

1. XIX.
2. CXVI.
3. LXXXV.
4. DCLXX.
5. MDCCCLXXV.
6. $\overline{\text{MLIX}}$CXX.

Express by letters—

7. Forty-eight.
8. Two hundred five.
9. One thousand six hundred eleven.
10. Eighteen hundred seventy-nine.

CONTRACTIONS.

To Add Two Columns at a Time.

795.—Ex. 1. Add, two columns at a time, 1235, 6714, 4566 and 4967.

1235
6714
4566
4967
17482

SOLUTION.—67 + 6 = 73, + 60 = 133, + 4 = 137, + 10 = 147, + 5 = 152, + 30 = 182; write 82. 1 + 49 = 50, + 5 = 55, + 40 = 95, + 7 = 102, + 60 = 162, + 2 = 164, + 10 = 174; write 174. *Ans.* 17482.

In practice; thus, 67, 73, 133, 137, 147, 152, 182; write 82. 1, 50, 55, 95, 102, 162, 164, 174; write 174. *Ans.* 17482.

796. Rule for Adding two Columns at a Time.—*To the lowest number add the ones of the next number above, then add the tens of that number; to the sum thus obtained add the ones of the next number above, then the tens of that number, and so on.*

PROBLEMS.

(1.)	(2.)	(3.)	(4.)	(5.)
36	4402	6645	47	4141
71	6307	5232	81	3226
58	1453	7070	92	1819
32	9205	3007	31	4234
43	1824	7984	35	1781
64	7132	2636	74	9693
59	1042	2273	60	2009

To Multiply by a Multiplier of Two Orders at Once.

797.—Ex. 1. Multiply 1246 by 32.

1246
32
39872.

SOLUTION.—6 × 32 = 192; write 2. 4 × 32 = 128, + 19 = 147; write 7. 2 × 32 = 64, + 14 = 78; write 8. 1 × 32 = 32, + 7 = 39; write 39. *Ans.* 39872.

798. Rule for Multiplying by a Multiplier of Two Orders at Once.—*Multiply each order of the multiplicand separately by the entire multiplier.*

PROBLEMS.

Multiply—

1. 7418 by 35; by 42.	3. 91367 by 44; by 61.
2. 6320 by 15; by 53.	4. 34205 by 67; by 88.

To Multiply by an Aliquot Part of 10, 100, etc.

799.—Ex. 1. Multiply 3465 by 125.

$$\begin{array}{r} 8)\underline{3465000} \\ 433125 \end{array}$$

SOLUTION.—Since 125 is one eighth of 1000, multiply by 1000, and take one eighth of the product. *Ans.* 433125.

800. Rule for Multiplying by an Aliquot Part of 10, 100, etc.—*Multiply by 10, 100, etc., and of the product thus obtained take such a part as the given multiplier is of the multiplier used.*

PROBLEMS.

Multiply—

1. 674 by $2\frac{1}{2}$; by 250.	4. 8910 by $16\frac{2}{3}$; by $333\frac{1}{3}$.
2. 342 by $8\frac{1}{3}$; by $33\frac{1}{3}$.	5. 7648 by 25; by 250.
3. 758 by $12\frac{1}{2}$; by 125.	6. 68024 by $33\frac{1}{3}$; by 125.

To Divide by an Aliquot Part of 10, 100, etc.

801.—Ex. 1. Divide 433125 by 125.

$$\begin{array}{r} 433.125 \\ \underline{8} \\ 3465.000 \end{array}$$

SOLUTION.—Since 1000 is 8 times 125, divide by 1000, and take 8 times the quotient. *Ans.* 3465.

802. Rule for Dividing by an Aliquot Part of 10, 100, etc.—*Divide by 10, 100, etc., as the problem may require, and multiply the quotient thus obtained by the number which shows how many times the given divisor is contained in the divisor used.*

PROBLEMS.

Divide—

1. 16850 by 25; by 125.	4. 7360 by $16\frac{2}{3}$; by $33\frac{1}{3}$.
2. 5700 by $2\frac{1}{2}$; by $8\frac{1}{3}$.	5. 7600 by 250; by $333\frac{1}{3}$.
3. 25300 by $12\frac{1}{2}$; by 250.	6. 552642 by 50; by 125.

DUODECIMALS.

803. A **Duodecimal** is a denominate number in which a unit of any denomination is equivalent to twelve units of the next lower denomination; or, it may be regarded as a series of fractions whose denominators are successive powers of 12.

NOTE.—Examples in Duodecimals can generally be more readily performed by reducing the Duodecimals to Common or Decimal Fractions, but, since the special rules are used by some mechanics in measuring surfaces and solids, it is thought best to give them here.

In duodecimals the foot is taken as the unit; twelfths of a foot are called primes; twelfths of a prime, seconds; twelfths of a second, thirds, etc.

Primes are marked $'$; seconds, $''$; thirds, $'''$; fourths, $''''$, etc., and the marks are called *indices*.

TABLES.

For Lengths.	For Surfaces.	For Volumes.
$1\,ft. = 12' = 12$ in.	$1\,ft. = 12' = 144$ sq. in.	$1\,ft. = 12' = 1728$ cu. in.
$1' = 12'' = 1$ "	$1' = 12'' = 12$ "	$1' = 12'' = 144$ "
$1'' = 12''' = \frac{1}{12}$ "	$1'' = 12''' = 1$ "	$1'' = 12''' = 12$ "
$1''' = 12'''' = \frac{1}{144}$ "	$1''' = 12'''' = \frac{1}{12}$ "	$1''' = 12'''' = 1$ "

ADDITION AND SUBTRACTION.

804. Duodecimals are added and subtracted in the same manner as compound numbers are.

Thus, 23 ft. $9'\,10''\,11''' + 15$ ft. $8'\,7''\,8''' = 39$ ft. $6'\,6''\,7'''$; 12 ft. $3'\,7''\,9'''\,8'''' - 8$ ft. $8'\,9'' = 3$ ft. $6'\,10''\,9'''\,8''''$.

MULTIPLICATION.

805.—Ex. 1. How many square feet in a board 12 feet 9 inches long and 2 feet 6 inches wide?

$$\begin{array}{lll} 12\,ft. & 9' & \\ 2 & 6' & \\ \hline 6\,ft. & 4' & 6'' \\ 25 & 6' & \\ \hline 31\,ft. & 10' & 6'' = \end{array}$$

$$31 + \tfrac{10}{12} + \tfrac{6}{144} = 31.875\ sq.\,ft.$$

SOLUTION.—9 in., or $9'$, $= \frac{9}{12}$ ft., and 6 in., or $6'$, $= \frac{6}{12}$ ft. $9' \times 6' = 54'' = 4'\,6''$. Write the $6''$, and add the $4'$ to the next product. 12 ft. $\times 6' = 72'$; $72' + 4' = 76' = 6$ ft. $4'$, which we write in the product. $9' \times 2 = 18' = 1$ ft. $6'$. Write the $6'$, and add the 1 ft. to the next product. 12 ft. $\times 2 = 24$ ft.; 24 ft. $+ 1$ ft. $= 25$ ft., which we write in the product. Adding the partial products, we have 31 ft. $10'\,6''$,

$$\begin{array}{r} 12.75 \\ 2.5 \\ \hline 6375 \\ 2550 \\ \hline 31.875 \text{ sq. ft.} \end{array}$$

and reducing the primes and seconds to the decimal of a foot, we have 31.875 sq. ft.

SOLUTION BY DECIMALS.—Expressing the inches as decimals of a foot, and multiplying (Art. 398), we obtain, more readily than by duodecimals, the same result as before.

806. Rule for Multiplication of Duodecimals.—*Multiply as in multiplication of integers, observing that twelve of each duodecimal denomination are one of the next higher.*

PROBLEMS.

1. What is the area of a plank 20 ft. 3 in. long and 1 ft. 8 in. wide? *Ans.* 33.75 sq. ft.

2. What are the cubic contents of a stick of squared timber 20 ft. long, 18 in. wide and 14 in. thick? *Ans.* 35 cu. ft.

3. How many square feet of flooring in a room 24 ft. 7 in. long and 16 ft. 4 in. wide? *Ans.* 401 sq. ft., 76 sq. in.

DIVISION.

807.—Ex. 1. Divide 19 ft. 10′ 11″ 8‴ by 2 ft. 4′.

$$19 \text{ ft. } 10' \; 11'' \; 8''' = 34412'''$$
$$2 \text{ ft. } 4' = 4032'''$$
$$34412 \div 4032 = 8\tfrac{2156}{4032} = 8 \text{ ft. } 6' \; 5''$$

SOLUTION.—The dividend is equal to 34412‴; the divisor to 4032‴. Dividing, we have a quotient 8 and a remainder 2156. Reducing this remainder to twelfths or primes, and dividing, we have 6′ and a remainder 1680′. Reducing this remainder to twelfths of primes, or to seconds, and dividing, we have 5″, and no remainder. Hence, the entire quotient is 8 ft. 6′ 5″.

808. Rule for Division of Duodecimals.—*Reduce the dividend and divisor to the lowest denomination found in either, and divide as in integers. The quotient will be an integer. If there be a remainder, reduce it to twelfths or primes, and continue the division; reduce the second remainder, if any, to twelfths or seconds, and proceed as before.*

PROBLEMS.

1. Divide 90 ft. 3′ 6″ by 5 ft. 6′. *Ans.* 16 ft. 5′.

2. Divide 55 ft. 9′ 0″ 10‴ by 5 ft. 7′. *Ans.* 9 ft. 11′ 10″.

ACCURATE INTEREST.

809. The United States Government pays **Accurate Interest**, reckoning 365 days to the year.

Usual interest makes each day's interest $\frac{1}{360}$ of a year's interest; and to equal exact interest must be diminished by $\frac{1}{73}$ of itself in a common year, or by $\frac{1}{61}$ of itself in a leap year.

810.—Ex. 1. What is the interest on a Government note of $1000 for sixty days, at 5%?

$$\frac{\$50 \times 60}{365} = \$8.22.$$

SOLUTION.—Interest on $1000 for one year = $50. Interest for 60 days, or $\frac{60}{365}$ of a year, $\frac{\$50 \times 60}{365} = \$8.219+$. *Ans.* $8.22.

811. Rule for Accurate Interest.—*Multiply the interest for one year at the given rate by the given number of days, and divide the product by 365.*

PROBLEMS.

1. What is the interest on a Government bond of $500 for 31 days, at 6%?

2. What is the interest on a ten-forty U. S. bond of $5000 from June 11 to August 21? *Ans.* $27.12.

3. What is the interest on U. S. securities of $18000 from April 4 to July 13, at $4\frac{1}{2}$%? *Ans.* $221.92.

TEST QUESTIONS.

812.—1. What characters are used in expressing numbers by ROMAN NOTATION? What are the principles of Roman notation?

2. What is the RULE for adding two columns at a time? For multiplying by a multiplier of two orders at once? For multiplying by an aliquot part of 10, 100, etc.? For dividing by an aliquot part of 10, 100, etc.?

3. What is a DUODECIMAL? How are duodecimals added and subtracted? What is the rule for multiplication of duodecimals? For division of duodecimals?

4. What is ACCURATE INTEREST? What does usual interest make each day's interest? What is the rule for accurate interest?

EXAMINATION PROBLEMS.

813. THE following **Problems** may be used at the discretion of the teacher in testing the proficiency of pupils as they progress in the book. The Articles in parentheses denote the portions of the text to which the problems relate.

(Articles 1—139.)

1. Express in words 6115789023665724.

2. Represent by figures fifteen quadrillions four hundred one trillions eleven millions seventeen.

3. Show by an example the use of 0 in writing numbers.

4. A has 795 dollars, B has 105 more than A, and C has as many as A and B. How many dollars have they all?

5. From one million take five hundred thousand five.

6. A man purchased a house for 7500 dollars; after paying 560 dollars for repairs, and receiving 475 dollars for rent, he sold it for 8000 dollars. How much did he gain?

7. The difference between two numbers is 1162, and the larger number is 9340. What is the smaller?

8. Illustrate by an example a method of proving results in Addition.

9. How many men are there in an army of 112 regiments, each of which consists of 947 men?

10. I bought 35 cows at 52 dollars a head, and 27 at 60 dollars a head, and sold the whole at 54 dollars a head. How much did I gain or lose?

11. The product of two factors is 224638568, and one of them is 729346. What is the other?

12. The quotient of the exact division of one number by another is 3168. What would have been the quotient if the divisor had been 6 times as large?

13. Show by an example that Division is the reverse of Multiplication.

14. What is the value of $(\overline{8+16} \times 6) \div 9 - \frac{(30-15)\times 3}{2}$?

15. How many tons of hay at 32 dollars a ton, can be exchanged for 44 tons of coal at 8 dollars per ton?

16. Which of the numbers 84, 282 and 798 is divisible by the largest prime number?

17. How much less is the greatest common divisor of 30 and 42 than their least common multiple?

18. What is the smallest sum of money with which I can purchase colts at 25 dollars each, cows at 40 dollars each, oxen at 100 dollars each, or horses at 125 dollars each?

(Articles 140—296.)

19. How much does $\frac{3}{4}+\frac{2}{3}$ exceed $\frac{8}{4}-\frac{2}{3}$?

20. The sum of two fractions is $\frac{37}{45}$, and one of them is $\frac{2}{9}$. What is the other?

21. What number divided by $3\frac{3}{5}$ will give $\frac{6}{7}$?

22. If $\frac{3}{4}$ of a yard of cloth costs $\frac{11}{20}$ of a dollar, what will be the cost of 1 yard?

23. Show by examples the difference between a Compound Fraction and a Complex Fraction.

24. What is $\frac{3}{32}$ of a ship worth if $\frac{11}{16}$ of it be worth 50000 dollars?

25. A man spent $\frac{2}{3}$ of $\frac{1}{8}$ of his money one day, $\frac{3}{4}$ of $\frac{1}{5}$ of it another day, and then had 3880 dollars remaining. How much money had he at first?

26. Show by examples that multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

27. What common fraction is equal to the sum of .655, $.33\frac{1}{3}$ and .9375?

28. Express by figures six hundred thousand six, and six million sixty thousand six hundred six billionths.

29. How much less than one million is one millionth?

30. A 63-gallon cask is $\frac{3}{5}$ full of wine; if 27.625 gallons should leak out, the wine remaining will be what decimal part of a full cask?

31. Show by an example how Integers and Decimals correspond in expression.

32. What will 56.75 acres of land cost at \$20.25 per acre?

33. What is the value of $\frac{13.15}{.005} + \frac{\frac{6}{25}}{4} - \frac{.05}{5.5}$?

34. A grocer paid \$586.50 for apples, giving \$2.25 a barrel for 124 barrels, and \$3.75 for the remainder. How many barrels did he buy?

35. How many times is .029 exactly contained in .3786, and what will remain?

36. A sum of money was divided among three boys; the first received .375 of the whole; the second, .6 of the whole; and the third, \$2.12½. What was the sum divided?

(Articles 297—407.)

37. How many inches are there in 1051 yards 2 feet 5 inches?

38. Reduce 3186938 seconds to days.

39. What will 28 square rods 129 square feet of land cost, at 12 cents per square foot?

40. In walking from one town to another a man took 29700 steps of 2 feet 8 inches each. How many miles did he walk?

41. Show by an example that Reduction Descending and Reduction Ascending are reverse processes.

42. What is the sum and the difference of 75 yards 1 foot 9 inches and 46 yards 2 feet 11 inches?

43. On July 17, 1872, how old was a man who was born February 18, 1819?

44. How many cords are there in 3 ranges of wood, each being 12 feet long, 4 feet wide and 6 feet 4 inches high?

45. What is the product of $16° 15' 16''$ multiplied by 11?

46. How much must be paid for 41 gal. 2 qt. 1¾ pt. of molasses, at 72 cents a gallon?

47. What part of a cubic yard is a cube whose edge is one-half of a yard?

48. What fraction of an ounce Troy is 15 pwt. $9\frac{3}{18}$ gr.?

49. How much must be paid for one-seventieth of 336 bu. 3 pk. 4 qt. of corn, at 80 cents a bushel?

50. How many cubic feet in a rectangular beam 24 feet 6 inches long, 1 foot 9 inches wide and 1 foot 2½ inches thick?

51. What decimal of a ton is $\frac{5}{8}$ of an ounce?

52. How much hay, at $30 a ton, can be bought for $131.25?

53. Express $\frac{6}{25}$ of a day in hours, minutes and seconds.

54. From a piece of land 24 rods square I sold $\frac{5}{16}$ of an acre to A, $\frac{2}{5}$ of an acre to B, and .675 of an acre to C. How much of the land was left?

55. A ship's chronometer, set at Philadelphia, longitude 75° 10′ W., pointed to 3 h. 40 min. 24 sec. A.M. when the sun was on the meridian. In what longitude was the ship?

(Articles 408—534.)

56. If I sell land at $75 per acre, and thereby gain 25%, how much per acre did the land cost me?

57. How much is 10% of 25% of 1680 bushels?

58. I bought a horse for $120, and sold him for $160. What per cent. did I gain?

59. My house is worth $\frac{3}{10}$ as much as my farm. What per cent. of the value of the farm is the value of the house?

60. A received from B $5100, with which he bought flour at $5 per barrel, deducting his commission of 2% on the cost. How many barrels did he buy?

61. What are the interest and the amount of $8500 for 2 years 7 months 21 days, at 7%?

62. William bought cows at $80 each, and one-fifth of them died. At what price must he sell the rest, to gain 5% on the whole?

63. How long must $600 remain on interest at 6% to gain $1408?

64. What is the interest of $12750 for 5 years 5 months 18 days, at 6%?

65. What must be the face of a note payable in 90 days, on which $5000 would be received from a bank, discounting at 5%?

66. How much will the compound interest exceed the annual interest on $1000 for 3 years 6 months, at 6%?

67. What is the difference between the true and the bank discount of $500 for 3 months, at 8%?

68. What principal on interest at 7%, from April 9, 1871, to September 5, 1873, will amount to $1477.59?

69. The difference between the interest of $600 and that of $750, at 5%, for a certain time, is $18.75. What is the time?

70. What sum, paid May 16, will settle a bill of $850.50 for goods bought April 19, on 60 days' credit, the rate of interest being 7%?

71. A note for $1740, dated June 15, 1869, with interest at 6%, was indorsed as follows: Jan. 1, 1870, received $100; July 15, 1870, received $112; June 1, 1871, received $200; and May 1, 1872, received $600. What was due Sept. 1, 1872?

(Articles 535—651.)

72. Two numbers are 108 and 27; what is the ratio of the second to the first?

73. Two numbers whose sum is 3410 are in the ratio of 5 to 6. What are the numbers?

74. How many men can perform a piece of work in 112 days which 12 men can perform in 84 days?

75. If 18 men can dig a trench 30 yards long in 24 days by working 8 hours a day, how many men can dig a trench 60 yards long in 64 days, working 6 hours a day?

76. Show by examples the difference between Simple and Compound Proportion.

77. If A invests in a certain enterprise $600 for 4 months, B $300 for 7 months, and C $200 for 9 months, what part of the profits should each of them have?

78. A, B and C are partners. A furnishes $\frac{1}{4}$ of the capital; B, $500; and C, $400. At the end of the year the profits are $4200. What sum should each receive?

79. I owe Jones two bills, one of $600 on a credit of 60 days, and the other of $800 on a credit of 30 days. These bills being without grace, in how many days should a note given for their amount be made payable?

80. The balance of an account is $420, and is due, by average, April 28. What was its cash value March 8, interest being at 8%?

81. How much must be invested in Government 4 per cents., at $93\frac{5}{8}$, to realize a quarterly interest of \$30?

82. What will be the cost of a draft of \$12500, at 60 days, exchange being at $100\frac{1}{2}$, and interest at 7%?

83. What rate is paid for money when $1\frac{1}{2}$% is charged for exchange on a 30-day note discounted at 6%?

84. I invested \$1460 in $4\frac{1}{2}$ per cents, at $100\frac{1}{4}$, and sold when they had fallen, losing \$100, inclusive of the double brokerage of $\frac{1}{8}$%. At what price did I sell?

(Articles 652—792.)

85. What integral power of 5 is nearer than any other to 100000?

86. Show by an example the difference between the Square Root and the Cube Root of a number.

87. What is the difference, carried to 3 orders of decimals, between $\sqrt[3]{3}$ and $\sqrt{2}$?

88. What difference is there between the area of a floor 30 feet square and that of three others each 10 feet square?

89. Two ships, A and B, sailed from a certain port at the same time. A sailed north, 8 miles an hour, and B sailed east, 6 miles an hour. How far apart were they at the end of the third hour?

90. What is the product of $7.\dot{3}$ multiplied by $1.\dot{9}\dot{2}$?

91. Show by examples the difference between Arithmetical and Geometrical Progression.

92. A room is 20 feet long, 16 wide and 10 feet high. What is the distance from an upper corner to the opposite lower corner?

93. How many feet, board measure, in a rectangular beam 14 feet 9 inches long, 1 foot 8 inches wide and 1 foot 4 inches thick?

94. A garden is 100 feet long and 80 feet wide. What must be the dimensions of a similar garden to contain just one-half as many square feet?